1. Introduction

Welcome to the series of E-learning modules on Time Reversal, Factor Reversal and Circular Test.

At the end of this session, you will be able to:

- Understand the tests applied to any formula to indicate whether it is satisfactory or not. The following test are discussed:
  - Time reversal test
  - Factor reversal test
  - Circular test
- Understand the method of testing the construction of index numbers using these tests

The choice of a formula for the construction of index number is a two sided problem. It involves the choice of a method of averaging as well as the choice about the system of weighting. Several formulae have been suggested for constructing index numbers and the problem is that of selecting the most appropriate one in a given situation.

The following tests are suggested for choosing an appropriate index: Unit test, time reversal test, factor reversal test and circular test.

2. Introduction to the Various Tests

Unit Test: It requires that the formula should be independent of the units in which, or for which prices and quantities are quoted. This test is satisfied by all index number methods except the simple (unweighted) aggregative index method.

Time Reversal Test: A test that may be used under the axiomatic approach which requires that if the prices and quantities in the two periods being compared are interchanged the resulting price index is the reciprocal of the original price index.
The time reversal test requires that the index for the later period based on the earlier period should be the reciprocal of that for the earlier period based on the later period. In other words, the two should be reciprocals of each other i.e., P naught one into P one naught is equal to one

Factor Reversal Test: The factor reversal test requires that multiplying a price index and a volume index of the same type should be equal to the proportionate change in the current values. Suppose the roles of the prices and quantities in a price index are reversed it will yield a quantity index of exactly the same functional form as the price index.

The factor reversal test used under the axiomatic approach requires that the product of this quantity index and the original price index should be identical with the proportionate change in the value of the aggregate in question. Also known as the “product test”. Symbolically P naught one into Q naught one is = Value Index.

Circular Test: It is concerned with the measurement of price changes over a period of years, when it is desirable to shift the base. If P naught one represents the price change of the current year on the base year and P one two, the price change of the base year on some other base and P two naught, the price change of the current year on this first base, then following equation should be satisfied: P naught one into P one two into P two naught = one.

3. Time Reversal Test

Now let us discuss the time reversal test in detail.

Time reversal test:

Prof. Irving Fisher has made a careful study of the various proposals of computing index numbers and suggested various test to be applied to any formula to indicate whether or not it is satisfactory. The two important ones are the time reversal test and the factor reversal test

Time reversal test is a test to determine whether a given method will work both ways in time, forward and backward. In the words of Fisher, “the test is that the formula for calculating the index number should be such that it will give the same ratio between one point of comparison and the other, no matter which of the two is taken as the base.”

In other words, when the data for any two years are treated by the same method, but with the bases reversed, the two index numbers secured should be reciprocals of each other so that their product is unity.

Symbolically, the following relation should be satisfied; P naught one into P one naught is equal to one where, P naught one is the index for time ‘1’ on ‘0’ as the base and P one naught is the index for time ‘0’ on time ‘1’ as base. If the product is not unity there is said to be a time bias in the method.

For example let us say that the price of wheat from 2008 to 2009 is increased from Rs 480 to Rs 560 per quintal, the price in 2006 should be one hundred thirty three one by three (133) percent of the price in 2008 and the price in 2008 should be seventy five % of the price in 2009. One figure is the reciprocal of the other, their product (1.33x0.75) is unity. This is obviously true for each individual price relative and according to the time reversal test; it should be true for the index number.

Let us now look at how many of these methods that we have discussed has satisfied the time reversal test.

First let us take the Laspeyer’s method. In this method we calculate the index by taking the summation of P one q naught divided by summation P naught q naught. Name this as equation 1. Now, when we apply the concept of time reversal P naught one is equal to the summation of P one q naught divided by summation P naught q naught, which is same as equation 1 and P one naught is equal to summation of P naught q one divided by summation of P one q one denoted in equation 2 when taken the product of equation 1 and 2 we should get P naught one into P one naught is equal to unity, here after substituting the equation we see that P naught one into P one naught is not equal to unity, And hence this method does not satisfy the test of time reversal.

Next let us take the Paasche’s method, in this method we calculate the index by taking the summation of P one and q one divided by summation of P naught q one as in equation 1. Now when we apply the concept of time reversal P naught one is equal to summation of P one and q one divided by summation of P naught q one as in equation

1 and P one q naught is equal to summation of P naught q naught divided by summation of P one q naught as in equation 2 when taken the product of equation 1 and 2 we should get P naught one into P one naught is equal to unity, here after substituting we see that P naught one into P one naught is not equal to unity, And hence this method does not satisfy the test of time reversal.

The next method we will discuss is the Fisher's ideal index number method. in this method we calculate the index number as Price index is equal to square root of summation of product of price of the current year and the quantity of the base year divided by summation of price and quantity of the base year multiplied by the summation of the product of the price and quantity of the current year divided by summation of price of the base year and current year quantity as equation 1, changing the time P one naught, the formula will be Price index is equal to square root of summation of product of price of the base year and the quantity of the current year divided by summation of price and quantity of the current year multiplied by the summation of the product of the price and quantity of the base year divided by summation of price of the current year and quantity of the base year as equation 2 .Now by taking the product of both the equation we will get unity as the answer which indicates that Fisher's ideal index number satisfies the test.

Similarly there are four more methods which do satisfy the test:
Simple geometric mean of price relative
Aggregates with fixed weights
The weighted geometric mean of price relatives if we use fixed weights
Marshall-Edgeworth method.

Thus a time reversal test used with index numbers that is satisfied when the new index is the reciprocal of the original index if the functions of the base period and given period are interchanged. The advantage of index numbers meeting the criteria of the test is that, a symmetric comparison of the two periods is obtained and the results are consistent whether one or the other period is used as a base.

4. Factor Reversal Test

Factor reversal test:
The factor reversal test is another test of consistency suggested by Fisher; it holds that, the product of a price index and the quantity index should be usual to the corresponding value index. In the words of Fisher “just as each formula should permit the interchange of the two times without getting inconsistent results, so it ought to permit interchanging the prices and quantities without giving inconsistent results. That is, the two results multiplied together should give the true value ratio.”

In other words, the test is that the change in price multiplied by the change in quantity should be equal to the total change in value. The total value of a given commodity in a given year is equal to the product of the quantity and price per unit. The ratio of the total value in one year to the total value in the preceding year (p1q1/p0q0).i.e., P one q one by p naught q naught.

For example, let us assume that the price and quantity doubles from one year to the next, that is, the price relative is 200 and the quantity relative is 200. Then, the value index should be 400. The total value in the second year would be four times the value in the first year.

A symbolical representation can be done. Let us assume that P one and P naught represent the prices and q one and q naught represents the quantities in the current year and base year respectively then P naught one represents the change in the price in the current year and Q naught one represents the change in the quantity in the current year then the product of P naught one and Q naught one is equal to the value ratio summation of P one q one by summation of P naught q naught.
Note: if the product is not equal to the value ratio, then, in this test it means there is an error in one or both of the index number.

The factor reversal test is satisfied only by the Fishers ideal index number.
Fishers ideal index is equal to the square root of the summation of product of price of the current year and the quantity of the base year divided by summation of price and quantity of the base year multiplied by the summation of the product of the price and quantity of the current year divided by summation of price of the base year and current year quantity, when in the equation we change P to q and q to P we will get the quantity index is equal to the square root of the summation of product of quantity of the current year and the product of the base year divided by summation of quantity and price of the base year multiplied by the summation of the product of the quantity and price of the current year divided by summation of quantity of the base year and current year price. By multiplying the two indexes we will get the result as the value ratios.

Circular test:
Another test of the adequacy of the index number formula is the circular test. If in the use of index number interest attaches not merely to a comparison of two periods, but to

the measurement of changes over a period of years, it is frequently desirable to shift the base.

A test of this shift-ability of base is called the circular test. This test is just an extension of the time reversal test.
The test requires that if an index is constructed for the year 'a' on base year 'b', and for the year 'b' on base year 'c', we ought to get the same result as if we calculated directly an index for 'a' on base year 'c' without going through 'b' as an intermediary.
Symbolically if there are three indices P naught one, P one two and P two naught then the circular test will be satisfied if the product of all the three indices is equal to unity.

The Laspeyer's index- First let us take the Laspeyer's method. In this method we calculate the index by taking the summation of P one q naught divided by summation P naught q naught. Now when we apply the concept of circular test considering three years 0,1 and 2 P naught one is equal to the summation of P one q naught divided by summation P naught q naught, P one naught is equal to summation of P naught q one divided by summation of P one q one, P two naught is equal to summation of P naught q two by p two q two when taken the product of the three years it is not equal to 1, And hence this method **does not** satisfy the Circular test.
Similarly it can be shown in that the Paasche's index and Fishers index do not satisfy the test as the product is not equal to one.

However the simple aggregative method and the fixed weighted aggregative method satisfy the test, can be seen from the following;
When the test is applied to the simple aggregative method we will get Summation P one by summation P naught into summation P two by summation P one into summation P naught by summation P two is equal to one.
Similarly when applied to the fixed weight aggregative method we will get, summation P one q by summation P naught q into summation P two q by summation P one q into summation of P naught q by summation P two q is equal to one.

The advantage of this test is that it reduces the computation every time a change in the base year has to be made. Index numbers can be adjusted from year to year without referring each time to the original base.
The disadvantage is that weights in the index numbers depend on the periods between which comparisons are being made, if these periods change, the weights change.

5. Circular Test

Let us take an example to understand the calculation for time reversal test and factor reversal test.
Calculate fisher's ideal index umber from the following data and prove that it satisfies both time reversal test and factor reversal test.

Figure 1

	2008		2009	
Commodity	Price	Expenditure	Price	Expenditure
A	8	80	10	120
B	10	120	12	96
C	5	40	5	50
D	4	56	3	60
E	2	100	25	150

Solution:
First, let us prepare the table for calculating the various values needed for the solution.

Figure 2

	2008		2009					
Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	8	80	10	120	800	640	1200	960
B	10	120	12	96	1440	1200	1152	960
C	5	40	5	50	200	200	250	250
D	4	56	3	60	168	224	180	240
E	2	100	25	150	2500	200	3750	300
Total					5108	2464	6532	2710

In the first column we have the commodities, the second column we have the base year price (P naught), the third column we have the, base year quantity (q naught) in the fourth and the fifth column we have the price (P one) and quantity (q one) of the current year. The sixth column indicates the product of P one q naught and the total is equal to five thousand one hundred and eight, the seventh column indicates the values of P

naught q naught the total is equal to two thousand four hundred and sixty four, the eighth column gives the product of P one q one the total is equal to six thousand five hundred and thirty two and the last column gives the value of P naught q one is equal to two thousand seven hundred and ten.

Substituting the values in the formulae for finding the index we will get square root of five thousand one hundred and eight by two thousand four hundred and sixty four into six thousand five hundred and thirty two by two thousand seven hundred and ten into hundred which is equal to square root of two point zero seven three into two point four one zero into hundred is equal to square root of four point nine nine five into hundred is equal to two point two three five into hundred is equal to two hundred and twenty three point five as the index number.

Now we will test the consistency by using the time reversal test and the factor reversal test. By substituting the values in the product of P naught one and P one naught we will get as the square root of five thousand one hundred and eight divided by two thousand four hundred sixty four into six thousand five hundred and thirty two divided by two thousand seven hundred and ten into two thousand seven hundred and ten by six thousand five hundred and thirty two into two thousand four hundred and sixty four divided by five thousand one hundred and eight which is equal to square root of 1 is equal to 1. Hence time reversal test is satisfied.

Next we will substitute the values to check the factor reversal test where we will take the products of P one naught and Q one naught we will get it as the square root of five thousand one hundred and eight divided by two thousand four hundred and sixty four into six thousand five hundred and thirty two divided by two thousand seven hundred and ten into two thousand seven hundred and ten divided by two thousand four hundred and sixty four into six thousand five hundred and thirty two divided by five thousand one hundred and eight which is equal to square root of six thousand five hundred and thirty to divided by two thousand four hundred and sixty four which is equal to the ratio six thousand five hundred and thirty two by two thousand four hundred and sixty four that is the value ratio. Hence the above data has satisfied the factor reversal test.

Here's a summary of our learning in this session:
We have understood the method of testing the constructed index numbers using

- Time reversal test
- Factor reversal test
- Circular test