1. Introduction

Welcome to the series of E-learning modules on Fisher's ideal index number and weighted average of price relative.

By the end of this session, you will be able to:

- Construct index numbers using:

  - Fisher's ideal index number and
  - Weighted average of price relatives

Introduction
Professor Irving Fisher has given a number of formulae for constructing index number and ideal index is one among these. The Fisher's ideal index is the geometric mean of the Laspeyer's and Paasche's indices. Thus, in the Fisher's method, we average geometrical formulae that err in opposite directions.

The Fisher's Ideal Index is given by the formula, Price index is equal to square root of (summation of product of price of the current year and the quantity of the base year) divided by (summation of price and quantity of the base year) multiplied by the (summation of the product of the price and quantity of the current year) divided by (summation of price of the base year and current year quantity) multiplied by (100) or it can be geometric mean of the product of Laspeyer's index (L) and Paasche's index (P).

The Fisher's Index is called the ideal index number based on the following points:

It is based on the geometric mean, which is theoretically considered to be the best average for constructing index numbers
It takes into account both current and base year prices and quantities

It satisfies both the test of adequacy - The time reversal test and factor reversal test
It is free from bias. As the ideal formula is crossed geometrically, that is, by averaging process that of itself has no bias, the two formulae (Laspeyer's & Paasche's) express the opposing type and weight biases. The result is the complete cancellation of biases

However, practically index is difficult to compute because:

It is excessively laborious
The data, particularly for the Paasche segment of the index are not readily available
In practice, statisticians will continue to rely upon simple although less exact index number formulae

2. Weighted Average of Relatives

Weighted average of relatives
In the previous module, the weighted aggregative method we discussed has not computed the index number using the price relatives. Like the unweighted relative method, it is also possible to compute the weighted average of relatives. We can also use either the arithmetic mean or the geometric mean in the calculation of the averages.

The steps involved in the computation of the weighted arithmetic mean of relatives index number are as follows:
Express each item of the period for which the index number is being calculated as a percentage of the same item in the base period
Multiply the percentages as obtained in step one for each item by the weight, which has been assigned to that item
Add the results obtained from the several multiplications carried out in step two
Divide the sum obtained in step three by the sum of the weights used. The result is the index number

Symbolically, we will represent the calculation as price index is equal to (summation P into V) divided by (summation V)
Where, P stands for the price relative and v stands for value weights.
Price relative is obtained by dividing the current year price and base year price and multiplying the quotient by 100.
Value weights are obtained by multiplying the price and the quantity.

Instead of using the arithmetic mean, the geometric mean may be used for assigning relatives. The weighted geometric mean of relatives is computed in the same manner as the unweighted geometric mean or relative index number except that weights are introduced by applying them to the logarithms of the relatives.

When this method is used, the formula for computing the index is: Price index is equal to the (antilog of summation V into log P) divided by (summation V). Where, P is equal to P one by P not into 100 and V is equal to p not into q0 for each item.

The steps involved in the computation of the weighted geometric mean of relatives index number are as follows:

Express each item of the period for which the index number is being calculated as a percentage of the same item in the base period
Find the logarithm of each percentage relative found in step one
Multiply the logarithms by the weights assigned
Add the results obtained in step three
Divide the sum obtained in step four by the sum of the weights used
Find the antilogarithms of the quotient obtained in step five. The result is the index number obtained through weighted geometric mean of relative index numbers

Merits of weighted average of price relative method

The following are the special advantages of weighted average of relative indices over weighted aggregative index:

When different index numbers are constructed by the average of price relative's method, all of which have the same base, they can be combined to form a new index
When an index is computed by selecting one item from each of the many sub-groups of items, the values of each sub-group may be used as weights. Then only the method of weighted average of relatives is appropriate
When a new commodity is introduced to replace the one formerly used, the relative for the new item may be spliced to the relative for the old one, using the former value of weights
The price or quantity relatives for each single item in the aggregate are in effect, these index often yields valuable information for analysis

3. Illustrations

Let us take examples to understand the concept of Fisher's ideal index and weighted average of relative index numbers.
Example 1
Given the following data prices and average monthly quantities of four items purchased by children, compute the price index for the year 2009 with the year 2000 as base year by Fisher's method.

Table 1

Items	2000		2009	
	Price	Quantity	Price	Quantity
Comic Books	8	1	10	2
Toffees	1	30	2	25
Ice Cream	5	5	6	10
Play Articles	10	1	15	1

Solution:
Let us first prepare the table for calculating the various averages.

Table 2

Items	2000		2009					
	Price (P0)	Quantity (q0)	Price (P1)	Quantity (q1)	**P0q0**	**P0q1**	**P1q0**	**P1q1**
Comic books	8	1	10	2	**8**	**16**	**10**	**20**
Toffees	1	30	2	25	**30**	**25**	**60**	**50**
Ice cream	5	5	6	10	**25**	**50**	**30**	**60**
Play articles	10	1	15	1	**10**	**10**	**15**	**15**
Total					**73**	**101**	**115**	**145**

In the table we have the items in the first column, the second and the third columns indicate the price (P_0) and quantity (q_0) of the base year 2000. Fourth and fifth columns indicate the price (P_1) and quantity (q_1) of the current year 2009. The sixth column is the calculation for the product of the price and quantity of the base year P not and q not, that is, 8, 30, 25, 10 and its summation is equal to 73. The seventh column will have the product of the price of the base year with the quantity of the current year that is P not and q one, that is, we will get 16, 25, 50, 10 and the summation is equal to 101. The eighth column in the table gives the value of the product of the price of the current year and the quantity of the base year. We will get the values 10, 60, 30, 15 and the summation is equal to 115. In the last column, we will calculate the product of the current years that is P one and q one, we get 20, 50, 60 and 15, which gives a grand total of 145.

In the next step, let us take the formulae for calculation of the Fisher's ideal price index number. Price index is equal to square root of (summation of product of price of the current year and the quantity of the base year) divided by (summation of price and quantity of the base year) multiplied by the (summation of the product of the price and quantity of the current year) divided by (summation of price of the base year price and current year quantity) multiplied by (100) or it can be geometric mean of the product of Laspeyer's index and Paasche's index. Wherein the value for P one q not is equal to 115, P not q not is equal to 73, P one q one is equal to 145 and the value of P not q one is equal to 101.

Now, by substituting these values in the formulae, we will get Price index is equal to the square root of (115) divided by (73) into (145) divided by (101) into (100) which is equal to square root of (1.575) into (1.436) into 100 is equal to square root of (2.2617) into (100) is equal to (1.504) into (100) is equal to (150.4).
The result indicates that the price has increased by 50.4% in 2009 as compared to 2000.

Example 2:
From the following data, compute the price index by applying the weighted average of price relatives method using (a) arithmetic mean and (b) geometric mean.

Table 3

Commodities	P_0 (Rs.)	q_0	P_1
Sugar	3	20 Kg	4
Flour	1.5	40 Kg	1.0
Milk	1.0	10 Lt	1.5

Solution:
Let us first calculate the weighted arithmetic mean of price relatives. For this, let us prepare the table for the required contents.

Table 4

Commodities	P_0 (Rs.)	q_0	P_1	V= P0xq0	P= (P1/p0) x 100	PV
Sugar	3	20 Kg	4	**60**	**133.33**	**8000**
Flour	1.5	40 Kg	1.0	**60**	**106.66**	**6400**
Milk	1.0	10 Lt	1.5	**10**	**150**	**1500**
Total				**130**		**15900**

The first column indicates the commodities, the second column indicates the price in the base year, the third column indicates the various quantities of the items in the base year, the fourth column indicates the price of the items in the current year, the fifth column indicates the value

weights (V), which is the product of P not q not is equal to 60, 60, 10 and the summation of V is equal to 130. Then, we calculate the price relative (P), which is equal to (current price) divided by (price not) into (100) is equal to 133.33, 106.66 and150. In the last column, we will get the values by multiplying the price relative (P) into value weights (V) and get the PV values as 8000, 6400, 1500, which gives the total summation PV equal to 15,900.
Let us now calculate the Price index is equal to (summation of Price relative Value weights) divided by (summation of value) is equal to (15,900) divided by (130) is equal to (122.31), which indicates that there has been a 22.3% increase in price over the base level.

Now, let us calculate the index number using geometric mean of price relatives. As a first step, we will prepare the table for getting the necessary values.

Table 5

Commodities	P_0 (Rs.)	q_0	P_1	V= P0xq0	P= (P1/p0) x 100	Log P	V log P
Sugar	3	20 Kg	4	**60**	**133.33**	**2.1249**	**127.494**
Flour	1.5	40 Kg	1.0	**60**	**106.66**	**2.0282**	**121.692**
Milk	1.0	10 Lt	1.5	**10**	**150**	**2.1761**	**21.761**
Total				**130**			**270.947**

The first column indicates the commodities, the second column indicates the price in the base year, the third column indicates the various quantities of the items in the base year, the fourth column indicates the price of the items in the current year, the fifth column will indicates the value weights (V), which is the product of P not q not is equal to 60, 60, 10 and the summation of V is equal to 130. In the sixth column, we calculate the price relative (P), which is equal to (current price) divided by (price not) into (100) is equal to 133.33, 106.66 and 150. In the seventh column, we will calculate the log of P we will get 2.1249, 2.0282 and 2.1761. Then, in the last column, we will get the product of V and log P, that is, 127.494, 121.692 and 21.761 and the summation is 270.947. Let us now calculate the Price index is equal to (antilog summation Value weights) into (log Price relative) divided by (summation of value) is equal to (Antilog of 270.947) divided by (130) is equal to (antilog of 2.084) is equal to (121.3).

4. Chain Base Index Numbers

Chain base index numbers:
The various formulae discussed for the construction of index numbers are based on the fixed base method. They reflect the relative change in the level of phenomenon in any period called current year, with its changes in same particular fixed year called the base year.

A series of index numbers are computed for each year using the preceding year as the base year. This index number calculated after taking the preceding year as base year is called the link relatives. Then, link relatives are chained together by successive multiplication to form a chain index.

Thus, a chain index is the figure for each year first expressed as a percentage of the preceding year. These percentages are then chained together by successive multiplication to form a chain index.

The following are the steps in construction of chain index numbers:

For each commodity, express the price in any year as a percentage of its price in the preceding year. This gives the link relatives
Chain together these percentages by successive multiplication to form a chain index
Chain base index number is equal to (Average link relative to current year) multiplied by the (preceding year chain base index) divided by (100)

The techniques of computing the index number by the fixed base and the chain base method are different. The fixed base index uses the original data, while the chain base index uses the link relatives. If there is only one series of observations, then the fixed base indices and the chain base indices will always be same.

Conversion of chain base index number to fixed base index numbers.

Fixed base index numbers can be obtained from the chain base index numbers by using the following formula:
Current year fixed base index is equal to (current year chain base index) multiplied by (previous year fixed base index) divided by (100)

Uses of the chain base index numbers:

In the chain base method, the comparisons are made with immediate past and accordingly the data for the two periods are compared
The comparisons are more valid and meaningful. The resulting index is more representative of the current trends in the habits, customs, taste and fashion of the society
Thus, chain base method has a great significance in economic and business fields
Weights of the various commodities can be adjusted frequently. This flexibility greatly increases the utility of the chain indices over the fixed base index numbers
Chain base method permits the introduction of new commodities, the old and absolute items may be deleted

5. Base Shifting

Base shifting:
Base shifting refers to the preparing of a new or more recent base period than the original one. Change of base year or reference period is known as shifting the base.
The formula for calculating the new index number series is: Shifted price index is equal to (original price index) divided by the (price index for the year to which it has to be shifted) multiplied by (100)

Shifting the base is necessary under the following situation:
It is required to compare series of index numbers with different base periods, when the base period is too old or too distant from the current period to make meaningful comparisons.

Splicing:
Splicing of index numbers means combining two or more series of overlapping index numbers to obtain a single index number on a common base.
Splicing of index number can be done only if the index numbers are constructed with the same items and have an overlapping year. A second series of index number is constructed by the same formula.

In order to secure continuity in comparisons, the two series are put together or spliced together to get a continuous series. In splicing of index numbers, we find a common factor by which the spliced index number series is multiplied to give a common base.

There are two methods of splicing- forward splicing and backward splicing.

Forward splicing: In this method, the old series, say A is brought forward to splice it with new series, say B. This is done by multiplying the various index numbers of the old series by the index number of the last year in the old series and dividing the obtained result by 100. This splicing is called forward splicing.

Backward splicing: In this method, the new series is pushed backward by dividing the various index numbers of the old series by the index number of the year in which change takes place and the result so obtained is multiplied by 100. This splicing procedure is known as backward splicing.

Deflating:
Deflating means adjusting, correcting or reducing a value, which is inflated. It is a technique of converting a series of value calculated at current prices into constant prices of a given year. This is a process of removing the effect of price changes from the current money values.

This is particularly desirable in the case of an economy, which has inflationary trends, because in such an economy, the increase in the prices of commodities over a period of year means a fall in their real incomes. Thus, it becomes necessary to adjust or correct the nominal wages in accordance with the rise in the corresponding price index to arrive at real income.

The purchasing power is given by the reciprocal of the index number. Consequently, the real income is income by the corresponding appropriate price index and multiplying the result by 100.
Real wages is the money or nominal wages divided by price index and multiplied by 100.
The real income is also known as deflated income.

The real wage so obtained may be converted into index number as follows:
Real wages or income index number is equal to (real wage of current year) divided by (real wage of base year) multiplied by (100), which is equal to (index of money wage) divided by (price index numbers) multiplied by (100).
This technique of deflation is used extensively to deflate the value series or value indices, production, wages income and so on.

Here's a summary of our learning in this session, where we have understood the method of constructing index numbers using:

- Fisher's ideal index number
- Weighted average of price relatives