1. Introduction

Welcome to the series of E-learning modules on measurement of trend by the moving average method.

By the end of this session, you will be able to

- understand the measurement of linear trend using the following methods
- Free hand or graphic method
- Semi average method
- Method of least squares
- Understand measurement of non Linear trend using
- Freehand or graphic method,
- Moving average method,
- Parabolic trend by a second degree polynomial equation obtained by the method of least squares

Introduction

The term 'secular trend' or simply "trend" is very popularly used in day-to-day conversation. For e.g.: we often talk that the population, prices, production, etc. are showing an upward trend. What we really mean thereby is that is we observe such variables over a long period of time we find an increasing tendency.

The general tendency of the data to grow or decline over a long period of time is called 'Secular Trend'.

Trends are classified in to 2 main categories mainly:

i. Linear / Straight Line methods

ii. Non-Linear Trends.

The methods of measurement for the straight line trends are the following:

- The free hand or graphic method
- The semi average method
- The method of least squares

The methods of measurement for the Non-linear trends are the following:

- Freehand or graphic method
- Moving average method
- A parabolic trend by a second degree polynomial equation obtained by the method of least squares

2. Graphic Method

Let us discuss each of this method in detail;

1. Graphic method:

It is the simplest method of studying trend. In this method, the given data are plotted on a graph paper and trend line is fitted to the data just by inspecting the graph of the series. As a rough guide, the line should be drawn in such a way that it passes between the plotted points in such a manner that the fluctuations in one direction are approximately equal to those in the other directions and that it shows a general movement.

When a trend line is fitted by the freehand method, an attempt should be made to make it conform as much as possible to the following conditions:

- The line should be smooth either a straight line or a combination of long gradual curves
- The sum of vertical deviations from the trend of the annual observations above the trend line should be equal to the sum of the vertical deviations from the trend of the observations below the trend line
- The sum of squares of the vertical deviations of the observations from the trend should be as small as possible
- The trend should bisect the cycles so that the area above the trend equals that below the trend, not only for the entire series but as much as possible for each full cycle. This last condition cannot always be met fully, but a careful attempt should be made to observe it as closely as possible

Merits and Limitations of the Freehand method:

Merits:

- It is a simplest method of measuring trend
- It is a very flexible method that can be used regardless of whether the trend is a straight line or curve.
- The trend line drawn by a statistician experienced in computing trend and having knowledge of the economic history of the concern or the industry under analysis may be better expression of the secular movement than a trend fitted by the use of formula which may have no other logical justification. It is typically used by experienced hands and not recommended for beginners

Limitations:

- This method is highly subjective because the trend line depends on the personal judgement of the investigator and hence different persons might draw different trend lines from the same set of data
- Since freehand curve fitting is subjective it cannot have much value if it is used as a basis for predictions
- Though it is simple and direct, it takes lot of time to construct a freehand trend if a careful job is done

Method of Semi-Averages

In this method the trend is calculated in the following steps:

Step 1: Divide the given data into 2 parts. In case of even numbers, the data is divided into 2

equal parts. In case of odd numbers, middle point will be eliminated and the remaining data will be divided into 2 equal parts.
Step 2: Once the data is divided into 2 parts, an arithmetic mean of each part is obtained. Thus we get 2 points.

Step 3: Each point is plotted at the mid-point of the class interval covered by the respective part
Step 4: The two points are joined by a straight line which gives the required trend. The line can be extended downwards or upwards to get the intermediate values or to predict the future values.

Merits:

i. Simple to understand compared to moving average and method of least squares
ii. Objective method of measuring trend as everyone who applies the method is bound to get the same result

De-Merits:

i. This method assumes straight line relationship between the plotted points irrespective of the fact whether a relationship exists or not
ii. Limitations of arithmetic average shall automatically apply. If there are extremes in either half or both halves of the series, then the trend line is not a true picture of the growth factor. This danger is greatest when the time period represented by the average is small.
iii. Consequently, trend values obtained are not precise enough for the purpose either of forecasting the future trend or of eliminating trend from original data.

3. Least Square Method

Method of Least Squares:
This method is most widely used in practice. A trend line is fitted to the data in such a manner that the following 2 conditions are satisfied.

i. $\Sigma(Y-Y_c) = 0$

The sum of deviations of the actual values of the Y and the computed values of Y is Zero

ii. $\Sigma(Y-Y_c)^2$ is least

The sum of the squares of the deviations of the actual and computed values is least from this line. That is why this method is called the method of least squares. The line obtained by this method is known as the "line of best fit".

The method of least squares can be used either to fit a straight a line trend or a parabolic trend.
The straight line trend is represented by the equation
$Y_c = a + bX$
where Y_c denotes the trend values to distinguish them from the actual Y values
a is the Y intercept or the value of Y when X = 0
b is the slope of the line associated with a change of one unit in X variable.
X variable in time series analysis represents time

$\Sigma Y = Na + b \Sigma X$ (Summation Y = N of a Plus b of Summation of X)
$\Sigma XY = a \Sigma X + b\Sigma X^2$ (Summation of XY = a Summation X Plus b Summation of X square)

Where N represents number of years for which data are given.
The first equation is nearly the summation of the given function.
The second equation is the summation of X multiplied by the given function.

Measuring the variable X – we can measure the variable X from any point of time in the origin such as the First year but the calculations are very much simplified when the midpoint is taken as the origin because in that case the negative values in the first half of the series will be balanced out by the positive values in the second half of the series and hence ΣX will be equal to Zero. Since ΣX is equal to Zero, the above two normal equations will take the form of

Summation Y is equal to N into 'a' and Summation XY is equal to 'b' Summation X square. $\Sigma Y = Na$ and $\Sigma XY = b\Sigma X^2$

Now we can determine the value of 'a' and 'b' easily.
'a' is equal to Summation Y by N – which is equal to Y Mean and
'b' is equal to Summation XY by Summation X square.

Merits:

i. It is a mathematical method of measuring trend and as such has no possibility of measuring subjectivity
ii. The line obtained is a line of best fit because this line is from where the sum of the positive and negative deviations is zero and the sum of the squares of deviations is the least.

Limitations:

i. It is a mathematical curve which will be useful to describe the general movements but analytical significance cannot be attached to them
ii. Mathematical trends cannot describe growth of population or a general rise in price
iii. Mathematical methods are not fool proof but they can be source of serious errors in statistical works and should be used rigidly controlled by separate logics analysis

4. Measurement of Non Linear Trend and Moving Average Method

Measurements of Non Linear Trend:
The straight line trends indicate the increase and decrease of a time series at a constant amount. It is the simplest form of describing the secular trend movement and the trend is frequently accurate. However there are situations where the straight line trend cannot fit the data adequately.
In such cases better description of the time series is given by a nonlinear curve and the following methods are used for measuring the non-linear trends:
Free hand or Graphic method as explained earlier involves an element of subjectiveness and is not recommended for general use.
Method of moving average – In this method the average value for a number of years (months or weeks) is secured and this average is taken as the normal or trend value for the unit of time falling at the middle of the period covered in the calculation of the average.

Method of Moving Average
The effect of averaging is to give a smoother curve reducing the influence of the fluctuation that pulls the annual figures away from the general trend. While applying this method, it is necessary to select a period for moving average such as 3 yearly averages, 5 yearly averages, 8 yearly average etc. the period of the average is generally decided keeping in mind the length of the cycle.

As the moving average is commonly applied to data that is characterized by cyclical movements, it is necessary to select a period for moving average which coincides with the length of the cycle otherwise the cycle will not be entirely removed. The danger is more severe when the time period is shorter.

When the period of moving average and the period of the cycle do not coincide, the moving average will display a cycle which has the same period as the cycle in the data, but having less amplitude than the cycle in the data.

Whenever the cycle in the data is of uniform length, we should take a moving average period equal to or greater than the average period of the cycle in the data. Generally the periods will range between 3 to 10 years for general business series.

The following formulae are used for a 3-year or 5 year moving average. 3 yearly moving average is equal to a + b + c by 3 , b + c + d by 3, c + d + e by 3 and so on… where as 5 yearly moving average is calculated as a +b + c +d + e by 5, b +c +d +e + f by 5 and so on… Where in the alphabet represents different time periods of the data.

If the period of moving average is odd then the moving average would coincide with an original period.
If the period of moving average is even then the moving average will be placed at the centre of time span for which they are computed and it will fall between 2 time periods

For an even period the moving average is placed at the centre of time span for which they are computed and it will fall between 2 time periods. This causes inconvenience as the moving average would not coincide with an original time period. In such situations, synchronizing of moving averages and original data is done through a process called Centring

Merits of Moving Average:

i. This method is simple as compared to the Method of Least Squares.
ii. It is a flexible method of measuring trend as calculations are not changed when more figures are added to the data.
iii. The period of moving average happens to coincide with the period of cyclical fluctuations in the data. Such fluctuations are automatically eliminated.
iv. It has the advantage of following the general movements of the data and that its shape is determined by the data rather than the statistician's choice of mathematical function.

Limitations of Moving Average Method:

i. Trend values cannot be computed for all the years, longer the period of moving average greater the number of years for which the trend value cannot be obtained.
ii. Great care has to be exercise in selecting the period of moving average as there are no hard and fast rules for the choice of the period.
iii. As moving average does not represent any mathematical function, it cannot be used in forecasting
iv. The best results would be obtained by a moving average whose periods are equal to the average length of the cycle in the given series; however it is difficult to determine the average length of the cycle until the cycles are isolated from the series.
v. Finally when the trend situation is not linear, the moving average lies either above or below the true sweep of the data.

Moving average is appropriate for trend computation only under the following conditions:

- When the purpose of investigation does not call for current analysis or forecasting
- When the trend is linear
- When the cyclical variations are regular both in period and amplitudes

However in practice these conditions rarely hold good for a data.

5. Second Degree Parabola and Illustration of Moving Average Method

Second degree parabola: is the simplest example of the non-linear tends. In the second degree parabola, the equation of which is written in the form of trend Y is equal to a plus bx plus cx square. where a, b and c have been derived, the trend value for any year may be computed by substituting in the equation the value of X for that year.

The values of a, b and c can be determined by solving the following three normal equations simultaneously.
The first equation is Summation Y is equal to 'N' into 'a' Plus 'b' summation 'X' Plus 'c' Summation 'X' Square. This equation is merely the summation of the given function.
The second equation is Summation XY is equal to Summation 'X' Plus 'b' Summation X Square Plus 'c' Summation 'X' Cube. This equation is the summation of x multiplied into the given function.
The third equation is Summation 'X' Square Y is equal to 'a' Summation X square Plus 'b' Summation 'X' cube Plus 'c' Summation 'X' power 4.
The first equation is merely the summation of the given function.
The second equation is summation of X multiplied into the given function.
The third equation is the summation of X square multiplied into the given function.

When time origin is taken between 2 middle years Summation X would be Zero. In such case the above equations are reduced to:
Summation Y is equal to 'N' into 'a' plus 'c' Summation X Square
Summation XY is equal to 'b' Summation X Square
Summation X square Y is equal to 'a' Summation X square plus 'c' Summation X power 4
The value of 'b' can be obtained from equation (ii) and that of 'a' and 'c' by solving (i) and (ii) simultaneously. Thus

'a' is equal to Summation Y minus 'c' Summation X square divided by N

'b' is equal to Summation X into Y divided by Summation X square

'c' is equal to N Summation X square Y – Summation X square into Summation Y divided by N Summation X power 4 minus whole square of Summation of X square.

Let us understand the moving average method in detailed by looking at this example:
Calculate 5 yearly and 7 yearly moving averages for the following data of a number of commercial industrial failures in a country during 1990 to 2005.

Figure 1

Year	Number of Failures	Year	Number of Failures
1990	23	1998	9
1991	26	1999	13
1992	28	2000	11
1993	32	2001	14
1994	20	2002	12
1995	12	2003	9
1996	12	2004	3
1997	10	2005	1

Solution:
Calculation of the 5 yearly and 7 yearly averages is done using the following table –

Figure 2

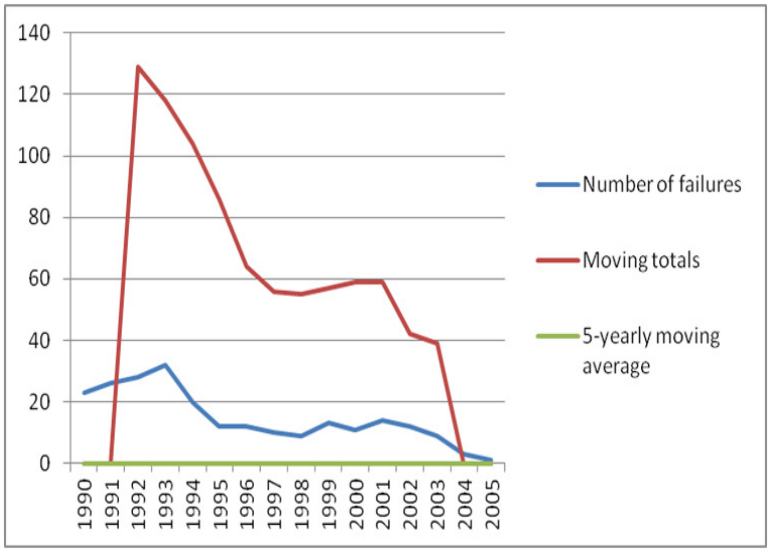
Year	No. of Failures	Moving Totals	5 yearly Moving Average
1990	23	-	-
1991	26	-	-
1992	28	129	25.8 or 26
1993	32	118	23.6 or 24
1994	20	104	20.8 or 21
1995	12	86	17.2 or 17
1996	12	64	12.6 or 13
1997	10	56	11.2 or 11

Year	No. of Failures	Moving totals	5 yearly Moving Average
1998	9	55	11.0 or 11
1999	13	57	11.4 or 11
2000	11	59	11.8 or 12
2001	14	59	11.8 or 12
2002	12	42	9.8 or 10
2003	9	39	7.9 or 8
2004	3	-	-
2005	1	-	-

In the table the first column represents the number of years from 1990 to 2005. The second column represents the number of commercial industrial failures. The third column provides the calculation of 5 yearly moving totals i.e. the first 5 years 1990 to 1994 is taken. Its total is 129 and is placed against the mid-year of the 5 data – 1992. Again take the total of the next 5 years eliminating the first year i.e. 1991 to 1995. Its total is 118 and is placed against the mid-year of the 5 data i.e. 1993. Similarly the other values of the data are obtained like 104, 86, 64, 56, 55, 57, 59, 59, 42 and 39.
In the fourth column we will calculate the 5 yearly moving averages for which we take the total 129 and divide it by 5 – we get a value 25.8 (rounded of to 26). Similarly the other values are obtained 24, 21, 17, 13, 11, 11, 11, 12, 12, 10 and 8. Thus the 5 yearly moving averages are calculated.

This is a graphical representation of the 5 yearly moving averages

Figure 3



The second requirement of the problem is to calculate the 7 yearly moving averages for which we will first calculate the 7 yearly moving totals and then the 7 yearly moving averages similar to the way we have done for the 5 yearly moving totals and 5 yearly moving averages.
So the 7 yearly moving totals will be 153, 140, 123, 108, 87, 81, 81, 78, 71 and 63. Their averages would be 22, 20, 18, 15, 12, 12, 12, 11, 10 and 9.

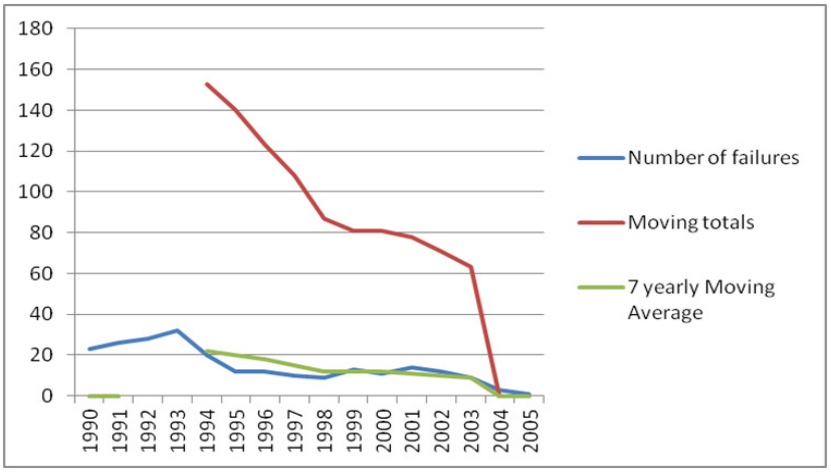
Figure 4

Year	Number of Failures	Moving Totals	7-yearly Moving averages
1990	23	-	-
1991	26	-	-
1992	28	-	-
1993	32	-	-
1994	20	153	21.9 or 22
1995	12	140	20.0 or 20
1996	12	123	17.6 or 18
1997	10	108	15.4 or 15

year	No. of Failures	Moving Totals	7 Yearly Moving Averages
1998	9	87	12.4 or 12
1999	13	81	11.6 or 12
2000	11	81	11.6 or 12
2001	14	78	11.1 or 11
2002	12	71	10.1 or 10
2003	9	63	9.0 or 9
2004	3	-	-
2005	1	-	-

Here is a graphical representation of the 7 yearly moving averages.

Figure 5



Here's a summary of our learning in this session where we have understood the Measurement of Linear trend using

- The free hand or graphic method, The semi average method, The method of least squares
- We have also understood the measurement of Non Linear trend using the following methods :
- Freehand or graphic method, Moving average method, a parabolic trend by a second degree polynomial equation obtained by the method of least squares.