1. Introduction

Welcome to the series of E-learning modules on Measurement of cyclical variation.

By the end of this session, you will be able to:

- Explain the measurement of cyclical variation

Let us start with an introduction:

The Term Cycle refers to the recurrent variations in time series that usually last longer than a year but not regular in length or amplitude.
Time series related to economies and business show some kind of cyclical variations.
Cyclical fluctuations are long term movements, which represent consistently recurring increase and decline in activity.

The study of such cyclical variations is extremely useful in,

- Framing suitable policies for stabilizing the level of business activity and
- For avoiding periods of booms and depressions as both are bad for an economy – particularly depression which brings about a complete disaster and shatters the economy

Though, we can measure the cyclical variations and the associated impacts on the economy, it is very difficult to predict and measure the economic fluctuations.
The reason being – Business cycles do not show regular periodicity as they differ in the timing and pattern.

Cyclical variations are mixed with erratic, random forces which make it difficult to isolate separately the effect of cyclical and irregular forces.
Business cycles are distinguished from seasonal variation in the following respects where in the cyclical variations are of a longer durations say more than a year.

Typical business cycles could vary from 2 to 10 years.
Moreover, they do not ordinarily exhibit regular periodicity as successive cycles vary widely in timing, amplitude and pattern.
Fluctuations in the 4 phases of business cycles are caused by multiple factors.

The period of prosperity, decline, depression and improvement viewed as four phases of a business cycle are generated by factors other than weather, social customs and those which create seasonal patterns.
Business cycles are perhaps the most important type of fluctuation in economic data.
Certainly they have received a lot of attention in economic literature.

Despite the importance of business cycles, they are the most difficult type of economic fluctuation to measure.
This is because successive cycles vary so widely in timing, amplitude, pattern and the cyclical rhythm is inextricably mixed with regular factors.
Because of these reasons it is impossible to construct meaningful typical cycle indexes or curves similar to those that have been developed for trends and seasonal.

The various methods used for measuring cyclical variations are:

- Residual method
- Reference cycle analysis method
- Direct percentage variation method
- Harmonic analysis method or fitting of sine functions

2. Residual Method

Let us now discuss about the residual method:

Amongst all the methods of arriving at estimates of the cyclical movements of time series, the residual method is most commonly used.
This method consists of eliminating the two components, seasonal variation and seasonal trend, thus obtaining the cyclical irregular movements.

The data are usually smoothed in order to obtain cyclical movements, which are sometimes termed the cyclical relatives, since they are always in percentages.
It is because cyclical, irregular or the cyclical movements remain as residuals that this procedure is referred to as the residual method.
Using the multiplicative model, Y of c is equal to T into S into C into I.

The steps involved in the computation of cyclical variation by the residual method may be summarized as follows:

Step-1:

- Calculate trend value T by moving average method and seasonal index S (which is to be taken in fractional form and not in percentage form), preferably by a moving average method.

Step-2:

- Divide time series (y) by trend (T_t) into seasonal variation (S_t). This step may be divided into two steps that is,
  - Divide time series by trend to get the product of seasonal variation, cyclical variation and irregular variation
  - Divide the product of seasonal variation, cyclical variation and irregular variation by seasonal variation to get cyclical variation and irregular variation

Step-3:
The moving average of suitable period of these cyclical variation and irregular variation values obtained in step two smoothens out the random component, thus leaving the cyclic component.
Thus, the residual method is, by far, the most commonly used method of measuring the cyclical variations.

Let us look at the limitations of the residual method:

The residual method would leave values reflecting only cyclical and irregular influences ,
If the trend ordinates perfectly depicted the pattern of secular change and if the seasonal index exactly reflected seasonal influence.
Because, such perfection is rarely encountered the computed values almost contain some trend and seasonal elements.

This condition will be more or less serious depending on how well or poorly the trend line and

the seasonal index represent secular and seasonal forces.
If a straight line trend is employed to describe an essential curve linear secular movement, figures presumably adjusted for trend will be grossly distorted.

The distortion would also occur if the seasonal index where not descriptive of the seasonal pattern at the time in question.
Thus, the residual method is based on the assumption that trend and seasonal can be accurately measured and therefore be removed at least in large part.

3. Reference Cycle Analysis or The National Bureau Method

Let us now discuss about Reference cycle analysis or the national bureau method:

The national bureau of economic research has developed a different method to analyze the cyclical variations, which it has used in the study more than 1,000 specific time series.
This method is of value in analysis past cycles only.

The national bureau procedure aims to answer two sets of questions:

1. Is there in a given series a pattern of change that repeats itself (with more or less variation) in successive cycles in business at large? If so, what are its characteristics?
   Question is concerned with the behavior of individual series during successive waves of expansion and construction in the general economic.

2. Is there in a given series a wave movement peculiar to that series? If so, what are its characteristics?
   Question is concerned with the periodic or semi-periodic fluctuations in individual series.

A procedure involving ‘references dates’ has been designed by the National Bureau of Economic Research as a device.
Device allows one not only to compare each series with a standard set of dates and to observe the behavior of individual series during expansion and contraction of general business but also to compare the results for the various individual series.

The first step in the selection of the references dates which are the dates of the peaks and troughs of the business cycles.
The reference dates which cover a duration of over one year and not over ten or twelve years were chosen after examination of large numbers of economic time series and after the study of the ‘contemporary’ reports of observers of the business scene.

The next step consists of processing the data of the individual series in order to obtain a cyclical pattern for each series for the period between two successive reference troughs.
Each period is the same for all series enabling one to compare the results for various series.

The processing follows the following steps:

- Data is adjusted for seasonal variations
- Seasonally adjusted data are divided into reference cycle segments, these segments corresponding to the intervals between adjacent reference troughs

For each segment, the monthly values are expressed as percentages of averages of all the values in the segment.
These are “reference-cycle-relatives”.
This step eliminates inter-cycle trend but does not eliminate intra-cycle trend.
Inclusion of intra-cycle trend is regarded as desirable since it is helps to reveal and to explain

what happens during a business cycle.

Each reference cycle segment is broken into nine stages to correspond to the same nine stages of business cycle and the reference cycles relatives are averaged for each of the nine stages.

The nine stages are as follows:

1. The 3 months centered on the initial trough
2. The first third of the expansion period
3. Second third of the expansion period
4. Last third of the expansion period
5. The 3 months centered on the peak
6. The first third of the contraction period
7. Second third of the contraction period
8. Last third of the contraction period
9. The 3 months centered on the terminal trough

The nine stage averages for each reference cycle segment serves to reduce the erratic movements in a series and gives a reference cycle pattern for particular series under considerations.
This method though complicated and cumbersome compared to the Residual technique, is relatively simpler and accurate in comparing the cyclical variations of individual series with those of general business.

It is free of errors that might be introduced where secular trend are improperly estimated. The latter advantage is indeed significant where series whose trend patterns are not clear are under analysis.

4. Direct Method & Harmonic Analysis

Let us discuss about Direct Method:

Statistical study of time series has largely developed along two rather distinct lines.
The first centers on the analysis of the specific relationship between several series.
IT follows the scientific method of approach setting up tentative hypothesis as to cause and effect and then using the statistical analysis to disprove or verify and measure the expected relations.

The second line of development has placed emphasis more largely upon the characteristics of individual series of data rather than upon the relationship of the series so as to show more clearly the response of the individual series to the recurring economic fluctuations.

Methods have been developed to eliminate the seasonal variations as well as the secular change from the basic data.
Both the methods have the same objective of forecasting the future changes but the first method treats specific changes and measure their causes whereas the second works largely in terms of long swings and general movements.

Let us now discuss about Harmonic analysis:

Harmonic analysis provides a sophisticated method of determining the cyclic component of the time series.
From mathematical analysis, we know that any function, (y_t) under some very general conditions, can be represented by a fourier series, that is a series of sums of sine and cosine functions.

Thus, for a time series (Y_t) with period oscillation lambda, we have,

Time series is equal to 'a' nought plus ['a' one sin of (2 pai by lambda into t) plus 'a' two of (2 pai by lambda into 2t) and so on] plus ['b' one cos of (2pai by lambda into t) plus 'b' two into cos of (2 pai by lambda into 2t) and so on]
Where,
'a' of 'I' is equal to 2 by 'n' summation of time series sin (2 pai by lambda into 'I''t').
'B' of 'I' is equal to 2 by n summation of time series cos of (2 pai by lambda into 'I''t') (where i is equal to 1, 2, etc) and,
'a' nought is equal to one by n summation of time series where, n is the number of terms in the time series.

For instance, if the period of oscillation is 12 months and y one, y two and so on up to y twelve is the series or average for a number of years then the constants a of i's and b of i's are given by a nought is equal to I by 12 summation of y of t by limits of 12 in the upper limit and t equal to one in the lower limit.

Where,
‘A’ of ‘I’ is equal to 2 by 12 summation of y of t sin of 2 pai by 12 into ‘I’‘t’, where (i=1,2,….6); and
‘B’ of ‘I’ is equal to 2 by 12 summation of Y of t cos of 2 pai by 12 into ‘I’‘t’, where (i=1,2,….,5).
So far lambda is regarded as a known constant.

5. Harmonic Analysis - Periodogram

An elegant method of determining lambda is through the Periodogram analysis.
Let us consider a time series in which the trend and the seasonal components have been eliminated.
In other words, the resulting series ‘Y of t’ consists of only two components, one periodic with period lambda and amplitude ‘S’ and the other, the random component E of i which is un-correlated with any cyclic movement, for long time at least.

Thus, y of t is equal to alpha sin of (2 pai by lambda) into t plus epsilon of t. Covariance (epsilon of i , epsilon of j) is equal to zero (I not equal to j) ; variance (epsilon of i) is equal to standard deviation square.
Covariance of (epsilon of i, sin of 2 pai by lambda into t) is equal to 0; covariance (epsilon of i , Cos 2 pai by lambda into t is equal zero.

Let us consider A is equal to 2 by n summation Y of t Cos (2 pai by mu into t),
B is equal to 2 by n summation Y of t Sin of 2 pai by mu into t; where, mu is arbitrary and defines: Amplitude square into mu is equal to A square plus B square which is known as intensity corresponding to the trial period mu.
Substituting and using the values, we get,

A is equal to 2 by n summation of [a sin of (2 pai by lambda into t) plus epsilon of t] cos (2 pai by mu into t).
Is equal to ‘a’ by n summation 2 sin of 2 pai by lambda into t cos of 2 pai by mu into t.
Is equal to ‘a’ by n summation 2 sin alpha t cos beta t, where alpha is equal 2 pai by lambda and beta is equal to 2 pai by mu.
Because A is equal a by n summation sin of alpha plus beta into t plus sin alpha minus beta into t.
Let amplitude S is equal to summation of sin of alpha plus beta into t.

Implies, that S into sin of alpha plus beta by 2 is equal to half summation of (2 sin (alpha plus beta) into t into sin of (alpha plus beta by 2).

Is equal to half summation of [Cos {(alpha plus beta) into t minus alpha plus beta by 2) minus cos { (alpha plus beta into t plus alpha plus beta by 2}].

Is equal to [{Cos of alpha plus beta by 2 minus Cos of 3 (alpha plus beta by 2} plus {Cos 3(alpha plus beta by 2) minus cos 5 (alpha plus beta plus 2)} plus etc Plus {Cos of (2n minus 1) (alpha plus beta) by 2 minus cos (2nplus 1) (alpha plus beta) by 2}].

Is equal to half [Cos alpha plus beta by 2 minus cos of (2n plus 1) into (alpha plus beta) by 2] is equal Sin {(n plus 1) into (alpha plus beta) by 2} into Sin n (alpha plus beta) by 2.

Implies that, S is equal to summation Sin (alpha plus beta) into t is equal to Sin {n into (alpha plus beta) by 2} into sin into {(n plus 1) into (alpha plus beta) by 2} divided by Sin (alpha plus beta by 2).

Similarly, we can get the value of summation sin (alpha minus beta) into t.
By substituting, we get ,
A is equal a by n into [sin {n of (alpha plus beta) by 2} into sin {n plus 1 into (alpha plus beta by 2}] divided by sin {(alpha plus beta by 2)} plus sin {n into (alpha minus beta) by 2 } into sin (n plus 1) into (alpha minus beta) by 2} divided by sin of {alpha minus beta by 2}.

If alpha is not equal beta, then A tends to 0 for large n, since then the expression in bracket is bounded for all alpha, beta and n.
However, if alpha tends to beta, that is alpha minus beta tends to 0, then for large n,
we get ‘A’ is equal to limit n to infinite ‘a’ by ‘n’ [some finite quantity] plus limit n to infinite ‘a’ by ‘n’ into Sin {n into (alpha minus beta) by 2} into sin {(n plus 1) into (alpha minus beta) by 2} by sin [(alpha minus beta) by 2].

Is equal to alpha sin {(n plus 1) into (alpha minus beta) by 2} for large n, (since limit Sin n theta by sin theta is equal to n).
Thus, for large n, if alpha is not equal beta which implies lambda is not equal to mu then A tends to 0.
If alpha tends to beta which implies lambda tends to mu, then A tends to alpha sin {(n plus 1) (alpha minus beta) by 2}.

Similarly, it can be shown that for large n,
If alpha is not equal to beta which implies lambda is not equal to mu then, B tends to 0
If alpha tends to beta which implies lambda tends to mu, then, B tends to alpha Cos {(n plus 1) into (alpha minus beta) by 2} .
Thus, if the arbitrary number ‘mu’ is exactly the period of oscillation (lambda) series, then S square into mu is equal to a square which implies S into mu is equal to a.
On the other hand if lambda is not equal to mu then s into mu is equal to zero.

Thus, S into mu remains small unless the trial period mu approaches the true period lambda in which case its value is equal to the amplitude ‘a’.
This conclusion forms the basis of periodogram analysis , which is summed up as follows;

From given time series y_1, y_2, …., y_n calculate A and B as defined in for different values for mu from 0 to n and compute S(mu) using the above equation.
The graph obtained on plotting S (mu) against mu is known as periodogram.
The values of mu corresponding to the significant maximum values of s provide the fundamental periods of oscillation, provided no mu i is a multiple of another mu; the corresponding value of S provide the corresponding amplitudes.

The obvious drawback of harmonic analysis lies in ‘huge calculations’ if by drawing the graph of the time series we can guess the true periods of oscillation, it may be necessary to compute S for only those value of the trial period mu which are in the neighborhood of approximate values.

Here’s a summary of our learning in this session, where we have understood:

- The method of measurement of cyclical variation