1. Computation of Crude & Standardized Death Rates

Welcome to the series of E-learning modules on Calculation of Fertility Rates, Mortality Rates, Life table, GRR & NRR.

By the end of this session, you will be able to:

- Explain the calculation of,
  - Fertility Rates
  - Mortality Rates
  - Life table
  - GRR and
  - NRR

Problem 1:
Compute the crude and standardized death rates of the two populations A and B, consider A as standard population from the data.

Figure 1

Age groups (yrs)	A		B	
	Population	Deaths	Population	Death
Under 10	20,000	600	12,000	372
10 -20	12,000	240	30,000	660
20 - 40	50,000	1250	62,000	1612
40 - 60	30,000	1050	15,000	525
Above 60	10,000	500	3,000	180

First column represents the age group ‘x’.
Second column represents the population of A for age ‘x’ & the third column represents the deaths of population at age ‘x’.
Fourth column represents the population of B for age ‘x’ & the fifth column represents the deaths of population at age ‘x’.

Solution:
Let us calculate the Crude Death Rate and the Standard Death Rate for the given data for which we prepare the table.

Figure 2

Age groups (yrs)	A			B			
	Population (P_x^a)	Deaths (D_x^a)	Death Rate Per 1000 (m_x^a)	Population (P_x^b)	Death (D_x^b)	Death Rate Per 1000 (m_x^b)	(m_x^b) (P_x^a)
Under 10	20,000	600	30	12,000	372	31	6,20,000
10 -20	12,000	240	20	30,000	660	22	2,64,000
20 - 40	50,000	1250	25	62,000	1612	26	13,00,000
40 – 60	30,000	1050	35	15,000	525	35	10,50,000
Above 60	10,000	500	50	3,000	180	60	6,00,000
Total	1,22,000	3,640		1,22,000	3,349		38,34,000

In the table the first column represents the age group ‘x’.
Second column represents the population of A for age ‘x’.
Third column represents the deaths of population at age ‘x’.
In the fourth column we will calculate the death rate per 1000 which is equal to 30, 20, 25, 35, and 50.
The fifth column represents the population of B for age ‘x’.
Sixth column represents the deaths in population B for age ‘x’
In the seventh column we calculate the death rate of population B per 1000 which is equal to 31, 22, 26, 35, 60 and in the last column we will take the product of the death rate of the population B and the population A for age ‘x’, we will get 6 lakh 20 thousand; 2 lakh 6 thousand; 13 lakh; 10 lakh 50 thousand and 6 lakh with a total of 38 lakh 34 thousand.

In the next step, let us calculate the Crude Death Rate and the Standard Death Rate. The Crude Death Rate for population A is equal to summation ‘x’ of death rate of population ‘a’ divided by summation ‘x’ population of A into 1000 is equal to 3thousand 640 divided by 1 lakh 22 thousand into 1000 is equal to 29 point 8. Similarly, for population B we calculate the Crude Death Rate by considering summation ‘x’ of deaths of population B divided by summation ‘x’ of population B into 1000 is equal to 3 thousand 349 divided by 1 lakh 22 thousand into 1000 is equal to 27 point 4.

Next we will calculate the Standard Death rates for both the population. the Standard Death rates for population A is equal to Crude Death Rate of population A which is equal to 29 point 8.(Since population A is taken as standard population).
The Standard Death rates of population B is equal to summation of the product of the death rate of the population B and the population A for age ‘x’ divided by summation of population A which is equal to 38 lakh 34 thousand divided by 1 lakh 22 thousand is equal to 31 point 4.

Figure 3

Particulars	Population A	Population B
Crude Death Rate	29.8	27.4
Standard Death Rate	29.8	31.4

Look at this table for comparison, the Crude Death Rate of population A is 29 point 8 and population B is 27 point 4 whereas the Standard Death Rate of population A is 29 point 8 and population B is 31 point 4 thus, we can conclude that the death rate in population B is greater than population A.

2. Computation of GFR, SFR & TFR

Problem2:
Compute the GFR, SFR and TFR from the data given below:

Figure 4

Age group of child bearing females	Number of women ('000)	Total Births
15 – 19	16.0	260
20 -24	16.4	2244
25 – 29	15.8	1894
30 -34	15.2	1320
35 – 39	14.8	916
40 – 44	15.0	280
45 - 49	14.5	145

First column represents the age sub-groups of population in years.
Second column showing the number of women in each sub group.
Third column shows the number of births in each subgroup.

Assume that the proportion of female births is 46.2%
Solution:

Figure 5

Age group of child bearing females	Number of women ('000)	Total Births
15 – 19	16.0	260
20 -24	16.4	2244
25 – 29	15.8	1894
30 -34	15.2	1320
35 – 39	14.8	916
40 – 44	15.0	280
45 - 49	14.5	145
Total	107.7	7059

Let us calculate the general fertility rate first for which we will take the table and total the columns.
First column shows the age of the population in years and sub grouped as 15-19, 20-24, and so on up to 45-49.
Second column shows the number of women in each sub group and the total of this column comes to 1 lakh 7 thousand 700 & the third column shows the number of births in each

subgroup with a total comes to 7 thousand 59.

Let us calculate the general fertility rate. The general fertility rate is equal to summation of number of birth divided by the summation of the female population into 1000 is equal to 7 thousand 59 divided by 1 lakh 7 thousand 700 is equal to 65 point 543 per thousand.

Next we will calculate the age specific fertility rate.

Figure 6

Age group of child bearing females	Number of women ('000)	Total Births	Age SFR
15 – 19	16.0	260	16.25
20 -24	16.4	2244	136.83
25 – 29	15.8	1894	119.87
30 -34	15.2	1320	86.84
35 – 39	14.8	916	61.89
40 – 44	15.0	280	18.67
45 - 49	14.5	145	10.00
Total	107.7	7059	450.35

The age specific fertility rate is equal number of births in each group divided by the total population on that specific group into 1000 which is equal to 16 point 25, 136 point 83, 119 point 87, 86 point 84, 61 point 89, 18 point 67, 10 for respective age groups and the summation is 450 point 35.

Next we will calculate the total fertility rate. The total fertility rate is equal to 5 into summation of the age specific rate is equal to 5 into 450 point 35 is equal to 2 thousand 251 point 75 per thousand.

Problem 3:

Figure 7

Age of women	15-19	20-24	25-29	30-34	35-39	40-44	45-49
Number of women	212,619	198,732	162,800	145,362	128,109	106,211	86,753
Age SFR (per 1000)	98	169.6	158.2	139.7	98.6	42.8	16.9

Calculate the general fertility rate, total fertility rate and the gross reproduction rate from the

following data, assuming that for every 100 girls, 106 boys are born.
The first row shows the age group.
The second row shows the number of women.
Third row shows the values for age specific fertility rate per 1000.

Solution:

Figure 8

Age group	Number of women nfPx	Age SFR nIx	Number of Births nBx = (2) x (3) /1000
15 – 19	212619	98	20837
20 - 24	198732	169.6	33705
25 - 29	162800	158.2	25755
30 - 34	145362	139.7	20307
35 – 39	128109	98.6	12632
40 - 44	106211	42.8	4546
45 - 49	86753	16.9	1466
Total	**1040586**	**723.8**	**119247**

In the usual notations, we have age specific fertility rate given by n of I of x is equal to n of B of x by n of fP of x into 1000 implies x of B of x is equal to n of I of x into n of fP of x by 1000.
This table shows the calculations for GRR and TFR.
In the first column, we show the age group, in the second column we show the number of women getting a total of 10 lakh 40 thousand 586, in the third column we shows the values for age specific fertility rate per 1000 that is n I of x with a summation total for 723 point 8 and the last column indicates the values for n B of x which is calculated by taking the product of the column 2 and column 3 divided by 1000 is rounded of to 20 thousand 837; 33 thousand 705; 25 thousand 755; 20 thousand 307; 12 thousand 631; 4 thousand 546 and 1 thousand 466 with a total of 1 lakh19 thousand 247.

GFR is equal to summation of n of B of x by summation of n of fP of x into 1000 is equal to 1 lakh 19 thousand 247 divided by 10 lakh 40 thousand 586 into 1000 is equal to 114 point 596.

TFR is calculated using the formula 5 into summation n of i of x is equal to 5 into 723 point 8 is equal to 3 thousand 619.
Since, it is given that for every 100 girls , 106 boys are born an approximation to GRR is given by GRR is equal to 100 by 100 plus 106 into TFR is equal to 100 by 206 into 3 thousand 619 is equal to 1 thousand 756 point 7961.

3. Computation of GRR & NRR

Problem 4:
Calculate the gross re-reproduction rate and the net reproduction rate from the given data in the table.

Figure 9

Age-group	Number of children born to 1,000 women passing through age-group	Mortality rate (per 1000)
16-20	150	120
21-25	1500	180
26-30	2000	150
31-35	800	200
36-40	500	220
41-45	200	230
46-50	100	250

First column represents the age group of the data.
Second column represents the number of children born to 1000 women passing through the age group.
Third column represents the mortality rate per 1000.

Sex ratio being males: females :: 52:48

Solution:
Let us prepare the table for the calculations of Gross Reproduction Rate and Net Reproduction Rate.

Figure 10

Age-group	Number of children born to 1,000 women passing through age-group	Number of female children ${}_nfB_x$	Survival Rate ${}_nfn_x$ = 1-(Mortality rate per woman)	Number of female children survived ${}_nfB_x \times {}_nfn_x$
16-20	150	150x.48 =72	1-0.12 = 0.88	63.36
21-25	1500	1500x.48=720	1-0.18 = 0.82	590.46
26-30	2000	2000x.48=960	1-0.15 = 0.85	816.00
31-35	800	800x.48=384	1-0.20 = 0.80	307.20
36-40	500	500x.48=240	1-0.22 = 0.78	187.20
41-45	200	200x.48=96	1-0.23=0.77	73.92
46-50	100	100x.48=48	1-0.25=0.75	36.00
Total	**5250**	**2520**		**2074.08**

In the table the first column represents the age group of the data.

The second column represents the number of children born to 1000 women passing through the age group which is 150, 1 thousand 500, 2 thousand and so on.
The third column represents the number of female children which is taken as 48% of the number of children born to 1000 women passing through age-group that is we take 48% of 150 is equal to 72, similarly, 48% of 1500 is 720 like this we calculate the number of female children for the other age groups as 960, 384, 240, 96 and 48 which gives a total of 2 thousand 520.
The fourth column represents the Survival rate that is 1 minus the mortality rate per women, the mortality rate is taken from the given table in the problem which is equal to 0 point 12; 0 point 18; 0 point 15; 0 point 20; 0 point 22; 0 point 23; and 0 point 25. Then we will get the survival rate as 0 point 88; 0 point 82; 0 point 85; 0 point 80; 0 point 78; 0 point 77 and 0 point 75.

In the last column we will calculate the number of female children survived which is the result of the product of column 3 and column 4 which is equal to 63 point 36; 590 point 46; 816; 307 point 20; 187 point 20; 73 point 92 and 36 which gives a total of 2 thousand 74 point 08.

Now we will calculate the gross reproduction rate and net reproduction rate.
The gross reproduction rate per women is equal to total number of female children born divided by 1000 is equal to 2 thousand 520 divided by 1000 is equal to 2 point52.
The net reproduction rate per woman is equal to total number of female children born and survived to 1000 women divided by 1000 is equal to 2 thousand 74 point 8 divided by 1000 is equal to 2 point 07.

4. Computation of the Life Table

Problem 5:
Given the following table for l of x, the number of rabbits living at age x, complete the life table for rabbits.

Figure 11

X	0	1	2	3	4	5	6
lx	100	90	80	75	60	30	0

The first row in the table shows the age x, as 0,1,2,3,4,5 and 6.
The second row shows the values of l of x which is equal to 100, 90, 80, 75, 60, 30 and 0.
X, Y, Z are three rabbits of age 1,2 and 3 years respectively. Find the probability that,

1. Atleast one of them will be alive for one year more
2. X, Y,Z will be alive for two years' time
3. Exactly one of the three is alive in two years and
4. All will be dead in two years' time

Solution:
Let us first complete the life table for the given data in the following table.

Figure 12

Age,x	lx	dx=lx-lx+1	qx =dx/lx	Lx=lx+lx+1/2	Tx= Σ Li i=x	E_x^o = Tx/lx
1	2	3	4	5	6	7
0	100	10	0.10	95	385	3.85
1	90	10	0.11	85	290	3.22
2	80	5	0.06	77.5	205	2.56
3	75	15	0.20	67.5	127.5	1.7
4	60	30	0.50	45	60	1
5	30	30	1.00	15	15	0.5
6	0	-	-	-	-	-

The first row in the table shows the age x, as 0,1,2,3,4,5 and 6.
The second row shows the values of l of x which is equal to 100, 90, 80, 75, 60, 30 and 0.
In the third column we will calculate the values of d of x, which is equal to l of x minus l of x+1, so we will get l of x is 100 for 0 age and l of x+1 is equal to 90 for age 1 so dx is equal to 10 similarly, we calculate the remaining values of the column as 10, 5, 15, 30 and 30.
In the fourth column we will calculate the value for q of x which is obtained by dividing d of x by l of x that is 10 divided by 100 is equal to 0 point 1 similarly the other values are obtained, that is 0 point 11; 0 point 06; 0 point 20; 0 point 50; 1.
In the fifth column we will calculate capital L of x which is obtained by using l of x plus l of x+1 by 2 is equal to 100 plus 90 divided by 2 which is equal to 95, 85, 77 point 5, 67 point 5, 45,

15.
In the next column we will calculate the value of T of x which is equal to summation of L of i where i is equal to x then we will get the values as 385, 290, 205, 127 point 5, 60 and 15.
In the last column we will get E_x^0 (exponential) which is obtained by dividing T of x divided by l of x which is equal to 385 by 100 which is equal to 3 point 85, similarly the other values are 3 point 22, 2 point 56, 170, 10 and 0 point 5. The life table is complete now.

Let us now find solutions for the other terms:
1. At least one of them will be alive for one year more
Solution:
Let p1 equal to probability that the rabbit X will die in one year which is equal to d1 by l1 is equal to 10 by 90.
p2 is equal to the probability that the rabbit Y will die in one year which is equal to d2 by l2 is equal to 5 by 80.
p3 is the probability that the rabbit Z will die in one year is equal to d3 by l3 is equal to 15 by 75.

Hence the probability that all will die in one year is given by the compound probability theorem by the expression p is equal to p1 into p2 into p3 is equal to 10 by 90 into 5 by 80 into 15 by 75 is equal to 1 by 720.
Therefore P (at least one of them will be alive for one year more) is equal to 1 minus p(nine of the rabbits will survive for one year more) is equal to 1minus p is equal to 1 minus 1 by 720 is equal to 719 by 720.

2. X, Y,Z will be alive for two years' time
Solution:
Let E1, E2 and E3 denote the events that the rabbits X, Y and Z survive for two years more respectively.
Then, P of (E1) is equal to l3 by l1 is equal to 75 by 90 is equal to 5 by 6 which implies that P of E1dash is equal to 1 minus 5 by 6 is equal to 1 by 6,
Similarly, P of (E2) is equal to l4 by l2 is equal 60 by 80 is equal to three by 4 which implies that P of E2 dash is equal to 1 minus 3 by 4 is equal to one by four.
Again, P of (E3) is equal to l5 by l3 is equal to 30 by 75 is equal to 2 by 5 which implies that p of E3 dash is equal to 1 minus 2 by 5 is equal to 3 by 5.
Hence the required probability that all the three rabbits will survive for two years is given by the compound probability theorem by expression: P of E1 intersection E2 intersection E3 is equal to P of E1 into P of E2 into P of E3 is equal to 5 by 6 into three by 4 into 2 by 5 is equal to 0 point 25.

3. Exactly one of the three is alive in two years
Solution:
Let E denote the event exactly one of the three rabbits will survive for two years more. Then the event E can materialize in the following mutually exclusive ways.
E1 intersection E2 dash intersection E3 dash happens , E1 dash intersection E 2 intersection E3 dash happens and E1 dash intersection E2 dash intersection E3 happens.
Hence by addition theorem of probability, we obtain P of (E) is equal to P of 'a' plus P of 'b' plus P of 'c' is equal to P of E1 intersection E2 intersection E3 dash plus P of E1 dash intersection E2 intersection E3 dash plus P of E1 dash intersection E2 dash intersection E3 is equal to P of E1 into P of E2 into P of E3 dash plus P of E1 dash into P of E2 into P of E3

dash plus P of E1 dash into P of E2 dash into P of E3 is equal to 5 by 6 into 1 by 4 into 3 by 5 plus 1 by 6 into 3 by 4 into 3 by 5 plus 1 by 6 into 1 by 4 into 2 by 5 is equal to 26 by 120 is equal to 0 point 2167

4. All will be dead in two years' time.

Solution:

In the notations of part (3) above, the required probability that all will die in two years is given by P of E1 dash intersection E2 dash intersection E3 dash is equal to P of E1 dash into P of E2 dash P of E3 dash is equal to 1 by 6 into 1 by 4 into 3 by 5 is equal to 1 by 40 is equal to 0 point 025.

5. Computation of Complete Expectation of Life

Problem 6:
Given that the complete expectation of life at ages 30 and 31 for a particular group are respectively 21 point 39 and 20 point 91 years and that the number of living at age 30 is 41 thousand 176.
find (i) the number that attains the age 31 and (ii) the number that will die without attaining the age 31.

Solution:
In the usual notations we want the value of $l_{31}$and we are given e_{30} is equal to 21 point 39, expectation 31 is equal to 20 point 90 and l_{30} is equal to 41 thousand 176 therefore e_{30} is equal to expectation 30 minus 1 by 2 is equal to 21 point 39 minus 0 point 5 is equal to 20 point 89 and e_{31} is equal to e_{30} minus 1 by 2 is equal to 20 point 91 minus 0.5 is equal to 20 point 41.
Here we shall use the formulae: P of x is equal to l of x plus 1 by l of x is equal to e of x by 1 plus e of x plus 1. Taking x equal to 30 we get, l_{31} by l_{30} is equal to e_{30} by 1 plus e_{31} which implies l_{31} is equal to l_{30} into e_{30} by 1 plus expectation 31 is equal to 41 thousand 176 into 20 point 89 by 20 point 41 is equal to 40 thousand 176
The number of persons who die without attaining the age 31 is equal to the numbers of persons dying between the age period 30 and 31 is given by d_{30} is equal to l_{30} minus l_{31} is equal to 41 thousand 176 minus 40 thousand 176 is equal to 1000.

Problem 7:
Fill in the blanks of the following table which are marked with question marks:

Figure 13

Age x	l_x	d_x	q_x	p_x	L_x	e_x^0
20	6,93,435	?	?	?	?	35,081,126
21	6,90,673	-	-	-	-	?

Solution:
Taking age 20 & age 21 as x plus 1 in the usual notations we get d_{20} is equal to l_{20} minus l_{21} is equal to 6 lakh 93 thousand 435 minus 6 lakh 90 thousand 673 is equal to 2 thousand 762.
q_{20} is equal to d_{20} by l_{20} is equal to 2 thousand 762 by 6 lakh 93 thousand 435 is equal 0 point 00398.
Then we calculate p_{20} is equal to 1 minus q_{20} is equal to 1minus 0 point 00398 is equal to 0 point 99602.
L_{20} is equal to l_{20} plus l_{21} by 2 is equal to 69 thousand 435 plus 6 lakh 90 thousand 673 by 2 is

equal to 6 lakh 92 thousand 54.

T_{21} is equal to T_{20} minus L_{20} is equal to 3 crore 50 lakh 81 thousand 126 minus 6 lakh 92 thousand 54 is equal to 3 crore 43 lakh 89 thousand 72.
Next we calculate e_{20} is equal to T_{20} by l_{20} is equal to 3 crore 50 lakh 81 thousand 126 by 6 lakh 93 thousand 435 is equal to 50 point 59.
e_{21} is equal to T_{21} by l_{21} is equal to 3 crore 43 lakh 89 thousand 72 by 6 lakh 90 thousand 673 is equal to 49 point 79.

The following table shows the completion of the values d20, q20, p20, l20, and T 21 as 2 thousand 762, 0 point 00398, 0 point 99602, 6 lakh 92 thousand 54 and 3 crore 43 lakh 89 thousand 72 respectively.

Figure 14

Age x	l_x	d_x	q_x	p_x	L_x	e_x^0
20	6,93,435	2762	0.00398	0.99602	692054	35,081,126
21	6,90,673	-	-	-	-	34389072

Here's a summary of our learning in this session, where we have understood:

- The Calculation of,
  - Fertility Rates
  - Mortality Rates
  - Life table
  - GRR, and
  - NRR