1. Introduction and Life Table (Part-1)

Welcome to the series of E-learning modules on Life Table – Components of life table and uses of life table.

At the end of this session, you will be able to:

- Explain the Life Table
- Explain the Components of life table
- Explain the Uses of life table

Let us start with an Introduction:

For public health purposes the force of mortality in a population is usually measured by means of such indices as crude death rate, infant mortality rate, and specific death rate at different ages by sex, etc.

Another effective and at the same time comprehensive method of describing mortality in a population is by means of life table.
A life table is composed of several sets of values showing how a group of infants all supposed to be born at the same time and experiencing unchanging mortality conditions would gradually die out.

In other words, the life table is a convenient method for summarizing the mortality experience of any population group – that is, it provides concise measures of the longevity of that population.
Such tables are usually worked out after each decennial census to represent mortality conditions either during the previous decennium or during shorter periods covering the date of the census.

Separate tables for males and females are usually prepared.
For detailed study it is not uncommon to construct tables for each geographical subdivision of a country or different population segments.
A life table can also be constructed to show how a group of babies would die if, hypothetically, one or more causes of death are eliminated.

In recent years life table techniques are being increasingly applied to follow up studies of chronic diseases or hospital patients.
The life table gives the life history of a hypothetical group or cohort as it is gradually diminished by deaths.

It is a conventional method of expressing the most fundamental and essential facts about the age distribution of mortality in a tabular form and is a powerful tool for measuring the probability of life and death of various age sectors.

2. Life Table (Part-2)

A life table provides answers for the following questions:

1. How will a group of infants all born at the same time and experiencing unchanging mortality conditions throughout the life time, gradually die out?
2. When in the course of time all these infants die, what would be the average longevity per person?
3. What is the probability that persons of specified age will survive a specified number of years?
4. How many persons, out of selected number of persons living at some initial age, survive on the average to each attained age?

The life table thus, gives a summary of the mortality experience of any population group during a given period and is very effective and comprehensive method for providing concise measures of the longevity of that population.

The data for constructing a life table are the census data and death registration data.
Life tables are generally constructed for various sections of the people which, as experience shows, have sharply different patterns of mortality.

Thus, there are life tables constructed for different races, occupational groups and sex.
Life tables are as well constructed on regional basis and other factors accounting differential mortality.

3. Components of Life Table

We give below a detailed discussion of the various terms and factors that enter in the construction of the life tables.

The components of the life table are:

l of x (Lx) is the number of persons living at any specific age ‘x’ in any year out of an assumed number of births, say, L Nougat (lo) usually called the cohort or radix of the life table.

d of x (dx) is the number of persons among the living persons l of x (lx) (attaining a precise age x) who die before reaching the age (x+1) obviously, we have d of x which is equal to number of persons living at any specific age x (lx) minus number of persons living at any specific age x+1 (l of x+1) is equal to minus delta l of x (-∆lx), where delta is the difference operator.

n p of x (nPx) is the probability that a person aged ‘x’ survives up to age x+n.

Thus, if l of x plus n is the number of persons living at the age x+n in any year, then, the probability of a person aged x is equal to person of living at the age x+n (l of x+n) divided by the person living at age ‘x’ which implies that the person living at age x+n is equal to the product of the person living at x and probability of the person surviving at the age x.

In particular, if we take the value of n is equal to 1 then, we have,

The probability of a person survive at the age of x is the ratio of the person living at the age x+1 divided by the person living at the age x which gives the probability of a person aged x will survive till his next birthday.

q of x (qx) is the complementary probability of survival that is q of x is the probability that a person of exact age x will die within one year following the attainment of that age.
Thus, we have q of x is equal to 1 minus p of x which is equal to d of x divided by l of x also we know that l of x plus 1 is equal to l of x minus d of x which can also be written as l of x into 1 minus q of x which can also be written as l of x into p of x.

Capital L of x (Lx) is the number of years lived in the aggregate by the cohort of l nougat persons between x and (x plus 1) or capital L of x may be interpreted as the average size of the cohort between ages x and x plus 1.
Thus, if deaths are assumed to be uniformly distributed over the whole year or equivalently, if we assume the linearity of l of x plus t for t belongs to (0,1).

Then we get,
Capital l of x is the integration of lower to upper limits tending from zero to one for l of x plus t of d t and l of x plus t is equal to l of x minus td of x.
Then capital L of x is equal to integration of lower to upper limit tending from zero to one of (l of x minus t d of x) into dt is equal to 'l' of x of absolute value of t tending from lower limit equal to zero to upper limit equal to 1 minus d of x into absolute value of 't' square divided by

2 with limits tending from 0 to 1.

By simplifying the equation is now 'l' of x minus one by two of d of x (by substituting t equal to 1 which is the upper limit)   which reduces to 'l'of x minus one by two into (l of x minus l of x+1) which is equal to half of 'l' of x plus 'l' of x plus 1 where Capital L of x is equal to L of x plus half.

T of x (Tx) is the total number of years lived by the cohort l nougat after attaining the age x that is the total future life time of the living persons who reach age x.
Thus, we have t of x is equal to l of x plus l of x plus 1 plus l of x plus 2 and so on,
Thus, if omega is the highest age at which any survivors are recorded in the mortality table,
That is if l omega is equal to zero then, t of x is equal to summation l of x plus I where i is equal to zero and upper limit is equal to omega minus x minus 1. We also observe that t of x is equal to l of x plus t of x plus 1.

4. Relationship Between Quantities

The following theorems establish the relationship between the various quantities defined above.

Theorem 1:

n of P of x is equal to P of x into p of x plus 1 up to P of x plus n minus 1
Proof:
We have by definition that n of p of x is equal to L of x plus n divided by l of x which is equal to l of x plus 1 divided by l of x into l of x plus 2 divided by l of x plus 1 and so on till l of x plus n divided by l of x plus n minus 1 which is equal to p of x into p of x plus 1 and so on till p of x plus n minus 1. Hence proved.

Theorem 2:

N of q of x is equal to d of x plus n minus 1 divided by l of x
Proof:
N of q of x is equal to probability that a person aged x dies in the nth year after attaining that age is equal to probability that person aged x survives till age x plus n minus 1 but dies in age period x plus n minus 1, x plus n which is equal to P of a person aged x survives for n minus 1 years into P of a person aged x plus n minus 1 dies within one year.
By using the compound probability theorem we will have n of q of x is equal to l of x plus n minus 1 by l of x into d of x plus n minus 1 divided b y l of x plus n minus 1 which is equal to d of x plus n minus 1 divided by l of x ,thus proved.

We have a corollary to the same theorem that is n of p of x minus n plus 1 of P of x is equal to l of x plus n by l of x minus l of x plus n plus 1 by l of x which is equal to l of x plus n minus l of x plus n plus 1 by l of x is equal to d of x plus n minus one by l of x which is equal to n plus 1 q of x.

Theorem 3:
If omega is the last age at which l of x vanishes that is if l of omega is equal to zero,
Then, l of x is equal to summation d of I where upper limit is omega minus 1 and I is equal to x.

Proof:
Summation d of I upper limit omega minus 1 where I is equal to x is equal to d of x plus d of x plus 1 plus so on till plus d of omega minus 1 which is equal to (l of x minus l of x plus 1) plus (l of x plus 1 minus l of x plus 2) plus so on till (l omega minus 2 minus l omega minus 1) plus l omega minus 1 minus l omega) is equal to l of x.
Because, l of omega is equal to zero.

Theorem 4:

T of x is equal to half l of x plus l of x plus 1 plus l of x plus 2 and so on
Proof:

By definition t of x is equal to summation l of x plus t with upper limit infinite and t equal to zero which is equal to summation upper limit infinite t equal to zero into half of l of x plus t plus l of x plus t plus 1 is equal to half of l of x plus summation of l of x plus t upper limit infinite t equal to 1.

5. Uses Of Life Tables

Let us now discuss about uses of life table:

Although, the basic objective of life tables is to give a clear picture of the age distribution of mortality in a given population group, it has also been used widely in a large number of spheres.
Today life table is widely accepted as important basic material in demographic and public health studies.
In the words of William Farr, life table is the 'Biometer' of the population.

We enumerate below some of the important applications of life tables:

1. For use by actuaries in Insurance:
   Life tables are indispensable for the solution of all questions concerning the duration of human life.
   These tables, based on the scientific use of statistical methods, are the key stone or pivot on which the whole science of life assurance hinges.

2. Life tables form the basis for determining the rates of premium to be paid by persons of different age groups, for various amount of life assurance. Life tables provide the actuarial science with a sound foundation, converting the insurance business from a mere gambling in human lives to the ability to offer well calculated safeguard in the event of death.

3. For population projections:
   Life tables are used by demographic devise measures such as the 'net reproduction rate' to study the rate of growth of population.
   They have also been used in the preparation of population projections by age and sex that is in estimating what the size of the population will be at some future date.

4. For comparison of different populations:
   Life tables for two or more different groups of population may be used for the relative comparison of various measures of mortality such as death rate, expectation of life at various ages, etc. of particular interest is the comparison of expectation of life the average longevity for members of a population.

5. Life tables are as well used by the government and the private establishments for determining the rates of retirement benefits to be given to its employee for formulating various programs for retired persons.

6. Since a life table depicts the distribution of the people according to age and sex, it is extremely useful in planning in respect of education and for predicting the school going population in connection with the school building programs.

7. Life table is also used,

- For making policies and programs relating to public health by the government and public administration

- To evaluate the impact of family welfare programs on the population growth
- For estimating the probable number of future widows and orphans in a community
- For computing the approximate size of future labour force and military forces, etc
- Study of trends in age distribution of population

8. In addition, life table techniques have been applied to the analysis of other types of demographic data.
   For example: In the computation of probability of marriage, specific age and sex, on the basis of census data classified by marital status.

9. To the student of population, life tables are a valuable instrument.
   They enable him to study population growth and forecast, the size and distribution of population at a future date, under certain assumptions.

However, the accuracy and usefulness of life tables depends mainly upon the accuracy and completeness of the registration of deaths and of the enumeration of the population at the census.
Deficiencies in death registration are likely to be greater than in the census enumeration.

The accuracy and the international comparability of life table values are particularly suspect at the higher ages.
Differences in the methods used for constructing the life table may affect. The reliability of the results can impair their international comparability.
The effect of such difference is however, probably much smaller than that of deficiencies in census and death registration.

Here’s a summary of our learning in this session, where we have understood:

- The Life Table – Components of life table and uses of life table