1. Introduction and Reproduction Rate

Welcome to the series of E-learning modules on Reproduction Rate, Gross Reproduction Rate and Net Reproduction Rate.

By the end of this session, you will be able to:

- Explain the concept of Reproduction Rate, Gross Reproduction Rate and Net Reproduction Rate

Let us start with an introduction on Reproduction Rate:

The fertility rates are unsuitable for giving an idea of the rate of population growth because they ignore the sex of the newly born children and their mortality.
If the majority of the births are those of boys the population is bound to decrease while the reverse will be the case if majority of births are girls.

Similarly, if mortality is ignored, a correct idea of the growth of population cannot be formed because it is possible, a number of female children may die before reaching the child-bearing age.

For measuring the rate of growth of population, we calculate the reproduction rates. Reproduction rates are of two type's Gross reproduction rate (GRR) and Net reproduction rate (NRR).

In order to have a better idea about the rate of population growth, in addition to the age and sex composition of the population we must take into account the sex of the new born children since it is ultimately the female births who the potential future mothers and result in an increase in the population.
The gross reproduction rate is a step in this direction.

2. Gross Reproduction Rate (Part - 1)

Let us discuss on gross reproduction rate:

The gross reproduction rate is defined as the sum of age specific fertility rates calculated from female births for each year of reproductive period.
Gross reproduction rate measures the rate at which a new born female would on an average add to the total female population, if they remained alive and experienced the age specific fertility rate till the end of the child bearing period.

It is the sum of fertility rate till the end of child bearing period.
It is the sum of age specific fertility rates calculated from female births for each single year of age.
It shows the rate at which mothers would be replaced by daughters and the old generation by the new, if no mother died or migrated before reaching the upper limits of the child bearing that is 49 years.

Another underlying assumption is that the same fertility rate continued to be in operation.
If the gross reproduction rate of a population is exactly 1, it indicates that the sex under consideration is exactly replacing itself.
If it is less than 1, the population would decline, no matter how the death rate may be.
And if it is more than 1, the population would increase, no matter how low the death rate may be.

The gross reproduction rate is computed by the following formulae,
GRR is equal to number of female births divided by the total number of births into total fertility rate.
We can also use the formula, GRR is equal to number of female children born to 1000women divided by 1000.

The Gross Reproduction Rate is used as a measure of the fertility in a population.
It is useful for computing fertility in different areas or in the same area at different time periods.
The gross Reproduction Rate could in theory range from 0 to about 5.

Gross reproduction Rate has an advantage over the total fertility rate because,
GRR in its computation take into account only the female babies who are the future mothers.
Whereas, in total fertility rate it include both male and female babies that are born.

An important limitation of the gross reproduction rate is that it ignores the current mortality.
All the girls born do not survive till they reach the child bearing age.
Hence the gross reproduction rate is misleading in that it inflates the number of potential mothers.
This defect is removed by the net reproduction rate.

The accuracy of gross reproduction rate depends on the accuracy with which age specific fertility rates can be computed. The principle sources of errors are under registration of births, mis-statements or inadequate statements of age of mother at registration and errors in enumeration or estimation of the female population by age-group.

3. Gross Reproduction Rate (Part - 2)

Remarks 1:
The computation of Gross Reproduction rate requires the availability of the following data:

i. The classification of the births according to the age of the mother at the time of birth
ii. The sex of the new born babies

Usually, such data are not available.
Hence, an approximate value of gross reproduction rate is obtained under the assumption that sex ratio at birth remains more or less constant at all ages of women in the reproductive period

Thus, we have the following definition that the Sex Ratio is equal to the Number of Female Births divided by Number of Male Births which is equal to a constant.

The same can be represented by a constant ‘b’ not belonging to ‘x’ which is represented by ‘f’ of ‘B’ of ‘x’ divided by ‘m’ of ‘B’ of ‘x’.
Thus, we have ‘f’ of ‘B’ of ’x’ divided by the Sum of ‘f’ of ‘B’ of ‘x’ and ‘m’ of ‘B’ of ‘x’ which is equal to ‘b’ divided by sum of ‘b’ + 1 which is equal to ‘C’
Thus, ‘f’ of ‘B’ of ‘x’ is equal to ‘C’ multiplied by (Sum of ‘f’ of ‘B’ of ‘x’ and ‘m’ of ‘F’ ‘B’ of ‘x’)

Where, ’B’ of ‘x’ is equal to Sum of ‘f’ of ‘B’ of ‘x’ and ‘m’ of ‘B’ of ‘x’ which is the Total number of Births to women of age ‘x’ during the given period in the given region.
Thus, we say that ‘f’ of ‘B’ of ‘x’ divided by ‘B’ of ‘x’ is equal to ‘C’.
‘C’ is equal to ‘f’ of ‘B’ of ‘x’ divided by ‘B’ of ‘x’ which is equal to Summation of x equal to Lambda1 to Lambda2 (‘f’ of ‘B’ of ‘x’) divided by Summation of x equal to Lambda1 to Lambda2 of (‘B’ of ‘x’) which is equal to ‘f’ of ‘B’ divided by ‘B’.

Where, (‘f’ of ‘B’) is the total number of Female Births and B is the total number of births.
Thus, we get (‘f’ of ‘B’ of ‘x’) is equal to ‘f’ of ‘B’ divided by ‘B’ multiplied by ‘B’ of ‘x’

Thus, the Gross Reproductive Rate is equal to ‘f’ of B divided by ‘B’ multiplied by Total Fertility Rate Which is equal to Number of Female births divided by Total number of births multiplied by Total Fertility Rate.

Remarks 2:
As a measure of fertility, Gross reproductive rate is quite useful for comparing the fertility indifferent regions or in the same region at different periods of time. Gross reproduction rate may be regarded as a measure of the extent to which a sex under consideration – female sex in this case is replacing itself, unity being the criterion for exact replacement.

Thus, if the Gross reproductive rate is less than unity, the population would decline no matter how low the death rate may be and the Gross reproductive rate is greater than unity then the population would increase no matter how high the death rate may be.

Remarks 3:
The accuracy of the Gross reproductive rate depends upon the accuracy of the computation of ‘f’ of ‘i’ of ‘x’, the main source of error being,

i. Under registration of births
ii. Under statement or inadequate statements of women’s age at the time of registration
iii. Errors in enumeration or estimates of female population by age group

Remarks 4:
Gross reproductive rate is computed on the hypothesis that one of the newly born female babies is subject to the risk of mortality till the end of the reproductive period of life.
This is very serious limitation of Gross reproductive rate since all the girls born do not survive till the end of child bearing span.

Accordingly, gross reproductive rate leads to fallacious conclusion as it inflates the number of potential mothers.
However, the drawback is to overcome the net reproduction rate which becomes a challenge in certain situations.

4. Net Reproduction Rate

Let us now discuss about Net reproduction rate:

Though, gross reproduction rate gives an idea about the growth of population, it excludes the effect of the mortality on the birth rate.
The rate estimates the average number of daughters that would be produced by women throughout their lifetime if they were exposed at each age to the fertility and mortality rates on which the calculation is based.

It thus indicates the rate at which the number of female births would eventually grow per generation if the same fertility and mortality rates remained in operation.
A net reproduction rate of 1 indicates that on the basis of the current fertility and female mortality, the present female generation is exactly maintaining itself.

Both fertility and mortality are taken into account while calculating net reproduction rate.
In its calculation it is assumed that 1000 mothers give birth to certain number of girls of whom a percentage dies in infancy and certain percentage does not marry.
Of married girls some become widow and it is only the balance who pass through the fertility period and add to the population growth.
The net reproduction rate represents the rate of replenishment of that population.

The net reproduction rate is obtained by multiplying the female specific fertility of each age by the population of female survivors to that age in a life table and adding up the products. An allowance is thus made for mortality.
Symbolically, the net reproduction rate (NRR) is equal to Summation of female births per person at each age with the limit of 15-49 into number of years lived at each age per women (l_x) divided by the original group of females (l_0).
The net reproductive rate can also be represented as summation of number of female births into survival rate divided by 100.

Let us now take into consideration the factor of mortality of mothers also in measuring the growth of population.

To formulate our ideas mathematically, to start with we construct a life table for females on the basis of age specific death rates for females, (fm_x).

The values in the L_x column of the life table, denoted by ‘n’ of ‘f’ into ‘L’ of ‘x’, gives the mean size of the cohort of fL_0 females in the age-interval x to x+n.

In the usual notations, let ’ n’ of ‘f’ into ‘B’ of ‘x’ (${}_nfB_x$) be the number of female births to the women in the age group x to x+n at any period t, (say) then’ n’ of ‘f’ into ‘L’ of ‘x’ (${}_nfL_x$) divided by ‘f ‘ into’ L’ nought (fL_0) into ‘n’ of ’ f’ into ‘B’ of ‘x’ (${}_nfB_x$) gives the average number of female children that would be born to the cohort ‘f’ into ‘L’ nought (fL_0) in the age group x to x+n.

The quantity ‘n’ of ‘f’ pai of ‘x’ (${}_nf\pi_x$) is equal to female living at the age x (${}_nfL_x$) divided by mean size of the cohort females in the age interval (fL_0) gives the life table probability of survival of a

female to the age-interval x to x+n and is called the survival rate.

This implies that out of k newly born female babies k into n of ‘f’ pai x will enter into the child bearing age interval x to x+n.

Hence, instead of multiplying the (female birth at age x divided by the female population at age x) by k alone as in GRR we multiply it by the factor k into f of pai x ($f_n\pi_x$) for each age interval x to x+n.

Finally, a new measure of population growth known as net reproduction rate (NRR) is given by NRR is equal to k summation n into female birth at age x divided by the female population at age x multiply by the survival rate n f of pai x ($_nf\pi_x$) for each age interval x to x+n.

NRR can be rewritten as ‘k’ summation of n into female age specific fertility rate into survival factor for all the age groups of reproductive span.

Remarks 1:
Since NRR takes into account the mortality of the new born female babies we get NRR is less than or equal to GRR with the sign of equality holding if and only if all the new born girls survive at least till the end of the reproductive period.
Thus, GRR provides an upper limit to NRR and hence in theory, NRR also ranges from 0 to 5 per annum.

Remarks 2
It may be pointed out that out of a number of girls born to 1000 women some die in infancy and some do not marry at all.
Of the married woman some become widow and it is only the balance who pass through the fertility period and thus add to the population growth.
Thus, NRR may be interpreted as the rate of replenishment of the population.

Remarks 3
If NRR = 1, we may conclude that if the current fertility and female mortality rates prevail, in future, then a group of new born girls will exactly replace itself in the next generation.
In other words on the average, each of the females in the life table cohort will be replaced by their daughters that is the present female generation will exactly maintain itself.

Thus, in this case the population has a tendency to remain more or less constant.
On the other hand if NRR is greater than unity then the population has a tendency to increase while NRR less than unity indicate a declining population.
Accordingly, NRR may be regarded as a good index of population growth.

Remarks 4
However, even the net reproduction rate as a measure of replacement of population cannot be much relied upon because of the following two reasons:

- It assumes that current mortality and fertility rates prevail in future, an assumption which is not true since in practice both these rates go on changing time to time
- It overlooks the factor migration. The population of a given region in any given period may be depleted more by emigration rather than by declining birth rate or it may increases as a result of fresh stock of immigrants who might be more virile

5. Conclusion and Example

Let us conclude:

In using gross and net reproduction rates as a means of analyzing the applications of observed fertility and mortality rates for future population development, it should be borne in mind that the age specific fertility and mortality rates recorded in a given country at a given time do not actually represent the experience of any real generation of women, and that may be influenced by factors which are by their nature necessarily temporary.

Let us understand the calculation of the gross reproduction rate and the net reproduction rate by taking the following example.
Calculate the gross re-reproduction rate and the net reproduction rate from the given data in the table.

Figure 1

Age-group	Number of children born to 1,000 women passing through age-group	Mortality rate (per 1000)
16-20	150	120
21-25	1500	180
26-30	2000	150
31-35	800	200
36-40	500	220
41-45	200	230
46-50	100	250

Let us prepare the table for the calculations of Gross Reproduction Rate and Net Reproduction Rate in the table the first column represents the age group of the data the second column represents the number of children born to thousand women passing through the age group and third column represents mortality rates per thousand

Sex ratio being males: females :: 52:48

Let us have a look at the solution:

Let us prepare the table for the calculations of Gross Reproduction Rate and Net Reproduction Rate.

Figure 2

Age-group	Number of children born to 1,000 women passing through age-group	Number of female children ${}_nfB_x$	Survival Rate ${}_nf\pi_x$ = 1-(Mortality rate per woman)	Number of female children survived ${}_nfB_x \times {}_nf\pi_x$
16-20	150	150x.48 =72	1-0.12 = 0.88	63.36
21-25	1500	1500x.48=720	1-0.18 = 0.82	590.46
26-30	2000	2000x.48=960	1-0.15 = 0.85	816.00
31-35	800	800x.48=384	1-0.20 = 0.80	307.20
36-40	500	500x.48=240	1-0.22 = 0.78	187.20
41-45	200	200x.48=96	1-0.23=0.77	73.92
46-50	100	100x.48=48	1-0.25=0.75	36.00
Total	**5250**	**2520**		**2074.08**

In the table the first column represents the age group of the data.

The second column represents the number of children born to 1000 women passing through the age group which is 150, 1500, 2000 and so on.

The third column represents the number of female children which is taken as 48% of the number of children born to 1000 women passing through age-group that is we take 48% of 150 is equal to 72, similarly 48% is 1500 is 720 like this we calculate the number of female children for the other age groups as 960, 384, 240, 96 and 48 which gives a total of 2520.

The fourth column represents the Survival rate that is 1 minus the mortality rate per women, the mortality rate is taken from the given table in the problem which is equal to .12, .18, .15, . 20, .22, .23, and .25. then we will get the survival rate as 0.88, 0.82, 0.85, 0.80, 0.78, 0.77 and 0.75.

In the last column we will calculate the number of female children survived which the result of the product of column 3 and column 4 which is equal to 63.36, 590.46, 816, 307.20, 187.20, 73.92 and 36 which gives a total of 2074.08.

Now, we will calculate the gross reproduction rate and net reproduction rate.

The gross reproduction rate per women is equal to total number of female children born divided by 1000 is equal to 2520 divided by 1000 is equal to 2.52.

The net reproduction rate per woman is equal to total number of female children born and survived to 1000 women divided by 1000 is equal to 2074.8 divided by 1000 is equal to 2.07.

Here's a summary of our learning in this session, where we have understood:

- The concept of Reproduction Rate, Gross Reproduction Rate and Net Reproduction Rate