Summary

Logistic curve
This is a particular form of complex types of growth curve. A symmetric logistic curve, also known as Pearl-reed curve is given by y is equal to time series y_t is equal to k divided by 1 plus exponential of (a plus bt), where b is less than zero, where a, b and k are constants and y_t is the value of the time series at the time t. This equation can also be written as y_t is equal to k divided by 1 plus e to the power a into e to the power bt is equal to k by 1 plus e to the power bt where b is less than 0 also from the equation we have, 1 by y_t is equal to 1 by k into 1 plius e to the power of a plus bt. Is equal to 1 by k plus 1 by k into e to the power of a into e to the power of bt is equal to A plus Bc to the power of t, where A is equal to 1 by k, B is equal to 1 by k into e to the power of a, c is equal to e to the power of b, are constants. Thus, the reciprocal of y follows modified exponential law.

Hence, the given time series observations y_t will follow logistic law if their reciprocal 1 by y_t follows modified exponential law.
The principle of least squares cannot be applied to fit the logistic curve to the given set of data. We discuss below the various methods of fitting the logistic curve to the given data.

Method of three selected point:
The given time series data is first placed on a graph paper and a trend line is drawn by the freehand method. Three ordinates y1, y2 and y3 are now taken from the trend line corresponding to selected equidistant points of time, say t is equal to t1 t is equal to t2 and t is equal to t3 respectively such that t2 minus t1 is equal to t3 minus t2. The sum or average or more than one neighbouring values can also be taken with advantage. Values must be equidistant. For population data, geometric mean may be used.

Substituting the values of t is equal to t1, t2 and t3 in the equation we get respectively, y1is equal to k by 1 plus e to the power of a plus bt1, y2 is equal to k by 1 plus e to the power a plus bt2, and y3 is equal to k by 1 plus e to the power a plus bt3 it also implies a plus bt1 is equal to log e of k by y1 minus 1; a plus bt2 is equal to loge of k/y2 minus 1; a plus bt3 is equal to loge of k by y3 minus 1, which implies b into t2 minus t1 is equal to log of k by y2 minus 1 divided by k by y1 minus 1 and b of t3 minus t2 is equal to log k by y3 minus 1 divided by k by y2 minus 1. Since the points are equidistant, that is t2 minus t1 is equal to t3 minus t2, we get log of k by y2 minus 1 divided by k by y1 minus 1 is equal to log k by y3 minus 1 divided by k by y2 minus 1 which implies that (k by y3 minus 1) into (k by y1 minus 1)is equal to(k by y2 minus 1 whole square) which implies y2 square of (k minus y3) into (y1 into y3 of (k minus y2 square) which implies y2 square of (k square minus k(y1 plus y3) plus y1 into y3) is equal to y1 into y3 of (k square plus y2 square minus 2y2) which implies k square (y2 square minus y1 into y3 is equal to k (y2 square of (y1 plus y3) minus 2 y1 into y2 into y3. Since k is not equal to zero k is equal to y2 square (y1 plus y3) minus 2 y1 into y2 into y3 divide by y2 square minus y1 into y3. Also from the above equation we get respectively b is equal to 1 by t2 minus t1 log e of [(k minus y2) into y1 divided by(k minus y1) into y2 and a is equal to log of (k minus y1 by y1) minus bt1.

Yules method:
Let us suppose that the value of k is approximately known or obtained by other methods. Then the logistic curve will contain two parameters a and b, and two variables t and y_t. Hence the principle of least squares can be used to estimate a and b. we will have as a plus bt is equal to log of k by y minus 1 or v is equal to a plus bt where v is equal to k by y minus 1 represents the linear trend between v and t, according to the principle of least squares, the normal equations for estimating a and b are summation v is equal to na plus b summation t and summation t v is equal to a summation t plus b summation t square.

Hotellings method:

A very elegant and ingenious method for fitting a logistic curve is given by Hottelling. We have differentiation of y is equal to minus by (1 minus y by k) which implies 1by y into dy by dt is equal to minus b (1 minus y by k). if the interval is not too large, then as an approximation to 1 by y, dy by dt, we take 1 by y, delta y_t by delta t. thus we get 1 by y_t, delta y_t by delta t is equal to minus b plus b by k into y_t or u is equal to A plus By where U is equal to 1 by y into delta y_t by delta t, A is equal to minus b and B is equal to b by k, where A and B and consequently b and k can be estimated from the equation by the principle of least squares. The constant 'a' is then obtained by assuming that the curve passes through the mean of Y and the mean of t. this is so far the best method, both from the point of simplicity and accuracy. Logistic curve is found very good for measuring the trend of the population data.

Method of successive Approximation:

If some approximate values of the parameter k, a, b are known, then a first correction for each of these can be obtained by the principle of least squares. Let us suppose that first correction for k,a and b are mu, lambda and delta respectively so that y_t is equal to k plus mu by 1 plus exponential [a plus lambda plus (b plus delta) into t] is equal to k plus mu by 1 plus exponential (a plus bt) into exponential (lambda plus delta t). Which is equal to k plus mu by 1 plus e to the power of a plus bt into (1 plus lambda plus delta t), higher posers of lambda and delta being ignored since lambda and delta are very small. Therefore y_t is equal to (k plus mu by 1 plus e to the power of a plus bt) [1 plus (lambda plus delta t0 into e to the power of a plus bt divided by 1 plus e to the power of a plus bt] whole to the power of minus 1 is equal to k plus mu by 1 plus e to the power of a plus bt [1 minus (lambda plus delta t0 into e to the power of a plus bt divided by 1 plus e to the power of a plus bt], when higher powers being neglected. Therefore y_t is equal to k divided by 1 plus e to the power of a plus bt plus mu divided by 1 plus e to the power of a plus bt minus k into lambda e to the power of a plus bt by (1 plus e to the power of a plus bt) whole square minus k into t into delta e to the power of a plus bt by 1 plus e to the power of a plus bt whole square ignoring the terms involving lambda delta and mu delta. The above equation can also be rewritten as say, y_t is equal to A t plus mu Bt plus lambda Ct plus delta Dt where At, Bt, Ct, Dt are known as k, a and b are known. mu lambda and delta can be obtained by the principle of least squares .