1. Computation by Using Method of Monthly Average

Welcome to the series of E-learning modules on Measurement of seasonal variation by the method of ratio to trend.

By the end of this session, you will be able to:
- Explain the computation of the Measurement of seasonal variation by the method of ratio to trend

Let us start with the problems:
Problem 1:
Use the method of monthly averages to determine the monthly indices for the following data of production of a commodity for the year 2002, 2003, 2004

Figure 1

Month	2002	2003	2004	Month	2002	2003	2004
January	12	15	16	July	16	17	16
February	11	14	15	August	13	12	13
March	10	13	14	September	11	13	10
April	14	16	16	October	10	12	10
May	15	16	15	November	12	13	11
June	15	15	17	December	15	14	15

First & fifth column represents the months of respective years.
Second, third, fourth & sixth, seventh, eighth columns represents the production of a commodity for the year 2002, 2003 & 2004 respectively.

Solution:
Computation of seasonal indices as shown below,

Figure 2

Month	2002	2003	2004	Total	Monthly average	Seasonal Index
January	12	15	16	43	14.33	1.048856
February	11	14	15	40	13.33	0.975663
March	10	13	14	37	12.33	0.90247
April	14	16	16	46	15.33	1.122049
May	15	16	15	46	15.33	1.122049
June	15	15	17	47	15.66	1.146203
July	16	17	16	49	16.33	1.195242
August	13	12	13	38	12.66	0.926624
September	11	13	10	34	11.33	0.829277
October	10	12	10	32	10.66	0.780238
November	12	13	11	36	12.00	0.878317
December	15	14	15	44	14.66	1.07301
Total				492	163.97	12
Averages				41	13.6644	1

First, we will take the given data that is, the month and the production of the commodity for each month for the years 2002, 2003, and 2004 in the first four columns of the table.
Next, in the fifth column we will take the totals of the production for each month for all the three years where we will get the values 43, 40, 37, 46, 46, 47, 49, 38, 34, 32, 36, 44 and the total of this coming up to 492. The average of this total will be 492 divided by 12 is equal to 41.

In the sixth column, let us take the monthly averages that is the total value in column 5 divided by 3 as we have considered 3 years period2002, 2003 and 2004 which is equal to 14 point 33, 13 point 33, 12 point 33, 15 point 33, 15 point 33, 15 point 33, 16 point 33, 12 point 66, 11 point 33, 10 point 66, 12, and 14 point 66 and the total coming up to 163 point 97 and its average x bar is 163 point 97 divided by 12 coming up to 13 point 664.
In the last column we will calculate the seasonal index for each month that is monthly average divided by the total average x bar is equal to 1 point 048856, 0 point 97566, 0 point 90247, 1 point 122049, 1 point 122049, 1 point 146203, 1 point 195242, 0 point 926624, 0 point 829277, 0 point 780238, 0 point 878317, 1point 07301 which total up to 12 and the average is 12 divided by 12 is equal to 1.

2. Computation by Using the Ratio to Trend Method

Problem 2:
Using the ratio to trend method determine the quarterly seasonal indices for the following data;

Figure 3

Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
1995	30	40	36	34
1996	34	52	50	44
1997	40	58	54	48
1998	54	76	68	62
1999	80	92	86	82

First column represents the years.
Second, third, fourth & fifth columns represent the quarterly values respectively.

Solution:
First of all we will determine the trend values for the quarterly averages by fitting a linear trend by the method of least squares.

Figure 4

Year (t)	Total of quarterly values	Average of Quarterly values (y)	X= t-1997	X²	xy
1995	140	35	-2	4	-70.00
1996	180	45	-1	1	-45.00
1997	200	50	0	0	0.00
1998	260	65	1	1	65.00
1999	340	85	2	4	170.00
Total		280	0	10	120.00

Let the straight line trend equation be; Y is equal to a plus b of x.
Here the origin year is 1997 for x unit and the quarterly averages the values for y then we will prepare the following table
The first column indicates the values year (t).
In the second column, we will get the total of the quarterly values that is 140, 180, 200, 260

and 340.

In the next column, we will calculate the averages of the quarterly values (y), we will get 35, 45, 50, 65, 85 and summation of y is equal to 280.

In the next column, we will calculate the x which is equal to t – 1997, we will get minus 2, minus 1, 0, one and two, let us square the value of x in the next column, we will get 4, 1, 0, 1 and 4 and summation of x square, we will get 10.

In the next column, we will calculate the product of xy we will get the values minus 70, minus 45, 0 65 and 170 & its summation will be 120.

Now by substituting this values in the equation we will estimate 'a' and 'b'.

Thus, we will get the first equation. Summation y is equal to n a plus b summation x and for the second equation, we will get summation xy is equal to 'a' summation x plus b summation x square.

Now by substituting the values we have calculated the value of 'a' is equal to 56 and the value of b equal to 12.

Hence, the straight line trend is given by the equation Y_c= 56 plus 12x.

Figure 5

Year (t)	Total of quarterly values	Average of Quarterly values (y)	X= t-1997	X²	xy	Trend value
1995	140	35	-2	4	-70.00	32
1996	180	45	-1	1	-45.00	44
1997	200	50	0	0	0.00	56
1998	260	65	1	1	65.00	68
1999	340	85	2	4	170.00	80
Total		280	0	10	120.00	

By substituting the x values, we will get the trend values as 32, 44, 56, 68 and 80. Also from the trend equation we observe that yearly increment in trend values b is equal to 12 which implies quarterly increment is equal to 12 by 4 is equal to 3. The positive value of b implies that we have an increasing trend.

Next, we will determine the quarterly trend values as follows;

Figure 6

2nd quarter of 1995	32-1.5	30.5
3rd quarter of 1995	32 + 1.5	33.5
4th quarter of 1995	33.5 + 3	36.5
1st quarter of 1995	30.5 – 3	27.5

For the year 1995, the average quarterly trend value is 32 which is, in fact, the trend value for the middle quarter that is half of the second quarter and half of the 3rd quarter, of 1995. Since the quarterly increment is 3 we obtain the trend value of the 2nd ,3rd and the 4th quarters of year 1995 as 32 minus 1 point 5 equal to 30 point 5; 32 plus 1 point 5 equal to 33 point 5 and 33 point 5 plus 3 equal to 36 point 5 respectively and consequently the trend value for the first quarter is 30 point 5 minus 3 is equal to 27 point 5.

Figure 7

2nd quarter of 1996	44-1.5	42.5
3rd quarter of 1996	44 + 1.5	45.5
4th quarter of 1996	45.5 + 3	48.5
1st quarter of 1996	42.5 – 3	39.5

Similarly, we will calculate the trend value for the year 1996, the average quarterly trend value is 44 which is, in fact, the trend value for the middle quarter that is half of the second quarter and half of the 3rd quarter, of 1996. Since the quarterly increment is 3 we obtain the trend value of the 2nd, 3rd and the 4th quarters of year 1996 as 44 minus 1 point 5 equal to 42 point 5; 44 plus 1 point 5 equal to 45 point 5and 45 point 5 plus 3 equal to 48 point 5 respectively and consequently the trend value for the first quarter is 42 point 5 minus 3 is equal to 39 point 5.

Figure 8

Year (t)	1st Qrt	2nd Qrt`	3rd Qrt	4th Qrt
1995	27.5	30.5	33.5	36.5
1996	39.5	42.5	45.5	48.5
1997	51.5	54.5	57.5	60.5
1998	63.5	66.5	69.5	72.5
1999	75.5	78.5	81.5	84.5

Similarly, we can get the trend values for other years as given in the following table-for the year 1997 as 51 point 5, 54 point 5, 57 point 5 and 60 point 5, for the year 1998 we will get 63 point 5, 66 point 5, 69 point 5 and 72 point 5, for the year 1999 as 75 point 5, 78 point 5, 81 point 5 and 84 point 5.

The next step we calculate the trend eliminated values, this values are calculated as the percent of trend values.

Figure 9

Year (t)	1st Qrt	2nd Qrt`	3rd Qrt	4th Qrt
1995	109.1	131.1	107.5	93.1
1996	86.1	122.4	109.9	90.7
1997	77.7	106.4	93.9	70.3
1998	85	114.3	97.8	85.5
1999	106	117.1	105.5	97
Total	463.9	591.3	514.6	436.6
Average (total /5)	92.78	118.26	102.92	87.32
Adjusted (Average x k)	92.07	117.36	102.14	86.66

We will get the values as 109 point 1; 131 point 1, 107 point 5 and 93 point 1 for the year 1995.
86 point 1; 122 point 4; 109 point 9 and 90 point 7 for the year 1996.
77 point 7; 106 point 4; 93 point 9 and 70 point 3 for the year 1997.
85, 114 point 3; 97point 8 and 85 point 5 for the year 1998 and
106; 117 point 1; 105 point 5 and 97 for the year 1999.
We get the totals of each quarter, in the first quarter we have 463 point 9, in the second quarter 591 point 3, in the third quarter 514 point 6 and in the fourth quarter 436 point 6.
Next we will take the average of the seasonal indices for each quarter by dividing the total by 5, we get 92 point 78; 118 point 26; 102 point 92 and 87 point 32 respectively.
The indices obtained above are adjusted to a total of 400 (since the sum of indices is equal to 403 point 08, which is greater than 400). By multiplying each of them by a constant factor k called correction factor given by k is equal to 400 by sum of indices is equal to 400 by 403 point 08 is equal to 0 point 9924. We get the adjusted seasonal indices as 92 point 07; 117 point 36; 102 point 14 and 86 point 66.

3. Computation by Using the Ratio to Moving Average Method

Problem 3:
Calculate seasonal indices by the ratio to moving average method from the following data:
First column represents quarters.

Figure 10

Quarter/Year	2001	2002	2003	2004
Q_1	75	86	90	100
Q_2	60	65	72	78
Q_3	54	63	66	72
Q_4	59	80	85	93

2nd ,3rd ,4th ,5th column represents the price in the year 2001,2002,2003,2004 respectively.

Solution:
Assuming multiplicative model of time series, the trend (Moving Average) values are eliminated by expressing the given values (y of t) as a percentage of trend values and are given in the sixth column of the table.

Figure 11

Year-Qrt	Price (yt)	4 quarterly moving totals	2-period of moving total	4 quarter moving average	Ratio to MA
2001 I	75				
II	60				
III	54	248			
IV	59	259	507	63.375	85.2071
2002 I	86	264	523	65.375	90.2486
II	65	273	537	67.125	128.1192
III	63	294	567	70.875	91.7108
IV	80	298	592	74	85.1351
2003 I	90	305	603	75.375	106.1360
II	72	308	613	76.625	117.4551
III	66	313	621	77.625	92.7536
IV	85	323	636	79.5	83.0189
2004 I	100	329	652	81.5	104.2945
II	78	335	664	83	120.4819
III	72	343	678	84.75	92.0354
IV	93				

The first column shows the year and quarter, the second column is Price, the third column is the 4 quarterly moving totals where we will get 248, 259, 264, 273, 294, 298, 305, 308, 313,

323, 329, 335 and 343.

In the next column we will calculate the 2 period of the moving totals, we will get 507, 523, 537, 567, 592, 603, 613, 621, 636, 652, 664 and 678.

The next column indicates the 4 quarter moving average, we get the values by dividing the values of column 4 by 8 we will get 63 point 375; 65 point 375; 67 point 125; 70 point 875; 74; 75 point 375; 76 point 625; 77 point 625; 79 point 5; 81 point 5; 83 and 84 point 750.

In the sixth column we will calculate the ratio to moving average that is the values in column 2 divided by the values in column 5 into 100 so we will get 54 divided by 63 point 375 into 100 is equal to 85 point 2071, similarly, we will get all the other values as 90 point 2485; 128 point 1192; 91 point 7108; 85 point 1351; 106 point 1360; 117 point 4551; 92 point 7536; 83 point 0189; 104 point 2945; 120 point 4819 and 92 point 0354.

Let us calculate the seasonal indices through multiplicative model.

Figure 12

Year	1 Qrt	2 Qrt	3Qrt	4 Qrt	
2001			85.2071	90.2486	
2002	128.1192	91.7108	85.1351	106.1360	
2003	117.4551	92.75362	83.0189	104.2945	
2004	120.4819	92.0354			
Total	366.0563	276.4998	253.3611	300.6790	
Average (total / 3)	122.019	92.167	84.454	100.226	398.865
Adjusted (Average * k)	122.3604	92.42467	84.69017	100.507	399.982 = 400

The values in the column 6 of the previous table is taken as the values for the quarters in the trend elimination value table then we take the total of each quarter that is, first quarter is 366 point 0562, second quarter is 276 point 4998, third quarter is 253 point 3611 and fourth quarter is 300 point 6790.

Next we will calculate the average by dividing the totals by 3 we will get 122 point 0187; 92 point 166; 84 point 6902 and 100 point 226 whose total is equal to 398 point 865.

The seasonal indices obtained as average (AM) above are adjusted to a total of 400, by multiplying each of them by a constant factor k which is calculated as k is equal to 400 divided by sum of seasonal indices is equal to 400 divided by 398 point 865 is equal to 1 point 0028, then we get the adjusted seasonal indices as 122 point 3603; 92 point 4246, 84 point 6902, 100 point 5066 is equal to 399 point 985 is equal to 400.

If we assume the additive model of the time series, then the trend eliminate values, (short-term and irregular fluctuations), are obtained on subtracting the trend (MA) values from the given time series values (y of t) by the formulae: Short –term fluctuations is equal to y of t – MA values is equal to S+I.

Figure 13

Year	1 Qrt	2 Qrt	3Qrt	4 Qrt	
2001			-9.375	-6.375	
2002	18.875	-5.875	-11.0000	4.6250	
2003	13.375	-5.625	-13.5000	3.5	
2004	17.000	-6.750			
Total	49.25	-18.25	-33.875	1.7500	
Average	16.417	-6.083	-11.292	0.583	-0.375
Adjusted	16.511	-5.989	-11.198	0.677	0.001 = 0

So the trend eliminated values are minus 9 point 375, minus 6 point 375; 18 point 875; minus 5 point 875; minus 11; 4 point 625; 13 point 375, minus 5 point 625; minus 13 point 500; 3 point 5; 17 and minus 6 point 750.

We get the total 49 point 25; minus 18 point 25, minus 33 point 875 and 1 point 750. Then we calculate the average by dividing the totals by 3 we will get 16 point 417; minus 6 point 083; minus 11 point 292 and 0 point 583 whose grand total is equal to minus 0 point 375.

Next we calculate the adjusting seasonal indices from the averages to a total zero by subtracting from each of them the constant factor k. the constant factor k is equal to sum of seasonal indices by 4 is equal to minus 0 point 375 by 4 is equal to minus 0 point 094. Thus the adjusted seasonal indices is equal to 16 point 511; minus 5 point 989; minus 11 point 198 and 0 point 677 is equal to 0 point 001 is equal to 0.

4. Computation by Using Moving Average Method

Problem 4:
Apply ratio to moving average method to ascertain seasonal indices from the following data:

Figure 14

Year & Month 2002	No.of persons visiting a place of interest	Year & Month 2003	No. of perons visiting a place of interest	Year & month 2004	No. of persons visiting a place of interest
Jan	90	Jan	100	Jan	110
Feb	85	Feb	89	Feb	93
Mar	70	Mar	74	Mar	78
April	60	April	62	April	66
May	55	May	55	May	58
June	45	June	47	June	40
July	30	July	30	July	35
August	40	August	43	August	45
Sept	70	Sept	65	Sept	72
Oct	120	Oct	127	Oct	130
Nov	115	Nov	118	Nov	118
Dec	118	Dec	120	Dec	124

1st, 3rd & 5th column represents the year of 2002,2003 & 2004 respectively.
2nd,4th & 6th column represents the number of persons visiting a place of interest.

Solution:

Figure 15

Year & Month	No.of persons visiting a place of interest	12 point moving totals	12 point M A	12 point MA Centered	Ratio to MA
2002 Jan	90				
Feb	85				
Mar	70				
April	60				
May	55				
June	45	898	74.83	75.3	39.9
July	30	908	75.67	75.8	52.8
August	40	912	76.00	76.2	91.9
Sept	70	916	76.33	76.4	157.1
Oct	120	918	76.50	76.5	150.3
Nov	115	918	76.50	76.6	154.0
Dec	118	920	76.66	76.7	130.4
2003 Jan	100	920	76.66	76.8	115.9
Feb	89	923	76.91	76.7	96.5
Mar	74	918	76.50	76.8	80.7
April	62	925	77.16	77.2	71.2
May	55	928	77.33	77.4	60.7
June	47	930	77.50	77.9	38.5
July	30	940	78.33	78.5	54.8
August	43	944	78.66	78.8	82.5
Sept	65	948	79.00	79.2	160.4
Oct	127	952	79.33	79.5	148.4
Nov	118	955	79.58	79.3	151.3
Dec	120	948	79.00	79.2	138.9
2004 Jan	110	953	79.41	79.5	117.0
Feb	93	955	79.58	79.9	97.6
Mar	78	962	80.16	80.3	82.2
April	66	965	80.41	80.4	72.1
May	58	965	80.41	80.6	49.6
June	40	969	80.75		
July	35				
August	45				
Sept	72				
Oct	130				
Nov	118				
Dec	124				

Let us compute the moving average for the given data, the first column indicates the year and month.

The second column indicates the values for the number of persons visiting the places of interest.

In column 3, we need to calculate the 12 point moving totals for which we take the values of the first twelve year and then add it up in this case we take the values for the first twelve months of the year 2002 which is equal to 898 and is placed between the month June and July of the year 2002.

Similarly, the other 12 point moving totals are calculated and we get values as 908, 912, 916, 918, 918, 920, 920, 923, 918, 925, 928, 930, 940, 944, 948, 952, 955, 948, 953, 955, 962, 965, 965 and 966.

In the fourth column, we take the average of the 12 point moving totals, this is calculated by taking the values in the third column divided by 12 which is equal to 74 point 83; 75 point 67; 76; 76 point 33; 76 point 5; 76 point 5; 76 point 66; 76 point 66; 76 point 91; 76 point 5; 77 point 16; 77 point 33; 77 point 5; 78 point 33; 78 point 66; 79; 79 point 33; 79 point 58; 79; 79 point 41; 79 point 58; 80 point 16; 80 point 41; 80 point 41 and 80 point 75.

Fifth column shows the centered values of the 12 point moving averages. In the last column we calculate the ratio to the moving averages which is equal to the values of the 2 column divided by the values in the fifth column into 100, we will get the following values they are 39 point 8; 52 point 8; 91 point 9; 157 point 1; 150 point 3; 154; 130 point 4; 115 point 9; 96 point 5; 80 point 7; 71 point 2; 60 point 7; 38 point 5; 54 point 8; 82 point 5; 160 point 4; 148 point 4; 151 point 3; 138 point 9; 117; 97 point 6; 82 point 2; 72 point 1 and 49 point 6.

As a continuation, we will calculate the adjusted seasonal indices. For this the following table is prepared

Figure 16

Month	2002	2003	2004	Average	Adjusted
Jan		130.4	138.9	134.7	135
Feb		115.9	117.0	116.5	117.0
Mar		96.5	97.6	97.1	97.0
Apr		80.7	82.2	81.5	81.7
May		71.2	72.1	71.7	72.0
Jun		60.7	49.6	55.2	55.0
Jul	39.9	38.5		39.2	39.0
Aug	52.8	54.8		53.8	54.0
Sep	91.9	82.5		87.1	87.0
Oct	157.1	160.4		158.8	159.0
Nov	150.3	148.4		149.4	150.0
Dec	154.0	151.3		152.7	153.0
Total				1197.4	1200

The first column of the table represents the names of the month. In the second, third and the fourth column we have the ratio to moving average values of the year 2002, 2003, 2004 taken from the previous table prepared.

Fifth column are the seasonal indices values calculated by taking the arithmetic averages of the ratio to moving average of each month, the values are 134 point 7; 116 point 5; 97 point 1; 81 point 5; 71 point 7; 55 point 2; 39 point 2; 53 point 8; 87 point 1; 158 point 8; 149 point 4;

152 point 7 and the total being 1197 point 4.

The next step is to calculate the adjusted seasonal indices which is given by the multiplying the seasonal indices into the correction factor. The correction factor 'k' is obtained by dividing 1200 by 1197 point 7 which is equal to 1 point 0021. Now the adjusted seasonal indices is calculated by taking the correction factor 1 point 0021 and multiplying the seasonal indices, we will get the values 135, 117, 97, 82,72,55, 39, 54, 87, 159, 150, 153 and the total is equal to 1200.

5. Computation by Using the Link Relative Method

Problem 5:
Compute the seasonal indices by the 'link relatives' method for the adjoining data relating to the average quarterly prices (Rs. Per.kg.) of a commodity for five years:

Figure 17

Quarter/ Year	1996	1997	1998	1999	2000
I	30	35	31	31	34
II	26	28	29	31	36
III	22	22	28	25	26
IV	36	36	32	35	33

1st column represents the quarter.
$2^{nd}, 3^{rd}, 4^{th}, 5^{th}$ & 6th column represents values for the period 1996,1997,1998,1999,2000 respectively.

Solution:
Let us calculate the seasonal indices by the link relative method.

Figure 18

Year	1 Qrt	2 Qrt	3 Qrt	4 Qrt
1996		86.7	84.6	163.6
1997	97.2	80	78.6	163.6
1998	86.1	93.5	96.6	114.3
1999	96.9	100.0	80.6	140.0
2000	97.1	105.9	72.2	126.9
Total	377.4	466.1	412.6	708.5
Average	94.34	93.22	82.52	141.70
Chain relative	100	93.22	76.93	109.00
Adjusted CR	100	92.52	75.53	106.90
Seasonal Indices	106.68	98.70	80.57	114.04

The link relative for any month is calculated by taking the current month's value dividing it by previous month's value into 100.
So we get the values for the first quarter as 97 point 2; 86 point 1; 96 point 9 and 97 point 1.

The values for the second quarter are 86 point 7; 80; 93 point 5; 100 and 105 point 9.
The values for the third quarter are 84 point 6; 78 point 6; 96 point 6; 80 point 6 and 72 point 2.
The fourth quarter values are 163 point 6;163 point 6; 114 point 3; 140; 126 point 9.
Then we take the totals of the link relatives for all the four quarters they are 377 point 4; 466.1; 412 point 6 and 708 point 5.
In the next row we will calculate the arithmetic average of the link relatives for which we take the totals and divide it by the number of data that is we take 377 point 4 divided by 4, thus, we get 94 point 34, similarly the other values are calculated & the values are 93 point 22; 82 point 52 and 141 point 70.
The next calculation is about the chain relatives which is calculated by considering the first quarter as 100, then in the second quarter we will take the product of the average link relative and the chain relative and divided it by 100 so we will get 93 point 22 into 100 divided by 100 is equal to 93 point 22, for the third quarter we will get 82 point 52 into 93 point 22 divided by 100 is equal to 76 point 93 and for the fourth quarter we will get 141 point 7 into 76 point 93 divided by 100 is equal to 109.

Next we will calculate the new second chain relative for the 1st quarter which is equal to link relative of the 1st quarter into chain relative of the last (4th) quarter by 100 is equal to 109 into 94 point 34 by 100 is equal to 102 point 8. Using this value we calculate the correction factor.

The formulae for calculating the correction factor is the correction factor is equal to one fourth of (second new CR minus 100) which is equal to 102 point 8 – 100) is equal to 2 point 8 divided by 4 is equal to 0 point 7.
The correction factor is multiplied to the chain relative to get the adjusted chain relatives so we get the values 100; 92 point 52; 75 point 53 and 106 point 90.

when we calculate the average of the adjusted chain relatives we take the total of the adjusted chain relative that is 374 point 95 and divided it by 4, as it is quarters we have considered, we will get 93 point 74, using this value we calculate the seasonal indices which is obtained by dividing the adjusted chain relative by 93 point 74 into 100, we will get the values 106 point 68; 98 point 70; 80 point 57 and 114 point 04.

Here's a summary of our learning in this session, where we have understood:
   • The computation of the measurement of seasonal variation by the method of ratio to trend