1. Computation of the Cost of Living Index using the Aggregate Expenditure Method and the Family Budget Methods

Welcome to the series of E-learning modules on Computation of Index numbers using averages of relatives, aggregative methods, and consistency tests.

At the end of this session, you will be able to:

- Explain the Computation of Index numbers using,
  - Averages of relatives
  - Aggregative methods
  - Consistency tests

Let us start with introduction problem:

Problem 1:
Compute the cost of living Index Number using both the aggregate expenditure method and the family budget method, from the following information:

Figure 1

Commodity	Unit consumption in base year	Price in base year	Price in current year
Wheat	200	1.00	1.20
Rice	50	3.00	3.50
Pulses	50	4.00	5.00
Ghee	20	20.00	30.00
Sugar	40	2.50	5.00
Oil	50	10.00	15.00
Fuel	60	2.00	2.50
Clothing	40	15.00	18.00

First column includes various commodities.
Second column includes unit consumption in base year.
Third column & fourth column includes price in base year and price in current year respectively.

Solution :
First let us calculate the cost of living index by the aggregate expenditure method for which we prepare the following table.

Figure 2

Commodity	Unit consumption in base year (q_0)	Price in base year(P_0)	Price in current year (P_1)	($P1\ q_0$)	($P_0\ q_0$)
Wheat	200	1.00	1.20	240	200
Rice	50	3.00	3.50	175	150
Pulses	50	4.00	5.00	250	200
Ghee	20	20.00	30.00	600	400
Sugar	40	2.50	5.00	200	100
Oil	50	10.00	15.00	700	500
Fuel	60	2.00	2.50	150	120
Clothing	40	15.00	18.00	720	600
Total				**3085**	**2270**

In the first column we represent the commodities given in the problem, wheat, Rice, Pulses, ghee, sugar, oil, fuel and clothing.
Column two represents the quantity of consumption in the base year denoted as ‘q’ naught which is the values as given in the table in the problem.
Column three represents the Price of the commodities on the base year denoted by ‘P’ naught and used the same values as in the table given.
Column four is the price of the commodity in the current year and is denoted by ‘P’ one and the values are as same as the given table.
In column five, we calculate the values of the product of P one and q naught we get the values 240, 175, 250, 600, 200, 750, 150 and 720 and the total of this column, summation P one q naught is equal to three thousand eighty five.
Similarly, when we take the product of P naught and q naught we get 200, 150, 200, 400, 100, 500, 120 and 600 and the total of this column, summation of P naught and q naught is equal to two thousand two hundred seventy.

Let us take the formula for aggregate expenditure method.
Cost of living index is equal to summation of P one q naught divided by summation P naught q naught into 100 is equal to three thousand eighty five divided by two thousand two hundred seventy into 100 is equal to 135 point 9.

Let us now calculate the cost of living index by family budget method for which we prepare the following table.

Figure 3

Commodity	Unit consumption in base year (q_0)	Price in base year(P_0)	Price in current year (P_1)	$P = P_1 / q_0$	$V = P_0 \times q_0$	PV
Wheat	200	1.00	1.20	120	200	24,000
Rice	50	3.00	3.50	116.67	150	17500
Pulses	50	4.00	5.00	125	200	25000
Ghee	20	20.00	30.00	150	400	60000
Sugar	40	2.50	5.00	200	100	20000
Oil	50	10.00	15.00	150	500	75000
Fuel	60	2.00	2.50	125	120	15000
Clothing	40	15.00	18.00	120	600	72000
Total					2270	308500

First column represents the commodities like wheat, Rice, Pulses, Ghee, Sugar, Oil, Fuel, and Clothing.
The second column represents the quantity consumed in the base year as per the value in the table.
The third column represents the price of the commodities in the base year.
The fourth column represents the price in the current year.
In the fifth column we will calculate the price relativity P. That is the price relativity is equal to P one by q naught into 100 we get the values 120, 116.67, 125, 150, 200, 150, 125 and 120.
In the sixth column, we have v which is the product of P naught and q naught which is equal to 200, 150, 200, 400, 100, 500, 120 and 600 and the summation of v is equal to two thousand two hundred seventy.
The seventh column is the product of P and V which is equal to twenty four thousand, seventeen thousand five hundred, twenty five thousand, sixty thousand, twenty thousand, seventy five thousand, fifteen thousand and seventy two thousand and the total of this column summation PV is equal 3 lack eight thousand & five hundred.

Next, we calculate the cost of living index using the family budget method.

Cost of living index is equal to summation PV by summation V is equal to three lakh eight thousand & five hundred by two thousand two hundred seventy is equal to 135 point 9.

Note: The Cost of living index is the same by both the methods.

2. Computation of the Index Number to Show the Changes

Problem 2:
The following table gives the annual income of a teacher and the general index number of price during 2001-2009.

Figure 4

Year	Income(Rs'000)	Price Index no.
2001	360	100
2002	420	104
2003	500	115
2004	550	160
2005	600	280
2006	640	290
2007	680	300
2008	720	320
2009	750	330

Prepare the index number to show the changes in the real income of the teacher.
First column represents the year.
Second column represents income which is in thousands.
Third column represents price index number.

Solution:
Let us prepare a table to see the changes in the real income of the teacher.

Figure 5

Year	Income (Rs'000)	Price Index No.	Real Income (Rs. '000)	Real Income Index No.
2001	360	100	360	100.00
2002	420	104	403.85	112.18
2003	500	115	434.78	120.77
2004	550	160	343.75	95.49
2005	600	280	214.29	59.52
2006	640	290	220.69	61.30
2007	680	300	226.67	62.96
2008	720	320	225	62.52
2009	750	330	227.27	63.13

In the first column, we have represented the year.
Second column shows the income of the teacher in thousands.
In the third column, we have shown the price index number,
In the fourth column, we have calculated the real income in thousands by taking the income divided by the price index into 100 and we get the real income as three sixty, four hundred three point eight five, four thirty four point seven eight, three forty three point seven five, two fourteen point two nine, two twenty point six nine, two twenty six point six seven, two twenty five and two twenty seven point two seven.
And the last column indicates the real income index number hundred, one hundred twelve point one eight, one twenty point seven seven, ninety five point four nine, fifty nine point five two, sixty one point three zero, sixty two point nine six, sixty two point five two and sixty three point one three.

Problem 3:
In a working class budget enquiry in towns A and B, it was found in 2004 that an average working class family's expenditure on food and other items was as follows:

Figure 6

Particulars	Town A	Town B
Food	64%	50%
Other items	36%	50%

Family's expenditure of town-A on food and other items are 64 % & 36 % respectively and of town-B are 50 % & 50 % respectively.

In 2007 the consumer price index stood at 279 for town A and 265 for town B (base year = 100). It was known that the rise in the prices of all articles consumed by the working class was the same for A and B. what was the 2007 index for (a) food and (b) other items?

Solution:
Suppose the index number in 2007 for food for both the towns A and B is X and that of other items is Y.

For town A, total of weighted relatives would be:
Food X into 64 equal to 64X; other items Y into 36 equal to 36Y.
But it is already given in the problem that town A is equal to 279 into 100 equal to twenty seven thousand nine hundred and 64X plus 36Y is equal to twenty seven thousand nine hundred which is equation 1.

Similarly, for town B, we have the total weighted relatives which would be Food X into 50 is equal to 50X and other items Y into 50 is equal to 50Y, We also know that in the problem that 265 into 100 is equal twenty six thousand five hundred and hence 50X plus 50 Y is equal to y six thousand five hundred which is equation 2.

Now by taking both the equations, 64X plus 36y is equal to twenty seven thousand nine hundred and 50X plus 50 Y is equal to twenty six thousand five hundred.
Now let us cross multiply the first equation by 50 and the second equation by 64.

We will get the new set of equations three thousand two hundred X plus thousand eight hundred Y is equal to thirteen lakh ninety five thousand and also we get the second equation as three thousand two hundred X plus three thousand two hundred Y is equal to sixteen lakh ninety six thousand.
By subtracting both the equations we get, minus thousand four hundred Y is equal to minus three lakh and thousand.
Thus, value of Y is equal to three lakh and thousand by thousand four hundred is equal to two fifteen.

Now, by substituting this value in the equation 64X plus 36Y is equal to twenty seven thousand nine hundred we get, 64X plus 36 into 215 is equal to twenty seven thousand nine hundred which is equal to 64X plus seven thousand seven hundred forty is equal to twenty seven thousand nine hundred thus, 64X is equal to twenty seven thousand nine hundred minus seven thousand seven hundred forty is equal to twenty thousand one sixty thus, X is equal to twenty thousand one sixty by sixty four is equal to three hundred fifteen.
Thus, the index for food is three hundred fifteen and for other items is two hundred fifteen.

3. Computation of the Fisher's Ideal Index Number and Time Reversal Test & Factor Reversal Test

Problem4
Calculate fisher's ideal index number from the following data and prove that it satisfies both time reversal test and factor reversal test.

Figure 7

	2008		2009	
Commodity	**Price**	**Expenditure**	**Price**	**Expenditure**
A	8	80	10	120
B	10	120	12	96
C	5	40	5	50
D	4	56	3	60
E	2	100	25	150

First column represents the commodities.
Second & third column represents the price & expenditure of 2008 period.
Fourth & fifth column represents the price & expenditure of 2009 period.

Solution:
First let us prepare the table for calculating the various values needed for the solution as shown below.

Figure 8

	2008		2009					
Commodity	P_0	q_0	P_1	q_1	P_1q_0	P_0q_0	P_1q_1	P_0q_1
A	8	80	10	120	800	640	1200	960
B	10	120	12	96	1440	1200	1152	960
C	5	40	5	50	200	200	250	250
D	4	56	3	60	168	224	180	240
E	2	100	25	150	2500	200	3750	300
Total					**5108**	**2464**	**6532**	**2710**

In the first column, we have the commodities, the second column we have the base year price (P naught), the third column we have the, base year quantity (q naught) in the fourth and the fifth column we have the price and quantity of the current year.
The sixth column indicates the product of P one q naught and the total is equal to five thousand one hundred eight.
The seventh column indicates the values of P naught q naught & the total is equal to two thousand four sixty four.
The eighth column gives the product of P one q one & the total is equal to six thousand five thirty two and the last column gives the value of P naught q one is equal to two thousand seven ten.

Substituting the values in the formulae for finding the index we will get, square root of five thousand one hundred eight by two thousand four sixty four into six thousand five thirty two by two thousand seven hundred ten into 100 which is equal to square root of two point zero seven three into two point four one zero into 100 is equal to square root of four point nine nine five nine three into 100 is equal to two point two three five into 100 which is equal to two hundred twenty three point five as the index number.

Now we will test the consistency by using the time reversal test and the factor reversal test.
By substituting the values in the product of P naught, p one and q naught, q one.
We will get as the square root of five thousand one hundred eight by two thousand four sixty four into six thousand five thirty two by two thousand seven hundred ten into two thousand seven hundred ten by six thousand five thirty two into two thousand four sixty four by five thousand one hundred eight which is equal to square root of 1 is equal to 1.
Hence time reversal test is satisfied

Next we will substitute the values to check the factor reversal test where we will take the products of P naught, p one and q naught, q one.
We will get it as the square root of five thousand one hundred eight by two thousand four sixty four into six thousand five thirty two by two thousand seven hundred ten into two thousand seven hundred ten by two thousand four sixty four into six thousand five thirty two by five thousand one hundred eight.
Which is equal to six thousand five thirty two by two thousand four sixty four. Hence, the above data has satisfied the factor reversal test.

4. Computation of Index Number using Simple Average of Relatives Method, Laspeyer's Method, Paasche's Method and Marshall Edgeworth's Methods

Problem 5
Using simple Average of relative's method, compute index number for the year 1997 using 1990 prices as base.

Figure 9

Commodities	I	II	III	IV	V	VI
Prices in 1990 (P_0)	15	18	16	14	25	40
Prices in 1997 (P_1)	30	24	20	21	35	30

In the table we will see that the first row is the commodities the second row shows the prices in 1990 and the third row shows the prices in the year 1997.

Solution:
Let us calculate the index number using the arithmetic mean:

Figure 10

Commodities	I	II	III	IV	V	VI	N=6
Prices in 1990 (P_0)	15	18	16	14	25	40	
Prices in 1997 (P_1)	30	24	20	21	35	30	
Price relative (P_1/P_0) X100	200	133.33	125	150	140	75	$\Sigma(P_1/P_0 \times 100) = 823.33$

In the table we will see that the first row is the commodities, the second row shows the prices in 1990 and the third row shows the prices in the year 1997.

Let us calculate the price relative of each commodity in row four, price relative is equal to current price (P one) divided by (P naught) multiplied by 100, while substituting the values for each commodities we will get the values two hundred, one thirty three point three three, one twenty five, one fifty, one forty and seventy five.
Take the total of the price relative column to get summation of P one divided by P naught multiplied by 100 which is equal to eight twenty three point three three.

Figure 11

Commodities	**I**	**II**	**III**	**IV**	**V**	**VI**	**N=6**
Prices in 1990 (P_0)	15	18	16	14	25	40	
Prices in 1997 (P_1)	30	24	20	21	35	30	
Price relative (P_1/P_0) X100	**200**	**133.33**	**125**	**150**	**140**	**75**	**$\Sigma(P_1/P_0$ x100)= 823.33**

The total number of commodities in this data is 6 therefor, N is equal to 6.
Let us calculate the value of the price index by substituting the values in the formula summation P one by P naught multiplied by 100 divided by N which is equal to eight twenty three point three three divided by 6 is equal to one hundred thirty seven point two two.

Problem 6:
Construct the index numbers of price from the following data by applying laspeyer's method, Paasche's method and Marshall edgeworth's method.

Figure 12

Year	**2009**		**2010**	
Commodity	Price (P_0)	Quantity(q_0)	Price (P_1)	Quantity (q_1)
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

The first column will show us the commodities.
Second and third columns are the price and quantity in the year 2009 & the fourth and the fifth column is the price and quantity of the commodities for the year 2010.

Solution:
As a first step let us prepare the table with the products of the prices and the quantities in various combinations.

Figure 13

Year	2009		2010					
Commodity	Price (P_0)	Quantity (q_0)	Price (P_1)	Quantity (q_1)	P_1q_0	P_0q_0	P_1q_1	P_0q_1
A	2	8	4	6	32	16	24	12
B	5	10	6	5	60	50	30	25
C	4	14	5	10	70	56	50	40
D	2	19	2	13	38	38	26	26
Total					200	160	130	103

The first column will show us the commodities, the second and third columns are the price and quantity in the year 2009 which we will consider as the base year and hence name it as P naught and q naught.
The fourth and the fifth column is the price and quantity of the commodities for the year 2010 considered as the current year and hence denoted as P one and q one.
The sixth column is the result of the product of the price of the current year(P one) with the quantity of the base year (q naught) that is 32, 60, 70 and 38 which gives the summation of P one q naught is equal to 200.
In the seventh column we are taking the product of the price and quantity of the base year that is P naught and q naught we will get 16, 50, 56 and 38 which gives the summation of P naught Q naught is equal to 160.
In the eight column we are taking the product of the price and quantity of the base year that is P one and q one, we will get 24, 30, 50 and 26 and we get the summation of P one Q one is equal to 130.
And in the last column we are taking the product of P naught and q one, we will get 12, 25, 40 and 26 which gives the summation of p naught and q one is equal to 103.

In the next step we will calculate the index numbers in each of the methods,
Laspeyers method:
Price index is equal to summation P one q naught divided by summation P naught q naught multiplied by 100 which is equal to 200 divided by 160 into 100 is equal to 125 (values taken from the table).

Paasche's method:
Price index is equal to summation P one q one divided by summation of P naught q one into 100 is equal to 130 divided by 103 into 100 is equal to 126.21 (values taken from the table)

Marshall edgeworth's method:
Price index is equal to summation of P one q naught plus summation of p one q one divided by summation of P naught q naught plus summation of P naught q one into 100 is equal to 200 plus 130 divided by 160 plus 130 is equal to 330 by 263 into 100 is equal to 125.48.

As mentioned, we can see that when there is not much variation between the base year and the current year the index numbers on an average will not show a huge difference in various methods we can see here that the price index is on an average close to 125.

5. Computation of the Price Index by Applying the Weighted Average of Price Relative's Method using, Arithmetic Mean & Geometric Mean

Problem 7:
From the following data compute the price index by applying the weighted average of price relative's method using (a) arithmetic mean and (b) geometric mean.

Figure 14

Commodities	P_0 (Rs.)	q_0	P_1
Sugar	3	20 Kg	4
Flour	1.5	40 Kg	1.0
milk	1.0	10 Lt	1.5

As shown in the table given below, first column indicates the commodities, the second column indicates the price in the base year, the third column indicates the various quantities of the items in the base year, the fourth column indicates the price of the items in the current year.

Solution:
Let us first calculate the weighted arithmetic mean of price relatives.
For this let us prepare the table for the required contents.

Figure 15

Commodities	P_0 (Rs.)	q_0	P_1	$V= P_0 x q_0$	$P= (P_1/p_0) \times 100$	PV
Sugar	3	20 Kg	4	60	133.33	8000
Flour	1.5	40 Kg	1.0	60	106.66	6400
milk	1.0	10 Lt	1.5	10	150	1500
Total				130		15900

The first column indicates the commodities, the second column indicates the price in the base year, the third column indicates the various quantities of the items in the base year, the fourth column indicates the price of the items in the current year, the fifth column will indicate the value weights (V) which is the product of P naught q naught is equal to 60, 60, 10 and the summation of V is equal to 130.
Then we calculate the price relative (P) which is equal to p one by price naught into 100 is equal to one thirty three point three three, one hundred six point six six, one fifty.
In the last column we will get the values by multiplying the price relative (P) into value weights (V) and get the PV values as eight thousand, six thousand four hundred, thousand five hundred which gives the total summation PV equal to fifteen thousand nine hundred.

Let us now calculate the Price index is equal to summation of Price relative Value weights divided by summation of value is equal to fifteen thousand nine hundred divided by 130 is equal to one twenty two point three one which indicates that there has been a 22.3% increase in price over the base level.

Now let us calculate the index number using geometric mean of price relatives. As a first step we will prepare the table for getting the necessary values.

Figure 16

Commodities	P_0 (Rs.)	q_0	P_1	V= P0xq0	P= (P_1/p_0) x 100	Log P	V log P
Sugar	3	20 Kg	4	60	133.33	2.1249	127.494
Flour	1.5	40 Kg	1.0	60	106.66	2.0282	121.692
milk	1.0	10 Lt	1.5	10	150	2.1761	21.761
Total				**130**			**270.947**

First column indicates the commodities, the second column indicates the price in the base year, the third column indicates the various quantities of the items in the base year, the fourth column indicates the price of the items in the current year.
The fifth column will indicate the value weights (V) which is the product of P naught q naught is equal to 60, 60, 10 and the summation of V is equal to 130.
In the sixth column we calculate the price relative (P) which is equal to p one divided by p naught into 100 is equal to one thirty three point 3,3. One hundred six point 6,6. One fifty.
In the seventh column we will calculate the log of P we will get two point 1,2,4,9. two point 0,2,8,2. two point 1,7,6,1 then the last column we will get the product of V and log P we will get one twenty seven point 4,9,4.one twenty one point 6,9,2. twenty one point 7,6,1 and the summation is two seventy point 9,4,7.
Let us now calculate the Price index is equal to antilog summation Value weights into log Price relative divided by summation of value is equal to Antilog of two seventy point 9,4,7 divided by 130 is equal to antilog of two point 0,8,4 is equal to one twenty one point three.

Problem 8:
Calculate the cost of living index using the weighted geometric mean.

Figure 17

Group	Index number	Weights
Food	350	10
Fuel & lighting	150	2
Clothing	200	2
House rent	150	2
Miscellaneous	225	4

First column represents the groups, the second column represents the values of the index & the third column represents the weights.

Solution:
Let us prepare the table for getting the various values, in column one we represent the groups, the second column is the values of the index number350,150, 200, 150 and 225, the third column is the weights 10, 2, 2, 2, and 4.

Figure 18

Groups	Index number	Weights	Log I	logIW
Food	350	10	2.5441	25.4410
Fuel & lighting	150	2	2.1761	4.3522
Clothing	200	2	2.3010	4.6020
House rent	150	2	2.1761	4.3522
Miscellaneous	225	4	2.3522	9.4088
Total		**ΣW = 20**		**ΣlogIW = 48.1562**

Now let us take the log of the index number which are Two point 5,4,4,1. Two point 1,7,6,1. two point 3,0,1,0. two point 1,7,6,1 and two point 3,5,2,2 in the fourth column and multiply it with the weights we will get log IW which is equal to twenty five point 4,4,1,0. Four point 3,5,2,2. Four point 6,0,2,0. Four point 3,5,2,2 and nine point 4,0,8,8. the total of this column we will get summation of log IW which is equal to forty eight point 1,5,6,2.

Let us now calculate the cost of living index which is equal to antilog of summation log IW divided by summation W which is equal to antilog of forty eight point 1,5,6,2. divided by 20 is equal to antilog of two point 4,0,7,8. is equal to two fifty five point 8.

Here's a summary of our learning in this session, where we have understood:

- The Computation of Index numbers using,
  - Averages of relatives
  - Aggregative methods
  - Consistency tests