Summary

- An optimal as well as feasible solution to an linear programming problem is obtained by choosing among several values of decision variables x_1, x_2, ….,x_n, the one set of values that satisfy the given set of constraints simultaneously and also provide the optimal (maximum or minimum) value of the given objective function

- For linear programming problems that have only two variables it is possible that the entire set of feasible solutions can be displayed graphically by plotting linear constraints to locate a best (optimal) solution. The technique used to identify the optimal solution is called the **graphical solution technique** for a linear programming problem with two variables

- Two graphical solution techniques or approaches to find the optimum solution to an linear programming problem are,
  - Extreme point enumeration approach
  - Iso-profit (cost) function approach

- A convex set is a polygon and by 'convex', we mean that if any two points of the polygon are selected arbitrarily, then a straight line segment joining the two points lies completely within the polygon

- The extreme points of the convex set give the basic solutions to the linear programming problem

Extreme point enumeration approach:
The solution method for a linear programming problem is divided into five steps:
- **Step 1:** State the given problem in the mathematical form
- **Step 2:** Graph the constraints, by temporarily ignoring the inequality sign and decide about the area of feasible solutions according to the inequality sign of the constraints. Indicate the area of feasible solutions by a shaded area, which forms a convex polyhedron
- **Step 3:** Determine the coordinates of the extreme points of the feasible solution space
- **Step 4:** Evaluate the value of the objective function at each extreme point
- **Step 5:** Determine the extreme point to obtain the optimum (best) value of the objective function

Iso-Profit (cost) function approach:
The steps of iso-profit (cost) function approach are as follows:
- **Step 1:** Identify the feasible region and extreme points of feasible region
- **Step 2:** Draw an iso-profit (iso – cost) line for a particular value of the objective function. The word iso here implies that the iso-profit (iso-cost) function is a straight line on which every points has the same total profit (cost)
- **Step3:** Move iso-profit (iso-cost) lines parallel in the direction of increasing (decreasing) objective function values
- **Step 4:** The feasible extreme point for which the value of iso-profit (iso-cost) is largest (smallest) is the optimal solution

Some special cases in linear programming:
- **Alternative (or multiple) optimal solution:**
  So far we have seen that the optimal solution of any linear programming problem occurs at an extreme point of the feasible region and the solution is unique, that is one solution yields the same value of the objective function. However the linear

programming problem may have more than one solution yielding the same objective function value

- There are two conditions that should be satisfied in order that an alternative optimal solution exists:
  1. The given objective function is parallel to a constraint that forms the boundary (or edge) of the feasible solutions region. In other words, the slope of the objective function is same as that of the constraint forming the boundary of the feasible solutions region and
  2. The constraint should form a boundary on the feasible region in the direction of optimal movement of the objective function. In other words, the constraints should be an active constraint

  Remarks:
  The constraint is said to be active or binding or tight, if at optimally, the left hand side of a constraint equal the right hand side. In other words, an equally constraint is always active. An inequality constraint may or may not be active.
  Geometrically, an active constraint is one that passes through one of the extreme points of the feasible solution space

- **An unbounded solution:**
  When the value of decision variables in linear programming is permitted to increase infinitely without violating the feasibility condition, then the solution is said to be unbounded. Here the objective function value can also be increased infinitely. However an unbounded feasible region may yield some definite value of the objective function

- **An infeasible solution:**
  If it is not possible to find a feasible solution that satisfies all the constraints, then the linear programming problem is said to have an infeasible solution or alternatively, inconsistence. Infeasibility depends solely on the constraints and has nothing to do with the objective function