E-Learning Module on Calculation of probability of completing the project within a specified time

Learning Objectives

By the end of this session, you will be able to:

- Determine the probability of completing a project on or before the schedule date
- Know how to update a project along with resource leveling and smoothing
- Crash project schedule time and establish a time-cost trade-off for the completion of a project ,

Introduction

PERT was developed in the context where many activities associated with the project had never been attempted previously. PERT was developed to handle projects where the time duration for each activity is no longer just a single time estimate, (that is decision makers best guess) but is a random variable that is characterized by some probability distribution – usually a beta distribution.

Introduction

To estimate the parameters of the beta distribution (the mean and variance), the PERT model requires three time estimates for each activity. From these times a single value is estimated for future consideration

Introduction

Optimistic time (t_0 or a): the shortest possible time (duration) in which an activity, can be performed assuming that everything goes well.

Pessimistic time (t_p or b): the longest possible time required to perform an activity under extremely bad conditions. However, such conditions do not include natural calamities like earthquakes, flood, etc.

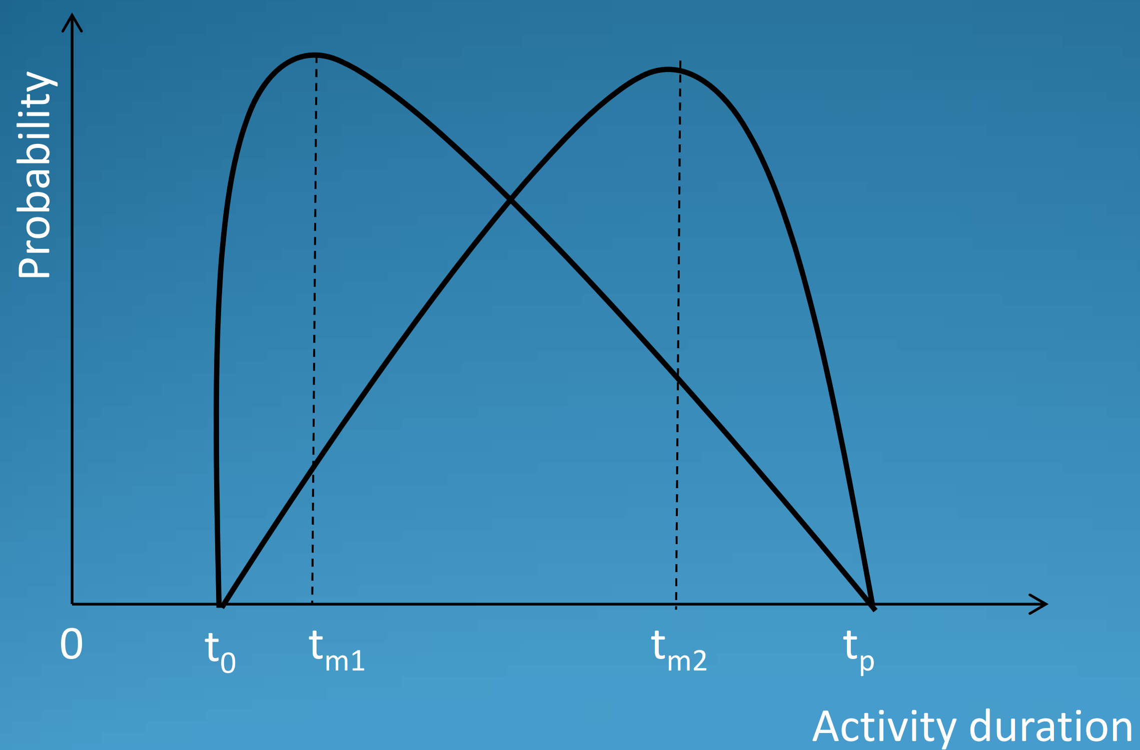
Introduction

Most likely time (t_m or m): the time that would occur most often to complete an activity, if the activity was repeated under exactly the same conditions many times. Obviously, it is the completion time that would occur most frequently (that is model value)

Introduction

The beta distribution is not necessary symmetric, the degree of skewness depends on the location of t_m to t_0 and t_p. Thus, the range specified by the optimistic time (t_0) and pessimistic time (t_p) estimates is assumed to enclose every possible estimate of the duration of the activity.

Introduction



Introduction

In beta-distribution the mid-point $(t_0 + t_p)/2$ is assumed to weigh half as much as the most likely point (t_m). Thus, the expected or mean (t_e or μ) time of an activity, that is also the weighted average of three time estimates, is computed as the arithmetic mean of $(t_0 + t_p)/2$ and $2t_m$. That is estimated time of an activity $(t_e) = (t_0 + t_p)/2 + 2t_m / 3 = t_0 + 4t_m + t_p / 6$.

Introduction

With uncertain activity time, variance can be used to describe the dispersion (variation) of the activity time values. The calculations are based on an analogy to the normal distribution where 99 percent of the area under normal curve is within + or - 3 standard deviation from the mean or fall within the range approximately 6 standard deviation in length.

Introduction

Therefore the interval (t_0, t_p) or range $(t_p - t_0)$ is assumed to enclose about 6 standard deviations of a symmetric distributions. Thus if σ_i denotes the standard deviation of the duration of activity i, then $6\sigma_i = t_p - t_0$ or

$\sigma_i = t_p - t_0 / 6.$

Variance of activity time $\sigma_i^2 = [1/6(t_p - t_0)]^2$.

Introduction

If we assume that durations of the activities are independent random variables, then the variance of the total critical path's duration is the sum of the variances on the critical path. Suppose σ_c is the standard deviation of the critical path.

Then: $\sigma_c^2 = \Sigma\sigma_i^2$ and σ_c = the square root of $\Sigma\sigma_i^2$

Estimation of project completion time:

Since we expect variation in the activity duration, therefore the chance of completing the project in a desired time and the duration necessary for obtaining any desired probability of actually meeting

Estimation of project completion time:

The probability distribution of times for completing an event can be approximated by the normal distribution due to the central limit theorem. Thus, the probability of completing the project in the schedule (or desired) time, T_s is given by:

Probability[$z = T_s - T_e / \sigma_i$] where T_e =expected completion time of the project, Z = number of standard deviations the schedule times (or desired completion times) lies away from the mean or expected date. $\sigma_I^2 = \sigma_1^2 + \sigma_2^2 + \ldots\ldots + \sigma_n^2$ is the sum of variances of critical activities.

Estimation of project completion time:

The desired completion time of the project can be calculated as: $T_s = Z\sigma + T_e$, where value of Z corresponds to the probability of project completion time. The computation of T_s enables a decision-maker to make certain commitments, knowing the degree of risk assumed.

Estimation of project completion time:

The expected completion time of the project is obtained by adding the expected time of each activity lying on the critical path. Since it is assumed that the two activities are independent, therefore the variance of the critical path can be known by adding the variance of critical activities.

Project Time-Cost trade off:

The cost of resources consumed by activities were not taken into consideration. The project completion time can be reduced by reducing (crashing) the normal completion time of critical activities. The reduction in normal time of completion will increase the total budget of the project.

Project Time-Cost trade off:

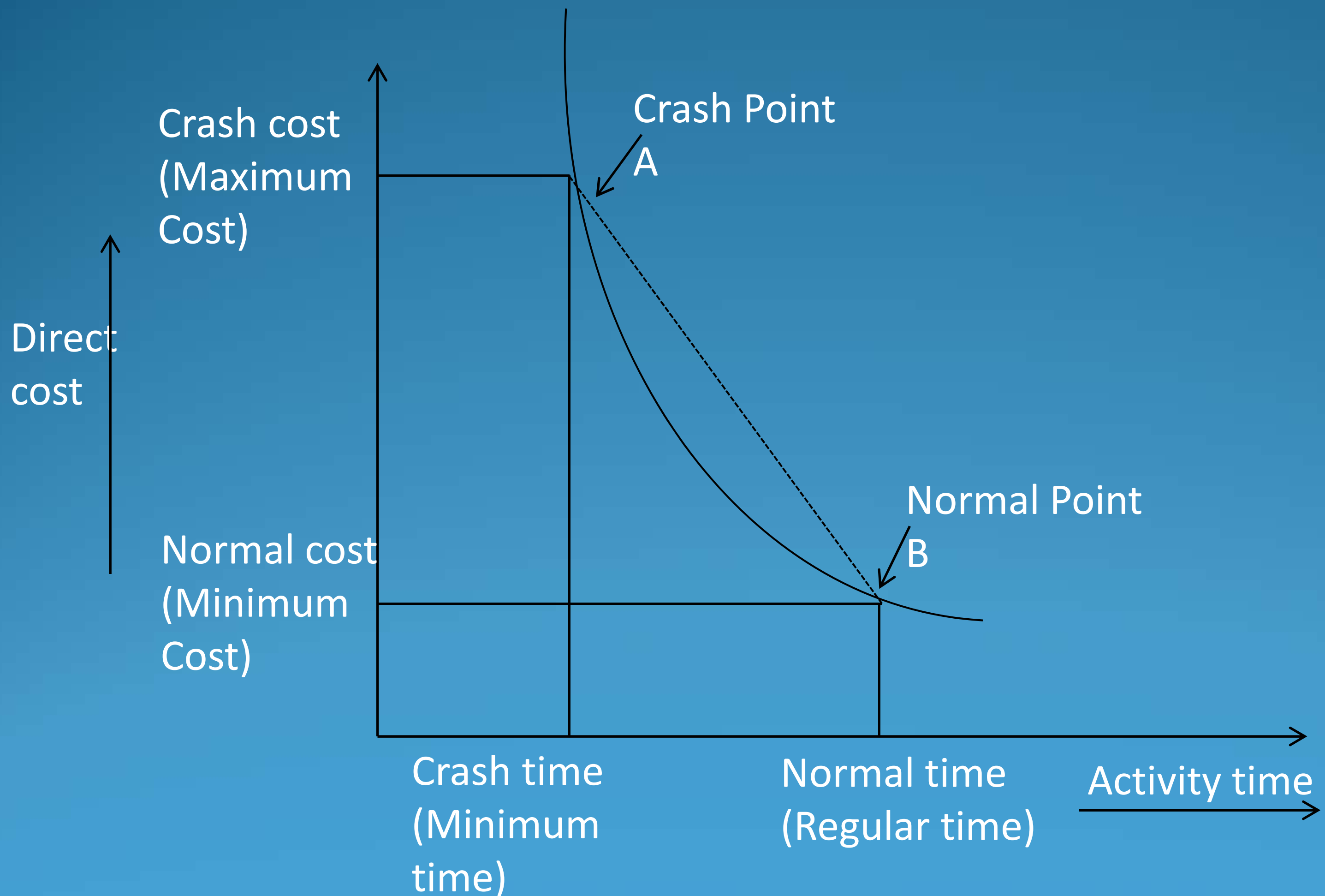
However, the decision-maker will always look for trade-off between the total cost of project and the total time required to complete it.

Project Time-Cost trade off:

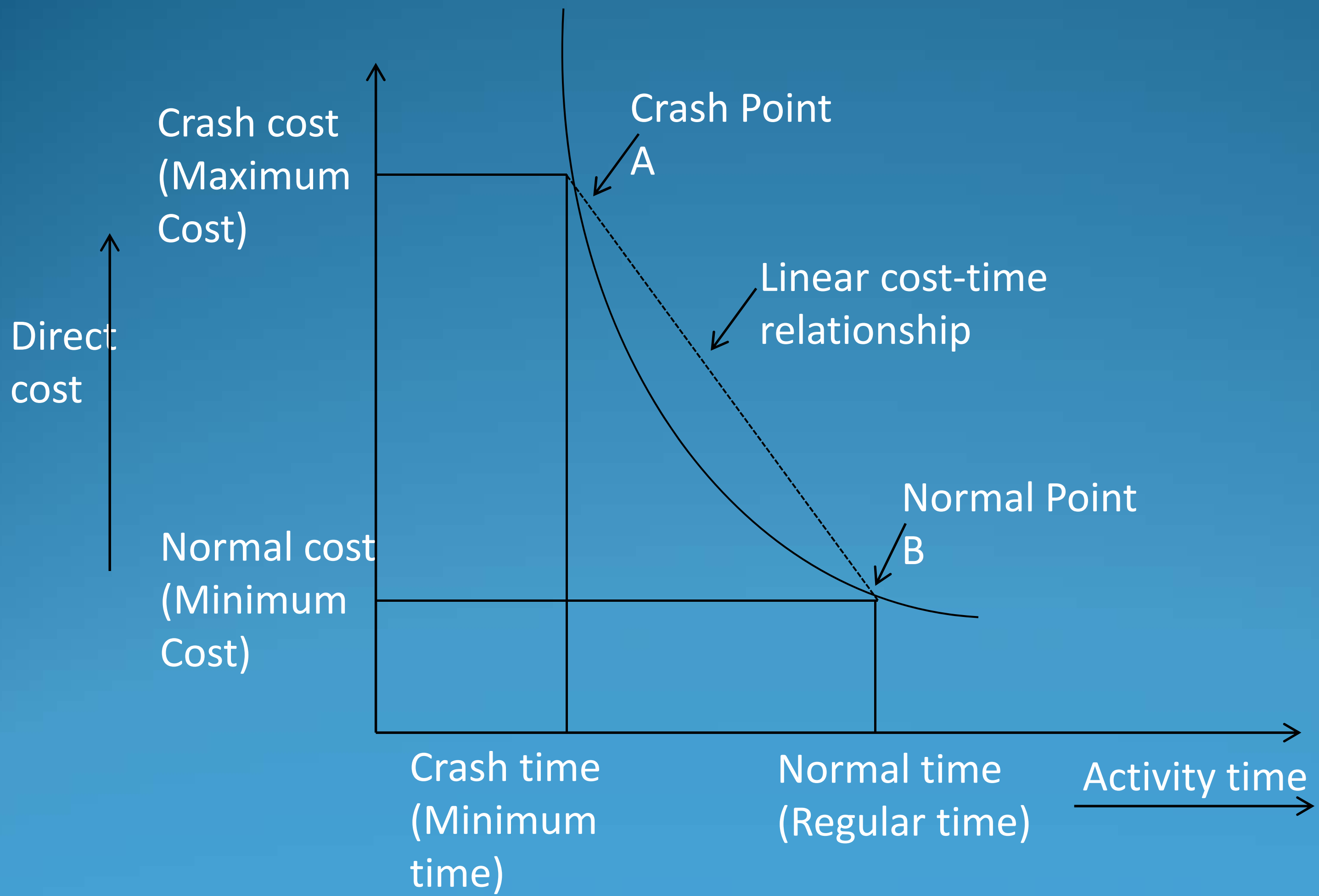
Project crashing:

Crashing the project means crashing a number of activities to reduce the duration of the project, below its normal time. Crashing is employed to reduce the project completion time by spending extra resources (cost).

Project Time-Cost trade off:



Project Time-Cost trade off:



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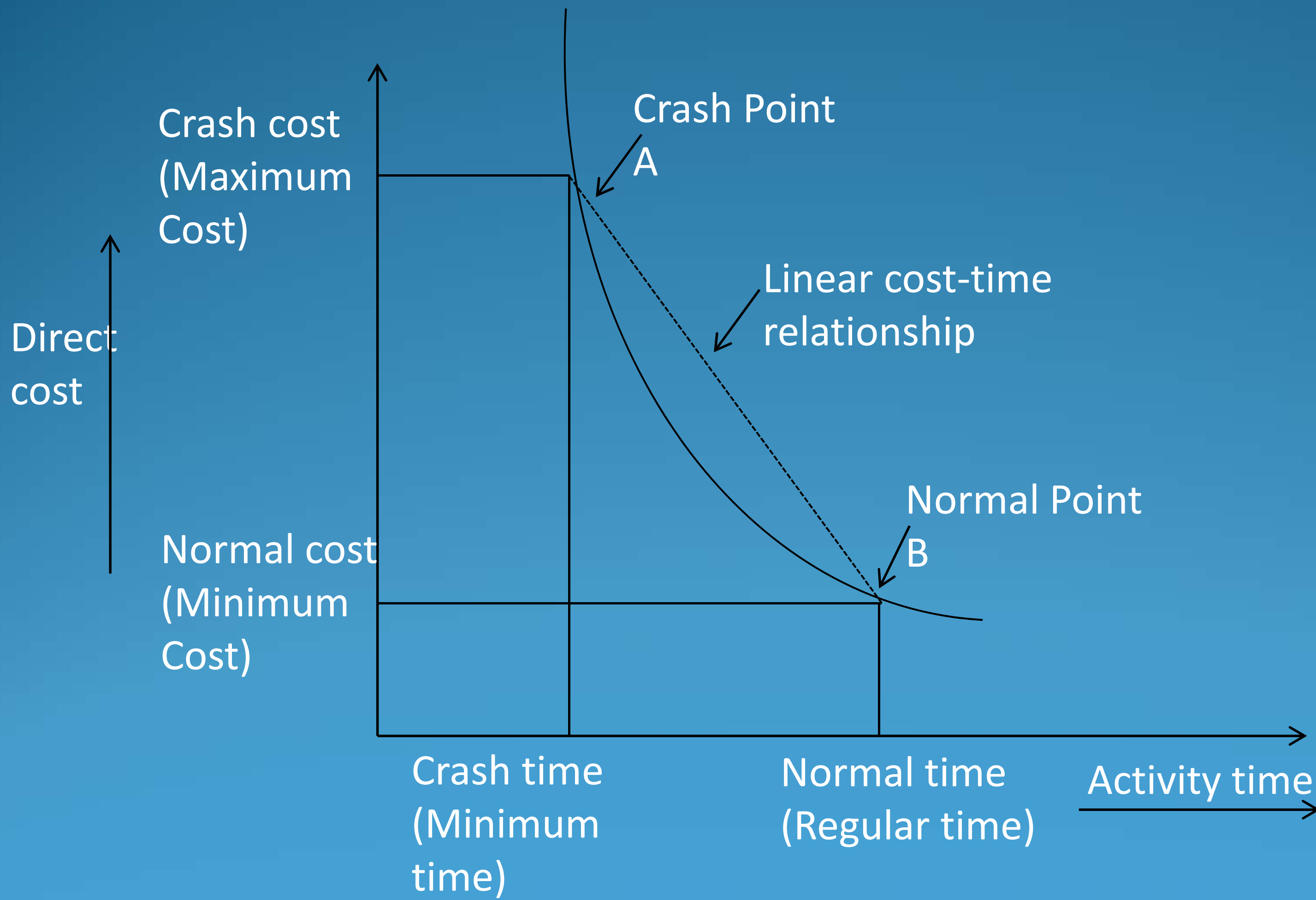


Project Time-Cost trade off:

Remark: crashing an activity means performing it in the shortest possible time by allocating to its necessary resources.

Project Time-Cost trade off:

Time-Cost trade off procedure

Step 1: determine the normal project completion time and associated critical path for the following two cases:

a)When all critical activities are completed with their normal time. This provides the starting point for crashing analysis.

b) When all critical activities are crashed. This provides the stopping point for crashing analysisv

Project Time-Cost trade off:

Time-Cost trade off procedure

Step 2: identify the critical activities and compute the cost slope for each of these by using the relationship cost slope = crash cost - normal cost divided by crash time - normal time. The value of cost slope for critical activities indicate the direct extra cost required to execute an activity per unit of time.

Project Time-Cost trade off:

Time-Cost trade off procedure

Step 3: for reducing the total project completion time, identify and crash an activity time on the critical path with lowest cost slope value to the point where

i. Another path in the network becomes critical or

ii. The activity has been crash to its lowest possible time.

Project Time-Cost trade off:

Time-Cost trade off procedure

Step 4: if the critical path under crashing is still critical, return to step 3. However, if due to crashing of an activity time in step3, other path(s) in the network also become critical, then identify and crash the activity(s) on the critical path(s) with the minimum joint cost slope.

Project Time-Cost trade off:

Time-Cost trade off procedure

Step 5: terminate the procedure when each critical activity has been crashed to its lower possible time, determine total project cost (indirect + direct cost) corresponding to different project durations.

Updating of the project progress

When a project is actually executed it may not exactly follow the time schedule developed for it. There are bound to be unexpected delays and difficulties in terms of delay in supply of materials, non-availability of some machine and/or breakdown of machines, non-availability of skilled man power, natural calamity, etc. in such cases it may be necessary to review the progress of network planning and scheduling.

Updating of the project progress

Such review of the progress helps in taking stock of the progress that has been made. It also helps in making the necessary change in the initial schedule in terms of time and resources required by the uncompleted activities in the project.

Updating of the project progress

There is no rule about the specific time that is required to update the project progress. The frequency of updating may be more when project duration is small because few slippages in detecting the progress will affect the project as a whole.

Updating of the project progress

This as a whole would help in absorbing such slippages to be less. But in case of large projects the frequency of updating may be less at the initial stages because a few initial slippages may be absorbed later in the project. However, to add dynamism to the nature and progress of work, updating may be carried out as frequently and as economically possible.

Updating of the project progress

The project progress can be updated in two ways:

i. Use the revised time estimate of incomplete activities and rom the initial event calculate the earliest and latest completion time of each event in the usual manner in order to know the project completion time.

Updating of the project progress

The project progress can be updated in two ways:

ii. Change the complete work to zero duration and represent all the activites already finished by an arrow called the elapsed time arrow. Events in the revised network diagram are renumbered. The completion times of remaining activities are taken as revised time.

Updating of the project progress

Resource allocation:

Resources such as men, material, money, machinery, etc. are limited and conflicting demands are made for the same ype of resources as a project progresses. A systematic method for the allocation of resources therefore becomes essential.

Updating of the project progress

Resource allocation:

The aim is to prevent the day-to-day fluctuation in the level of the required resources and obtain a uniform resource requirement during the project duration. There are two approaches of maintaining resources that are required to complete a project. Resources leveling and resource smoothing.

Updating of the project progress

Resource allocation:

Resource leveling the analysis that aims at the stabilizing the rate of resource utilization (relatively constant) by various activities at different times without changing the project duration, is called resource leveling.

Updating of the project progress

Resource allocation:

In order to stabilize the use of the existing level of resources the total float of non-critical activities is used. By shifting a non-critical activity between its earliest start time and latest allowable time, the project manager may be able to lower the maximum resource requirement.

Updating of the project progress

Resource allocation:

The following two general rules are normally used in scheduling non-critical activities:

i. If the total float of a non-critical activity = its free float, then it can be scheduled anywhere between its earliest start and latest completion times.

ii. If the total float of a non-critical activity is more that its free float, then its starting time can be delayed relative to its earliest start time by no more than the amount of its float, without affecting the scheduling of its immediately succeeding activites.

Updating of the project progress

Resource Smoothing:

The analysis that aims to reduce the peak demand for resources and to reallocate them among the activities of a project in a manner that the total project duration remains the shortest is known as resource smoothing (or loading).

Updating of the project progress

Resource Smoothing:

Step 1: calculate the earliest start time and latest finish times of each activity and then draw a time scaled version (or squared) of the network. In this network a critical path is drawn along a straight line and non-critical activities on both sides of this line. Resource requirement of each activity is given along the arrows.

Updating of the project progress

Resource Smoothing:

Step 2: Draw the resource histogram by taking earliest start times or latest start times of activites on the x-axis and required cumulative resource on y-axis.

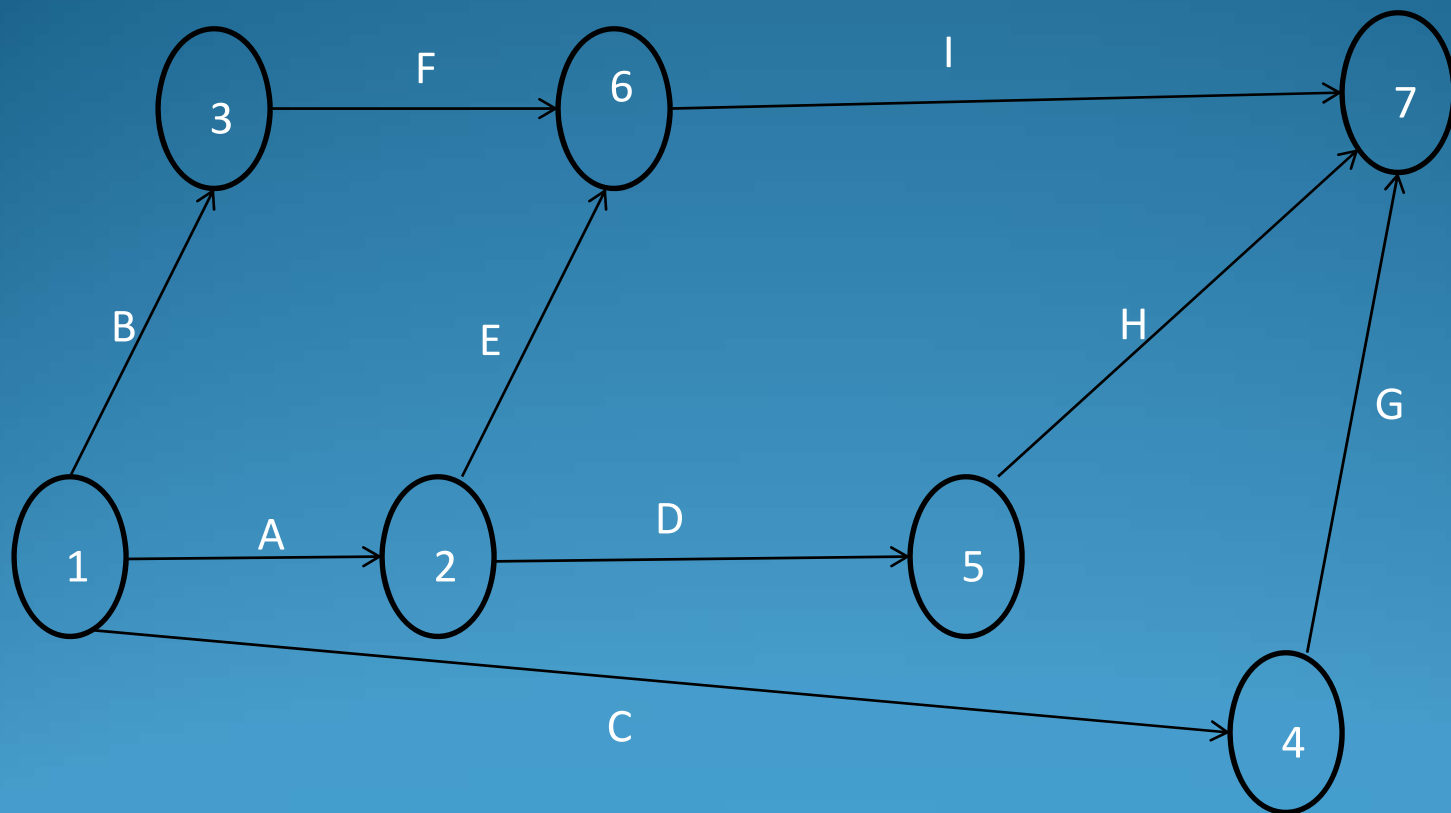
Step 3: shift the start time of those non-critical activities that have the largest float in order to smoothen the resources.

Example

The following network diagram represents activities associated with a project

Activities	A	B	C	D	E	F	G	H	I
Optimistic Time	5	18	26	16	15	6	7	7	3
Pessimistic Time	10	22	40	20	25	12	12	9	5
Most likely Time	8	20	33	18	20	9	10	8	4

Example



Example

Determine the following;

i. Expected activity time and variance

ii. The earliest and latest expected completion times of each event

iii. The critical path

iv. The probability of expected completion time of the project if the original schedule time of completing the project is 41.5 weeks

v. The duration of the project that will have 95 percent chance of being completed.

Solution

Using the following formula the expected activity time (t_e or μ) and variance (σ^2) calculations are done and exhibited in the table.

The formula is $t_e = 1/6\ (t_0 + 4t_m + t_p)$ and $\sigma_i^2 = \{1/6\ (t_p - t_0)\}^2$

Solution

Activity	t_0	t_p	t_m	t_e	σ^2
1-2	5	10	8	7.8	0.696
1-3	18	22	20	20.0	0.444
1-4	26	40	33	33.0	5.429
2-5	16	20	18	18.0	0.443
2-6	15	25	20	20.0	2.780
3-6	6	12	9	9.0	1.000
4-7	7	12	10	9.8	0.694
5-7	7	9	8	8.0	0.111
6-7	3	5	4	4.0	0.111

Solution

The earliest start time and the latest expected time is calculated using the forward pass method and the backward pass method considering the expected time of . The calculations are as follows:

Solution

Forward pass method

$E1 = 0;$

$E2 = E1 + t1,2 = 0 + 7.8 = 7.8;$

$E3 = E1 + t1,3 = 0 + 20 = 20;$

$E4 = E1 + t1,4 = 0 + 33 = 33;$

$E5 = E2 + t2,5 = 7.8 + 18 = 25.8;$

Solution

Forward pass method

$E6 = Max\{E1 + t1,6\} = MAX \{E2 + t2,6; E3 + t3,6\} = MAX \{7.8 + 20; 20 + 9\} = 29;$

$E7 = MAX\{Ei + ti,7\} = MAX\{E4 + t4,7; E5 + t5,7; E6 + t6,7\} = MAX \{33 + 9.8; 25.8 + 8; 29 + 4\} = 42.8$

Solution

Backward pass method

L7 = E7 = 42.8;

L6 i= L7 - t6,7 = 42.8 - 4 = 38.8;

L5 = L7 - t5,7 = 42.8 - 8 = 34.8;

L4 = L7 - t4,7 = 42.8 - 9.8 = 33;

L3 = L6 - t3,6 = 38.8 - 9 = 29.8

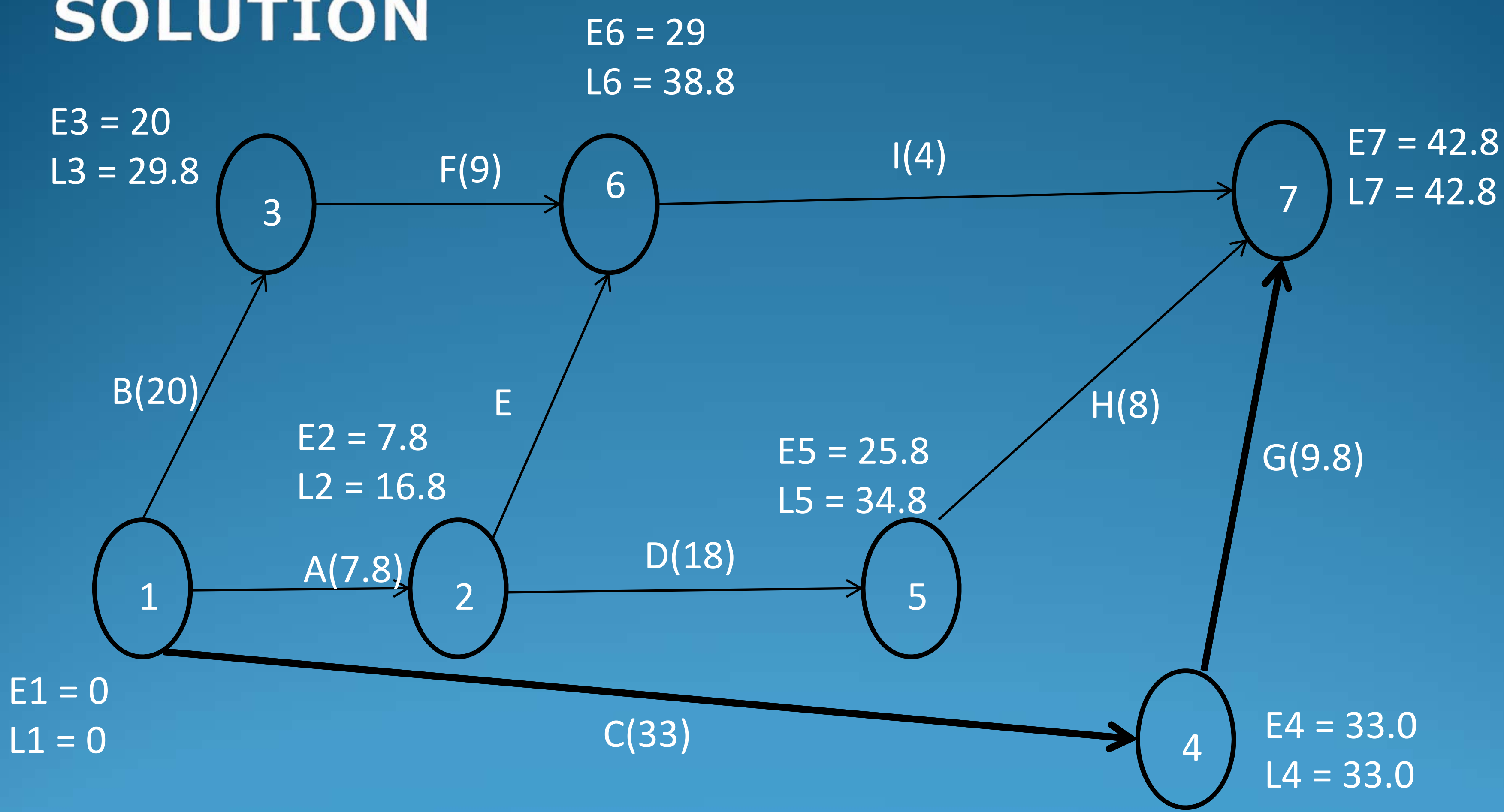
Solution

Backward pass method

L2 = Min {Lj - t2,j} = Min{L5 - t2,5; L6 - t2,6} = Min{34.8 -18; 38.8 - 20} = 16.8;

L1 = Min {Lj - t1,j} = Min{L2 - t1,2; L3 - t1,3; L4 - t1,4} = Min{16.8 -7.8; 29.8 - 20; 33-33} = 0

SOLUTION


E6 = 29
L6 = 38.8
E3 = 20
L3 = 29.8
F(9)
I(4)
E7 = 42.8
L7 = 42.8
3
6
7
B(20)
E
H(8)
E2 = 7.8
L2 = 16.8
E5 = 25.8
L5 = 34.8
G(9.8)
D(18)
A(7.8)
1
2
5
E1 = 0
L1 = 0
C(33)
4
E4 = 33.0
L4 = 33.0

Solution

Expected length of critical path

$T_e = t_c + t_g = 33 + 9.8 = 42.8$ weeks.

Variance of critical path length $\sigma^2 = \sigma_c^2 + \sigma_g^2 = 5.429 + 0.694 = 6.123$ weeks.

Since $T_s = 41.5$ and $T_e = 42.8$ and square root of σ = square root of 6.123 = 2.474, the probability of meeting the schedule time is given by; Prob ($z \leq T_s - T_e/\sigma$) = P ($Z \leq 41.5 - 42.8$ divided by 2.474) = prob ($Z \leq - 0.52$).

Solution

Thus, the probability that the project can be completed in ≤ 41.5 weeks is 0.3048. in other words, the probability that the project will get delayed beyond 41.5 weeks is 0.6952. Given that Prob ($z \le Ts - Te/\sigma$) = 0.95 we get the value from the normal distribution table as 1.64. thus, 1.64 = Ts minus 42.8/2.47 or Ts = 1.64 x 2.474 + 42.8 = 46.85 weeks.