Course Details	
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Paper Name	Operation Research
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E-Learning Module on Replacement Model for Items Which Deteriorate With Time

Learning Objectives

By the end of this session, you will be able to:

- Realize the need to study replacement and maintenance analysis techniques
- Apply replacement policy for items whose efficiency deteriorates with time and for items that fail completely

Introduction

The problem of replacement is felt when the job performing units such as men, machines, equipment’s, parts, etc. become less effective or useless due to either or sudden, or gradual deterioration in their efficiency, failure or breakdown.

Introduction

By replacing those with new ones at frequent intervals, maintenance and other overhead costs can be reduced.

However, such replacements would increase the need of capital cost for new ones.

Introduction

Example-1:
A vehicle tends to wear out with time due to constant use. More money needs to be spent on it on account of increased repair and operating cost. A stage comes when it becomes uneconomical to maintain the vehicle and it is better to replace it with a new one

Inference:
Here the replacement decision may be taken to balance the increasing maintenance cost with the decreasing money value of the vehicle, with the passing of time.

Introduction

Example-2:
In case of highway tube lights where time of failure is not predictable for individual tubes, they are replaced only after their individual failure. However, it may be economical to replace such tubes on a schedule basis before their failure.

Inference:
Here the replacement decision may be taken to balance between the wasted life of a tube before failure and cost incurred when a tube completely fails during service.

Introduction

Thus, the basic problem in such situations is to formulate a replacement policy in order to decrease an age (or period) at which the replacement of the given machinery by equipment is most economical, keeping in view of all possible alternatives.

Introduction

i. Items such as machines, vehicles, tyres, etc, whose efficiency deteriorates with age due to constant use and which need increased operating and maintenance costs, in such cases the deterioration level is predictable and is represented by
   - Increased maintenance by operational cost
   - Its waste or scrap value and damage to item and safety risk

Introduction

ii. Items such as light bulbs and tubes, electric motors, radio, television parts, etc, which do not give an anticipation of failures to specify the probability of failure for any future time period. With this probability distribution and the cost information, it is desired to formulate optimal replacement policy in order to balance the wasted life of an item, replaced before failure against the costs incurred when item fails in service.

Introduction

iii. The existing working staff in an organization gradually reduces due to retirement, death, retrenchment and other reasons

Replacement of items whose efficiency deteriorates with time

When operational efficiency of an item deteriorates with time (gradual failure), it is economical to replace the same with a new one.

Replacement of items whose efficiency deteriorates with time

Example:

The maintenance cost of a machine increases with time and a stage is reached when it may not be economical to allow the machine to continue in the system.

Besides, there could be a number of alternative choices and one may like to compare the available alternatives on the basis of the running costs (average maintenance and operating costs) involved.

Various Techniques – Model 1

Replacement policy for items whose running cost increases with time and value of money remains constant during a period:

The cost of maintenance of a machine is given as a function increasing with time, whose scrap value is constant.

Various Techniques – Model 1

a) If time is measured continuously then the average annual cost will be minimized by replacing the machine when the average cost to date becomes equal to the current maintenance cost

b) If time is measured in discrete units, then the average annual cost will be minimized by replacing the machine when the next period's maintenance cost becomes greater than the current average cost

Various Techniques – Model 1

Proof:

The aim here is to determine the optimal replacement age of a piece of equipment whose running cost increases with time and the value of money remains constant (that is value is not counted) during that period.

Various Techniques – Model 1

C	Capital or purchase cost of new equipment
S	Scrap (or salvage) value of the equipment at the end of t years
R(t)	Running cost of equipment for the year t
n	Replacement age of equipment

Various Techniques – Model 1

a. **When time 't' is a continuous variable**, if the equipment is used for t years, then the total cost incurred over this period is given by:

TC = Capital (or purchase) cost – scrap value at the end of t years plus running cost for t years

$= C - S + \int_0^n R(t).dt$

Therefore, the average cost per unit time incurred over the period of n years is

$ATC_n = 1/n \{ C - S + \int_0^n R(t).dt \}$ (eqn 1)

Various Techniques – Model 1

To obtain the optimal value n for which ATC_n is minimum, differentiate ATC_n with respect to n, and set the first derivative equal to zero.

That is for minimum of

ATC_n=d/dn of ATC_n

$=-1/n^2\{C-S\}+R(n)/n -1 / n^2 \int$ n to 0 of R(t).d(t)

= 0, or R(n)

= 1/n { C-S+∫ n to 0 of R(t).d(t)}, $n \neq 0$ (eqn 2)

Various Techniques – Model 1

Hence the following replacement policy can be derived with the help of the second equation.

Policy: Replace the equipment when the average annual cost for n years is equal to the current by annual running cost.
That is:

R(n) =1/n { C -S + ∫ n to 0 R(t).dt}

Various Techniques – Model 1

b. **When time ‘t’ is a discrete variable**, the average cost incurred over the period n is given by,

$$ATC_n = 1/n \{ C - S + \Sigma R(t)\} \text{ (eqn 3)}$$

If C - S and ΣR(t) are assumed to be monotonically decreasing and increasing, respectively, hence there will exist a value of n for which ATC_n is minimum.

Various Techniques – Model 1

Thus, we shall have inequalities:

$ATC_{n-1} > ATC_n < ATC_{n+1}$ or $ATC_{n-1} - ATC_n > 0$ and $ATC_{n+1} - ATC_n > 0$.

Equation (3) for period n +1, we get:

$ATC_{n+1} = 1/n+1\{C- S+\Sigma\ R(t)\}$

$= 1/n+1\ \{ C-S +\Sigma R(t)+R(n+1)\}$

Various Techniques – Model 1

Therefore,

$ATC_{n+1}-ATC_n=n/n+1.ATC_n+A(n+1)/(n+1)-ATC_n$

$= R(n+1)/(n+1)+ATC_n\{(n/n+1)-1\}$

$= R(n+1)/n+1-ATC_n/n+1$

Various Techniques – Model 1

Since ATC_{n+1} - $ATC_n > 0$,

we get, $R(n+1)$ / $n+1$ - ATC_n / $n+1 > 0$.

$R(n+1) - ATC_n > 0$ or $R(n+1) > ATC_n$.(eqn 4).

Similarly, ATC_{n-1} - $ATC_n > 0$, implies that $R(n+1) < ATC_{n-1}$

Various Techniques – Model 1

Policy 1:

If the running cost of next year; R(n+1) is more than the average cost of n^{th} year, ATC_n, then it is economical to replace at the end of n years.

i.e., R(n+1) is > 1/ n { c - S + summation R(t)}

Various Techniques – Model 1

Policy 2:

If the present years running cost is less than the previous year's average cost ATC_{n-1}, then do not replace it.

i.e., R(n) < I / n - 1 {C - S + summation R(t)}

Various Techniques – Model 1

Example:

A firm is considering the replacement of a machine, whose cost price is Rs 12,200 and its scrap value is Rs 200. From experience the running (maintenance and operating) costs are found to be as follows: when should the machine be replaced?

Year	1	2	3	4	5	6	7	8
Running Cost (Rs)	200	500	800	1200	1800	2500	3200	4000

Various Techniques – Model 1

Solution:

Year of service	Running cost	Cumulative running cost	Depreciation cost	Total cost	Average cost
1	200	200	12000	12200	12000
2	500	700	12000	12700	6350
3	800	1500	12000	13500	4500
4	1200	2700	12000	14700	3675
5	1800	4500	12000	16500	3300
6	2500	7000	12000	19000	3167
7	3200	10200	12000	22200	3171
8	4000	14200	12000	26200	3275

Various Techniques – Model 1

Solution:

It may be noted that the average cost per year, ATC_n is minimum in the sixth year.

Also the average cost in the seventh year is more than the cost in the sixth year Hence, the machine should be replaced after every six years.

Various Techniques – Model 2

Replacement policy for items whose running cost increases with time but value of money changes with constant rate during a period; Value of money criterion; If the effect of the time value of money is to be considered, then the replacement decision analysis must be based upon an equivalent annual cost.

Various Techniques – Model 2

For example:
If the interest rate on Rs100 is 10 per cent per year, then the value of Rs 100 to be spent after one year will be Rs 110.

This is also called value of money. Also, the value of money that decreases with constant rate is known as the depreciation ratio or discounted factor.

Various Techniques – Model 2

The discounted value is the amount of money required at the time of the policy decision in order to build up funds at compound interest large enough to pay the required cost when due.

Various Techniques – Model 2

For example:

If the interest rate on Rs 100 is ‘r’ percent per year, then the present value (or worth) of Rs 100 to be spent after n years will be,

$$d = (100 / 100 + r)^n$$

Where, d is the discount rate or depreciation value.

Various Techniques – Model 2

After having an idea of discounted cost, the objective should be to determine the critical age at which an item should be replaced so that the sum of all discounted cost is minimum.

Various Techniques – Model 2

Example:
Let the value of the money be assumed to be 10 percent per year and suppose that machine A is replaced after every three years, whereas machine B is replaced every six years.
The yearly costs (in Rs) of both the machines are given below: Determine which machine should be purchased.

Year	1	2	3	4	5	6
Machine A	1000	200	400	1000	200	400
Machine B	1700	100	200	300	400	500

Various Techniques – Model 2

Discounted cost at 10% Rate (Rs)		
Year	Cost	Present worth
1	1000	1000 x 1 = 1000
2	200	200 (100/100+10)= 200 x 0.9091 = 181.82
3	400	400 $(100 /100+10)^2$ = 400 x 0.8264 = 330.56
Total		Rs 1512.38

The average yearly cost of machine A is 1512.38/3 = 504.13.

Various Techniques – Model 2

Discounted cost at 10% Rate (Rs)		
Year	Cost	Present worth
1	1700	1700 x 1=1700
2	100	100 x (10/11)=100 x 0.9091 = 90.91
3	200	200 x (10/11) = 200 x 0.8264 = 165.28
4	300	300 x (10/11) = 300x 0.7513 = 225.39
5	400	400 x (10/11) = 400 x 0.6830 = 273.20
6	500	500 x (10/11) = 500 x 0.6209 = 310.45
Total		2765.23

The average yearly cost of machine B is 2,765.23 /6 =Rs.460.87.

Various Techniques – Model 2

With the data on average yearly cost of both the machines, the apparent advantage is in purchasing machine B. But, the periods for which the costs are considered are different.

Therefore, let us first calculate the total present worth of machine A for six years.

Various Techniques – Model 2

Total present worth,

= 1000 + 200 x 0.9091 + 400 x 0.8264 + 1000 x 0.7513 + 200 x 0.6830 + 400 x 0.6209

= Rs 2648.64.

This is less than the total present worth of machine B. Thus, machine A should be purchased.

Present worth factor criterion

1. The running cost of an equipment that deteriorates over a period of time increases monotonically and the value of the money decreases with a constant rate, if 'r' is the interest rate, then Pwf $=(1 + r)^{-n}$

   It is also called as the present value of one rupee spent in n years from time now onwards.

   But if n =1, Pwf =d $=(1+r)^{-1}$ where 'd' is called the discount rate or depreciation value.

Present worth factor criterion

2. The money to be spent is taken on loan for a certain period at a given rate under the condition of repayment in installments

Present worth factor criterion

In replacement of items on the basis of the present worth factor, includes the present worth of all future expenditure and revenues for each replacement alternatives. An item for which the present worth factor is less is preferred.

Present worth factor criterion

Let, C =Purchase cost of an item
R =Annual running cost
n =Life of the item in years
r =Annual interest rate
S =Scrap (or salvage) value of the item at the end of its life

Then the present worth of the total cost during n years is given by,
Total cost =C + R (Pwf for r% interest rate for n years) - S (Pwf for r% interest rate for n years).

Present worth factor criterion

If the running cost of the item is different for its different operational life, then the present worth of the total cost during n years is given by:

Total cost = C plus R (Pwf for r% interest rate for I years) minus S (Pwf for r% interest rate for I years)

where I = 1, 2,….,n.

General Cost Function

If the maintenance cost increases with time and the money value decreases with constant rate. That is the depreciation value is given, its replacement policy would then be based on the following:

1. Replace if the running cost of next period is greater than the weighted average of previous cost
2. Do not replace if the running cost of the next period is less than the weighted average of the previous costs

General Cost Function

Proof:
Suppose that the item (machine or equipment) is available for use over a series of time periods of equal length, say one year.
Let us use the following notations:

- C = purchase price of a new item
- R_n= running cost of the item at the beginning of n^{th} year ($R_n+1 > R_n$)
- r = annual interest rate
- d = depreciation value per unit of money during a year { 1/(1 plus r)}

General Cost Function

Let us assume that the item is replaced after every n years of service and has no resale value.

The replacement policy can be formulated by calculating the total amount of money required for purchasing and running the item for n years.

General Cost Function

The period for which the total money is minimum will be considered best. The present worth (discounted value) of all future costs of purchasing and running the item with a policy of replacing it after every n years is given by

$D_n = [c+\Sigma d^{i-1} \cdot R_i][1/1-d^n]$ (sum of infinite G.P..D<1) ……equation 5.

General Cost Function

The value of D_n in the equation 5 is the money required to pay all future costs of purchasing and running the equipment, with a replacement policy of every n years.

Obviously the minimum value of D_n will be preferred. In other words, if n is an optimal replacement interval, then D_n will be minimum because of the inequality $D_{n+1} > D_n < D_{n-1}$.

General Cost Function

From this, two inequalities $D_{n+1} - D_n > 0$ and $D_n - D_{n-1} < 0$ can be established.

The condition for minimum value of D_n can be expressed as $(D_n - D_{n-1}) < 0 < (D_{n+1} - D_n)$ which can be written as $[1\text{-}d^n / 1\text{-}d \,.\, R_n – P(n)] < 0 < [1\text{-}d^n/1\text{-}d.R_{n+1} - P(n)]$ which is also written as $[1\text{-}d^n / 1\text{-}d \,.\, R_n < P(n) < 1\text{-}d^n/1\text{-}d.R_{n+1}]$ which can also be written as $R_n < C + (R_1 + dR_2 + d^2R_3 + \ldots.. + d^{n-1}R_n)/\ 1+d+d^2+\ldots..+d^{n-1} < R_{n=1}$.

General Cost Function

This expression between R_n and R_{n+1} represents the weighted average W(n) of all costs up to the period (n - 1) with weights 1, d, d^2,…., d^{n-1}, respectively.

The given weights are actually the discounted factors of the cost in the previous years.

Hence the value, if n, satisfying the above relationship will be the best age for replacing the given item.