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E-Learning Modules on Problems with Restrictions on Capital and Problems with Restriction on Space

Learning Objectives

By the end of this session, you will be able to:

- Explain the EOQ model with warehouse space constraints
- Explain the EOQ model with investment constraints

Introduction

Effective inventory management allows a distributor to meet or exceed customers’ expectations of product availability with the amount of each item that will maximize their company’s net profit or minimize its total inventory investment.

Introduction

Effective inventory management

Outstanding inventory control

Outstanding inventory management

Difference between inventory control and inventory management is as follows:

Introduction

Inventory control is managing the inventory that is already in your warehouse, stockroom, or store. That is:

- Knowing what products are “out there” and how much you have of each item
- Knowing exactly where each piece of each product is located in your warehouse
- Ensuring that all inventory remains in salable or usable condition
- Storing products to minimize the cost of filling customer orders

Introduction

Inventory management is determining when to order products, how much to order, and the most effective source of supply for each item in each warehouse. That is:

- Ensuring that you have the right quantity of the right item in the right location at the right time.
- Inventory management includes all of the activities of forecasting and replenishment

Introduction

The inventory models with one item are considered to be independent of all items being carried in the inventory.

Such an arrangement of controlling inventory of each item is only possible if there are no constraints (limitation) on the total average inventory.

Introduction

These include:

- The total warehouse space
- The total investment in inventories
- The total number of orders to be placed per year for all items
- Number of deliveries which can be accepted
- Size of delivery that can be handled, etc

Thus, some modification to the optimal order quantity determined has to be made in order to take care of such constraints.

Introduction

The EOQ thus needs to be calculated separately for each item to minimize the total inventory cost under the given constraints of limited warehouse capacity and finance.

Introduction

The following **assumptions** and **notations** will be used to develop inventory models under various constraints.

Assumptions:

- Production or supply is instantaneous with no lead time
- Demand is uniform and deterministic
- Shortages are allowed

Introduction

Notations:

n	Total number of items being controlled simultaneously
f_i	Floor area (storage space) required per unit of item I (i=1, 2, …., n)
W	Warehouse space limit to store all items in the inventory
λ	Non-negative lagrange multiplier

Introduction

Notations:

D_i	Annual demand for i^{th} item
Q_i	Order quantity for item I in inventory (i=1, 2, 3, ….., n)
M	Upper limit of average number of units for all items in the stock
C_i	Price per unit of item I (i=1, 2, …., n)
F	Investment limit for all items in the inventory (Rs.)

The EOQ Model with Warehouse Space constraint model:

The use of inventory control procedures is critical to maintaining accurate, reliable numbers for your operation.

Control can be a complicated balancing from the time a stock order is placed, received at your warehouse, counted, verified, labeled, put away, picked and shipped out or picked up.

The EOQ Model with Warehouse Space constraint model:

Knowing where your product is with accuracy at all times is vital to your success.

You can control your stock of products by installing a system of inventory control procedures.

Follow your inventory from the first moment the product arrives at your door until it leaves your facility.

The EOQ Model with Warehouse Space constraint model:

Assume that if the Warehouse space required for per unit of an item I is f_i, then the total floor area required by all the inventory items must be less than or equal to the Total floor area of the warehouse.

This can be represented by,

$$\Sigma(i=1 \text{ to } n)\ f_i \times Q_i \leq W$$

The EOQ Model with Warehouse Space constraint model:

This constraint indicates that even if all items reach their maximum inventory levels simultaneously, the warehouse space should be sufficient to store the inventory of these multiple items.

One of the key assumptions in the warehouse space management is that it is assumed that all the items are received together.

The EOQ Model with Warehouse Space constraint model:

Thus, the problem to minimize the total variable inventory cost for each item together under warehouse capacity constraint – the same can be represented by,

Minimum of Total Variable Inventory (TVC) cost is

$$\Sigma(i=1 \text{ to } n)\ [[\ (D_i/Q_i\ )x\ C_{oi}]+[(Q_i\ /2)\ x\ C_{hi}]$$

Subject to the constraints,

$$\Sigma(i=1 \text{ to } n)\ f_i\ Q_i \leq W \quad \text{and } Q_i \geq 0$$

for all values of i.

Kuhn-Tucker necessary and sufficient conditions for optimal value of TVC:

In order to determine the order quantities for different items so as to achieve minimum value of TVC, we use Lambda (λ) as the non-negative Lagrange multiplier.

This non-negative Lagrangian fraction is given by

$$L(Q_i, \lambda) = TVC + \lambda \left[\Sigma(i=1 \text{ to } n)\ f_i \times Q_i - W \right]$$

Kuhn-Tucker necessary and sufficient conditions for optimal value of TVC:

Necessary conditions for L to be minimum are

$\delta L/\delta Q_i = D_i/(Q)^2 \times C_{oi} + \frac{1}{2} \times C_{hi} + \lambda f_i$ which is equal to 0 or in other ways to represent

$(Q_i)^*$= Square root of (2 times $D_i \times C_{oi}$) /(C_{hi} + $2 \times \lambda \times f_i$)

$\delta L/\delta\lambda = \Sigma$ (i=1 to n) $f_i \times Q_i - W=0$.

Kuhn-Tucker necessary and sufficient conditions for optimal value of TVC:

λ indicates an additional cost related to the storage area used by each unit of the item.

Since $(Q_i)^*$ and λ values are interdependent, a trial and error method is used assuming different values of λ in order to satisfy constraint equation on the availability of storage space, W.

Kuhn-Tucker necessary and sufficient conditions for optimal value of TVC:

Step 1:

For $\lambda = 1$, compute the EOQ for each item separately by using the formula,

$(Q_i)^*$ = Square root of $(2D_i \cdot C_{oi}) / (C_{hi} + 2 \times \lambda \times f_i)$, where I = 1,2,3,…n

Step 2:

If $(Q_i)^*$ (I = 1,2,3,…n) satisfies the condition of total storage space available, then stop. Go to Step 3.

Kuhn-Tucker necessary and sufficient conditions for optimal value of TVC:

Step 3:

Increase the value of λ if the value of left hand side of space constraint is more than available storage space otherwise decrease the value of lambda.

This means that the only way of finding appropriate solution is to adjust λ iteratively until the space required comes exactly or very close to the available storage space.

EXAMPLE

A shop produces three machines parts I, II, III in lots. The shop has only 650 square feet of storage space. The appropriate data for the three items are given in the below table:

Item	I	II	III
Demand rate (unit/year)	5,000	2,000	10,000
Procurement cost (Rs/order)	100	200	75
Cost per unit (Rs)	10	15	5
Floor space required (sqft/unit)	0.7	0.8	0.4

The shop uses an inventory carrying charge of 20 percent of average inventory valuations per year. If no stock outs are allowed, determine the optimal lot size for each item under the given storage constraint.

SOLUTION

For Lambda = 1, computing (Qi)* (I = 1,2,3) for each item as follows:

$(Q_1)^*$ = Square root [(2 x 5000 x 100) / (0.2 x 10 + 2 x 0.7)] = 542 units

$(Q_2)^*$ = Square root [(2 x 2000 x 200) / (0.2 x 15 + 2 x 0.8)] = 417 units

$(Q_3)^*$ = Square root [(2 x 10000 x 75) / (0.2 x 5 + 2 x 0.4)] = 913 units

Therefore, the total space requirements would be,

Summation (i=1 to n) of f_i x Q_i*

=0.7 x 542 + 0.8 x 417 + 0.4 x 913

= 1078 units

(which is greater than the 650 sqft available storage space)

SOLUTION

The requirement is much above the maximum available storage space.

Computing the above for lambda = 5, we get,

$(Q_1)^* = 333$ units

$(Q_2)^* = 270$ units

$(Q_3)^* = 548$ units

In this case the storage space is 668.3 sqft which is slightly more than total available space of 650 sqft.

If we use the value of Lambda = 5.4, the total space is 649.6, which is below the total available space.

EOQ model with Investment Constraint:

The need to control costs is of major concern to all types of business.

One of the primary controllable costs for business, both retail and manufacturing, is associated with inventory investment and management.

EOQ model with Investment Constraint:

These costs include both acquisitions costs and the related holding period costs.

Holding period costs includes items such as insurance, storage and taxes.

The holding cost can also include the time value of money.

EOQ model with Investment Constraint:

Thus, firms that finance inventory purchases may have higher inventory holding cost and therefore, gain more by minimizing the holding cost.

In a competitive market cost containment goals can be attained by keeping inventory to a minimum level.

In effect, maintaining inventory levels adequately minimizes the total cost.

EOQ model with Investment Constraint:

Inventory represents a substantial investment capital for many firms. Thus the decision maker places a limit on the amount of inventory to be carried.

The inventory control policy must then be adjusted to meet this objective if the total investment exceeds the limit.

EOQ model with Investment Constraint:

Suppose the inventory is controlled by a reorder level control policy (Q-systems) under the conditions of demand and lead time certainty.

If a monetary limit is placed on all items carried, then we can state that

$$\Sigma(i=1 \text{ to } n)\ C_i \times Q_i \leq F$$

EOQ model with Investment Constraint:

The problem is to minimize the total variable inventory cost under the investment constraint

Min of TVC = Σ (i=1 to n) [[[(D_i / Q_i)x C_{oi}]+ [(Q_i /2) x C_{hi}]

Subject to the constraint that,

$\sum$ (i=1 to n) C_i $Q_i \leq F$ and $Q_i \geq 0$ for all values of i.

Kuhn-Tucker necessary and sufficient conditions for optimal value of TVC:

Let λ be the non-negative Lagrange multiplier, then the Lagrangian function becomes

$L(Q_i,\lambda) = TVC + \lambda \{\Sigma \text{ (i=1 to n) } C_i \times Q_i - F\}$, where $\lambda \geq 0$

The necessary conditions for L to be minimum are

$\delta L/\delta Q_i = (-)\ D_i/(Q_i)^2.C_{oi} + ½ \times C_{hi} + \lambda C_i = 0$ or in other ways to represent

$(Q_i)^* =$ Square root of $(2$ times $D_i \times C_{oi}) / (C_{hi} + 2 \times \lambda \times C_i)$

$\delta L/\delta\lambda = \Sigma \text{ (i=1 to n) } C_i \times Q_i - W = 0$

Example:

A shop produces three machines parts I, II, III in lots. The demand rate for each item is constant and can be assumed to be deterministic. No back orders are to be allowed. The pertinent data for the items is given in the following table.

Example:

Item	I	II	III
Carrying cost (Rs per unit per year)	20	20	20
Setup Cost (Rs/setup)	50	40	60
Cost per unit (Rs)	6	7	5
Yearly demand (units)	10000	12000	7500

Determine the approximate economic order quantities for three items subject to the condition that the total value of average inventory levels of these items does not exceed Rs. 1000.

Solution:

For Lambda = 1, computing (Qi)* (I = 1,2,3) for each item as follows:

$(Q_1)^*$ = Square root [(2 x 10000 x 50) / (20 + 2 x 6)] = 177 units
$(Q_2)^*$ = Square root [(2 x 12000 x 40) / (20 + 2 x 7)] = 168 units
$(Q_3)^*$ = Square root [(2 x 7500 x 60) / (20 + 2 x 5)] = 173 units

Solution:

Based on the above values, the investment over the average inventory at any time is given by

$\sum(i=1 \text{ to } 3)\ C_i \times (Q_i/2)$

$= 6\ (177/2) + 7 \times (168/2) + 5 \times (173/2)$

$= \text{Rs. } 1551.5 > \text{Rs. } 1000$ (fund available)

Solution:

This requirement is more than the maximum fund available.

Therefore, conducting another trial with λ value = 4, we get the Corresponding investment on average inventory to be Rs. 1112.5 which is also slightly higher than the Fund available (Rs. 1000)

Solution:

If we use the value of Lambda = 4.7, then we get the corresponding investment on average inventory to be Rs. 999.50 which is very close to the available fund (Rs. 1000)

EOQ model with Average Inventory Level constraint

Since the average number of units in the inventory of an item (i) is $Q_i /2$ and it is required that the average number of units of all items together held in the inventory should not exceed the number M.

Therefore we must have,

$$\frac{1}{2} \Sigma \text{ (I =1 to n) } Q_i \leq M$$

EOQ model with Average Inventory Level constraint

The problem of minimizing the total variable inventory cost under the total average inventory constraint can be formulated as
Min TVC =Σ(i = 1 to n) [[(D_i by Q_i). C_{oi}]+ [(Q_i by 2).C_{hi}]

Subject to the constraint,
½ Σ (I =1 to n) $Q_i \leq M$ and $Q_i \geq 0$ for all i.

EOQ model with Average Inventory Level constraint

$L(Q_i, \lambda) = TVC + \lambda \{ \frac{1}{2} \times \Sigma \text{ (i=1 to n) } Q_i - F\}$, where $\lambda \geq 0$

The necessary conditions for L to be minimum are

$\delta L/\delta Q_i$ =(minus) $D_i/(Q_i)^2 \cdot C_{oi}$ plus $\frac{1}{2} \cdot C_{hi}$ plus $\lambda/2$ which =0 or in other ways to represent

EOQ model with Average Inventory Level constraint

$(Q_i)^*$=Square root of (2 times D_i . C_{oi}) / (C_{hi} + λ)

$\delta L/\delta\lambda$ =[½ x Σ (i=1 to n) Q_i - M =0.

EOQ model with Average Inventory Level constraint

Problem Example:

A shop produces three machines parts I, II, III in lots. The shop has only 560 square feet of storage space. The appropriate data for the three items are given in the below table

EOQ model with Average Inventory Level constraint

Problem Example:

Item	I	II	III
Demand rate (unit/year)	5,000	2,000	10,000
Procurement cost (Rs/order)	100	200	75
Cost per unit (Rs)	10	15	5
Floor space required (sqft/unit)	0.7	0.8	0.4

Determine the optimal number of units of each item, separately, so as to satisfy the given constraint.

EOQ model with Average Inventory Level constraint

Solution:

Compute the values as done earlier.

For Lambda = 1, computing $(Q_i)^*$ (I = 1,2,3) for each item as follows:

$(Q_1)^*$ = Square root [(2 x 5000 x 100) / (0.2 x 10 + 2 x 0.7)]

= 542 units

$(Q_2)^*$ = Square root [(2 x 2000 x 200) / (0.2 x 15 + 2 x 0.8)]

= 417 units

$(Q_3)^*$ = Square root [(2 x 10000 x 75) /(0.2 x 5 + 2 x 0.4)]

= 913 units

EOQ model with Average Inventory Level constraint

Solution:

For Lambda = 5.4, we get,

$$Q_1 = 324 \text{ units}$$
$$Q_2 = 262 \text{ units}$$
$$Q_3 = 531 \text{ units}$$

The average inventory level then becomes,

$$\tfrac{1}{2}(324+262+531) = 559 \text{ units}$$

which is very close to the required average inventory level of 560 units.

Thus, Q_1, Q_2 and Q_3 are the optimal quantities of the 3 items respectively.

EOQ model with Number of Orders Constraint:

There is a general approach what is not optimal but is considered reasonable.

It is used to find EOQ in case of multi-item inventory problem with a constraint on the number of orders to be placed per year

EOQ model with Number of Orders Constraint:

This approach can also be used where ordering cost per order and carrying cost per unit per time period are not known. However it works with the following assumptions:

1. Ordering cost and carrying cost are same for all items
2. Orders are received in lots
3. Demand is constant
4. Stockouts are not permitted

EOQ model with Number of Orders Constraint:

Under the above assumptions the Total Number of orders per year (N = D / Q) for all items can be determined as:

Number of orders per year = N x (Square root of DC) divided by Sigma x Square root of DC)

Where, DC = Demand in rupees and N
= specified number of orders.

EOQ model with Number of Orders Constraint:

Problem Example:

A company has to purchase four items A, B, C and D for the next year. He projected demand and unit price (in Rs.) are as follows

Item	Demand(units)	Unit Price (Rs)
A	60000	3
B	40000	2
C	1200	24
D	5000	4

If the company wants to restrict the total number of orders to 40 for all the four items, how many orders should be placed for each item?

EOQ model with Number of Orders Constraint:

Solution:

Item	Demand (units)	Unit Price (Rs)	√DC
A	60000	3	424.26
B	40000	2	282.84
C	1200	24	169.70
D	5000	4	141.22
Total			1018.22

EOQ model with Number of Orders Constraint:

Solution

Item	Demand (units)	Unit Price (Rs)	√DC	√DC / Σ√Dc
A	60000	3	424.26	0.416
B	40000	2	282.84	0.277
C	1200	24	169.70	0.166
D	5000	4	141.22	0.138
Total			1018.22	

EOQ model with Number of Orders Constraint:

Solution

Item	Demand (units)	Unit Price (Rs)	√DC	√DC / Σ√Dc	No. of orders per year
A	60000	3	**424.26**	**0.416**	**16.64=17**
B	40000	2	**282.84**	**0.277**	**11.08=11**
C	1200	24	**169.70**	**0.166**	**6.64 = 7**
D	5000	4	**141.22**	**0.138**	**5.2 = 5**
Total			**1018.22**		