Course Details	
Subject	Statistics
Year	3rd Year B.Sc.
Paper no	15
Paper Name	Operation Research
Topic no	29&30
Topic name	Critical Path Identification, Slack Time and Float
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E-Learning Module on Critical Path Identification, Slack Time and Float

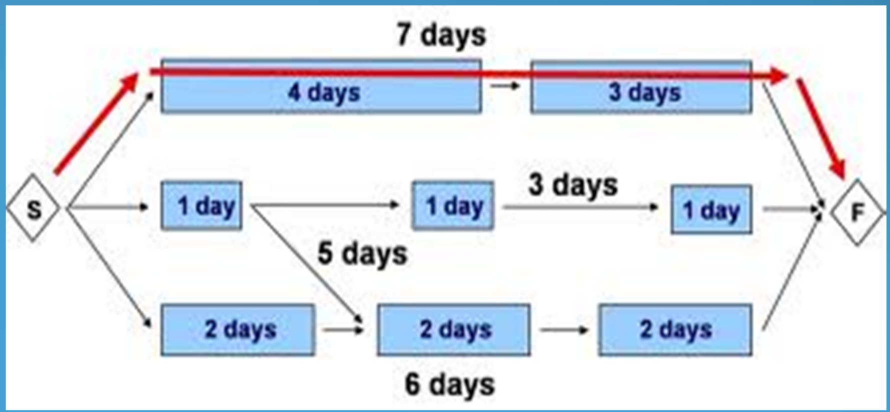
Learning Objectives

By the end of this session, you will be able to:

- Determine critical path
- Identify floats associated with non-critical activities and events
- Explain total project completion time

Introduction

Critical path is the longest path through the project network; the activities on the path are the critical activities therefore any delay in their completion must be avoided to **prevent delay** in project completion.



Introduction

The objective of critical path analysis is to estimate the total project duration and to assign **starting** and **finishing times** to all activities involved in the project.

This helps to check the actual progress against the schedule duration of the project.

Introduction

The duration of individual activities may be uniquely determined (in case of CPM)nor may involve the three time estimates (in case of PERT), out of which the expected duration of an activity is computed.

Introduction

Factors to prepare project scheduling

- Total completion time of the project
- Earlier and latest start time of each activity
- Critical activities and critical path
- Float for each activity, that is the amount of time by which the completion of a non-critical activity can be delayed, without delaying the total project completion time

Notations

E_i	Earliest occurrence time of an event i. This is the earliest time for an event to occur immediately after all the preceding activities have been completed, without delaying the entire project
L_i	Latest allowable time of an event i. This is the latest time at which an event can occur without causing a delay in already determined projects completion time

Notations

ES_{ij}	Early starting time of an activity (I, j). This is the earliest time an activity can possibly start without affecting the project completion.
LS_{ij}	Late starting time of an activity (I, j). This is the latest possible time an activity can start without delaying the project completion.

Notations

EF_{ij}	Early finishing time of an activity (I,j). This is the earliest time an activity can possibly finish without affecting the project completion.
LF_{ij}	Late finishing time of an activity (I, j). This is the latest time an activity must finish without delaying the project completion.
t_{ij}	Duration of an activity (I, j)

Events

It has already been mentioned that in a network diagram there should only be one initial event and one end event.

The other events are numbered consecutively with integer 1, 2, …., n, such that, $i < j$ for any two events i and j connected by an activity which starts at i and finishes at j.

Calculation

For calculating the earliest occurrence and latest allowable times of events, the following two methods are used, that is:

- Forward pass method
- Backward pass method

Forward pass method (for earliest event time):

In this method, calculations begin from the initial event 1, proceed through the events in an increasing order of event numbers and end at the final event, say N.

Forward pass method (for earliest event time):

At each event we calculate its earliest occurrence time (E) and earliest start and finish time for each activity that begins at that event.

When calculations end at the final event N, its earliest occurrence time gives the earliest possible completion time of the entire project.

Forward pass method (for earliest event time): Summarized

1. Set the earliest occurrence time of initial event 1 to zero. i.e., $E_1 = 0$, for i=1

2. Calculate the earliest time for each activity that begins at event i(=1). This is equal to the earliest occurrence time of event, i(tail event). i.e., $ES_{ij} = E_i$, for all activities (I, j) starting at event i

Forward pass method (for earliest event time): Summarized

3. Calculate the earliest finish time of each activity that begins at event i. This is equal to the earliest start time of the activity + the duration of the activity.
   i.e., $EF_{ij} = ES_{ij} + t_{ij} = E_i + t_{ij}$, for all activities (I, j), beginning at event i

Forward pass method (for earliest event time): Summarized

4. Proceed to the next event, say j; j > i

5. Calculate the earliest occurrence time for the event j. This is the maximum of the earliest finish time of all activities ending into that event, i.e., $E_j = \text{Max}\ \{ E_i + t_{ij}\}$, for all immediate predecessor activities

Forward pass method (for earliest event time): Summarized

6. If j = N (final event), then earliest finish time for the project, that is, the earliest occurrence time E_s for the final event is given by: E_N = max $\{EF_{ij}\}$ = Max $\{ E_{N-1} + t_{ij}\}$, for all terminal activities.

Backward Pass Method (for latest allowable Event Time)

In this method, calculations begin from the final event N.

Proceed through the events in the decreasing order of event numbers and end at the initial event 1.

Backward Pass Method (for latest allowable Event Time)

At each event, we calculate the latest occurrence time (L) and latest finish and start time for each activity that is terminating at that event and this procedure continues till the initial event.

Backward Pass Method (for latest allowable Event Time)- Summarized

1. Set the latest occurrence of last event, N equal to its earliest occurrence time (known from forward pass method). i.e., $L_N = E_N$, $j = N$

2. Calculate the latest finish time of each activity which ends at event j. This is equal to latest occurrence time of final event. i.e., $LF_{ij} = L_j$; for all activities (I, j) ending at event j.

Backward Pass Method (for latest allowable Event Time)- Summarized

3. Calculate the latest start times of all activities ending at j. This is obtained by subtracting the duration of the activity from the latest finish time of the activity. i.e., $LF_{ij} = L_j$ and $LS_{ij} = LF_{ij} - t_{ij} = L_j - t_{ij}$, for all activity (I, j) ending at event j.

Backward Pass Method (for latest allowable Event Time)- Summarized

4. Proceed backward to the event in the sequence, that decreases j by 1

5. Calculate the latest occurrence time of event I (i<j). This is the minimum of the latest start times of all activities from the event. i.e., L_i = Min { LS_{ij}} = min {L_j - t_{ij}}, for all immediate successor activities

Backward Pass Method (for latest allowable Event Time)- Summarized

6. If j=1 (initial event), then the latest finish time for project, that is latest occurrence time L_i for the initial event is given by $L_1 = \text{Min}\{ LS_{ij}\} = \text{Min}\{ L_{j-1} - t_{ij}\}$, for all immediate successor activities

Float (slack) of an Activity and Event

The float (slack) or free time is the length of time in which a non-critical activity and/or an event can be delayed or extended without delaying the total project completion time.

Float (slack) of an Activity and Event

Slack of an Event: The slack(s) also called float of an event is the difference between its latest occurrence time (L_i) and its earliest occurrence time (E_i).

Float (slack) of an Activity and Event

i.e., Event float = $L_i - E_i$, it is a measure of how long an event can be delayed without increasing the project completion time.

L = E	Such events are called critical events
L ≠ E	The slack on these events can be negative (L<E) or positive (L > E)

Float (slack) of an Activity and Event

Slack of an activity: It is the amount of time that an activity can be delayed without delaying project completion, it is calculated as the difference between the latest finish time and the earliest finish time for the activity.

Float (slack) of an Activity and Event

In other words the computation of activity floats tell us how long an activity time may be increased without increasing the project completion time.

Mainly three types of floats are defined for each non-critical activity of the project.

Float (slack) of an Activity and Event

Total float: This is the length of time by which an activity can be delayed when all preceding activities are completed at their earliest possible time and all successor activities can be delayed until their latest permissible time.

Float (slack) of an Activity and Event

The time within which an activity must be scheduled is computed from LS and ES values for each activity's start event and end event.

i.e., for each activity (I, j),

Total float = the latest allowable time for the event at the end of the activity - the earliest time for an event at the beginning of the activity - the activity duration.

i.e., Total float (TF_{ij}) = (L_j – E_i) –t_{ij} = LS_{ij} – ES_{ij} = LF_{ij} – EF_{ij}.

Float (slack) of an Activity and Event

Free float: For calculating the total float, only a particular activity was considered with respect to its tail and head event occurrence times or by considering latest start and finish time of an activity with respect to its earliest start and finish time.

Float (slack) of an Activity and Event

However, we may need to know how much an activity's completion time may be delayed without causing any delay in its immediate successor activities.

Thus, the free float of a non-critical activity is defined as the time by which the completion of an activity can be delayed without causing any delay in its immediate succeeding activities.

Float (slack) of an Activity and Event

Free float values for each activity (I, j) are computed as follows:

Free float (FF_{ij}) = $(E_j - E_i) - t_{ij}$ = Min { ES_{ij}, for all immediate successors of activity (I, j)} – EF_{ij}.

Float (slack) of an Activity and Event

Independent float: This is the amount of acceptable delay in the completion of an activity so that it neither affects its predecessor nor the successor activities.

Float (slack) of an Activity and Event

Thus, independent float is the amount of time available when preceding activities are completed at their latest permissible times and all the following activities can still be completed at their earliest possible time.

Float (slack) of an Activity and Event

Independent float values for each activity are computed as follows:

Independent float $(IF_{ij}) = (E_j - L_i) - t_{ij} = \{ES_{ij} - LS_{ij}\} - t_{ij}$.

The negative value of independent float is considered to be zero.

Float (slack) of an Activity and Event - Remark

1. Latest occurrence time of an event is always greater than or equal to its earliest occurrence time (i.e. $L_i \geq E_j$). $TF_{ij} \geq (E_i - E_j) - t_{ij}$ or $TF_{ij} \geq FF_{ij}$.

   This implies that the value of free float may range from zero to total float and can never exceed the total float value.
   i.e., independent float ≤ free float ≤ total float.

Float (slack) of an Activity and Event - Remark

2. The calculation of various floats can help the decision maker in identifying the underutilized resources, flexibility in the total schedule and possibilities of redeployment of resources.

Float (slack) of an Activity and Event - Remark

3. Once the float of an activity is disturbed, the float of all other activities of the project is changed and should be recalculated.

Float (slack) of an Activity and Event - Remark

4. Few conclusions which can be drawn from the total float value of an activity are as follows:
   a. Total float value is negative when L - E < 0 – Project completion is behind the schedule data, that is, the resources are not adequate and activity may not finish in time. This needs extra resources or certain activities need crashing in order to reduce negative float value.

Float (slack) of an Activity and Event - Remark

4. Few conclusions which can be drawn from the total float value of an activity are as follows:
   b. Total float value is Zero when L - E = 0 – Resources are just sufficient to complete the activity. Any delay in activity execution will necessarily increase the project cost

Float (slack) of an Activity and Event - Remark

4. Few conclusions which can be drawn from the total float value of an activity are as follows:
   c. Total float value is positive when L - E >0 - Project completion is ahead of the schedule date, that is resources are surplus. These resources can be deployed elsewhere or execution of the activity can be delayed.

Critical Path

Certain activities in a network diagram of a project are called critical activities because delay in their execution will cause further delay in the project completion time.

Thus, all activities having zero total float value are identified as critical activities.

Critical Path

The critical path is the sequence of critical activities that form a continuous path between the start of a project and its completion.

This is critical in the sense that, if any activity in these sequences is delayed , the completion of the entire project will be delayed.

The critical path is shown by a thick line or double lines in the network diagram.

Critical Path

The length of the critical path is the sum of the individual times of all the critical activities lying on it and defines the longest time to complete the project.

Critical Path

1. For all activities (I, j) lying on the critical path, the E-values and L-values for tail and head event are equal. i.e., $E_j = L_j$ and $E_i = L_i$

2. On critical path, $E_j - E_i = L_j - L_i = t_{ij}$.

3. Finding the critical path is important for directing the decision maker's attention because delay in any one of these activities will increase the project completion time.

Critical Path - Example

An established company has decided to add a new product to its line.

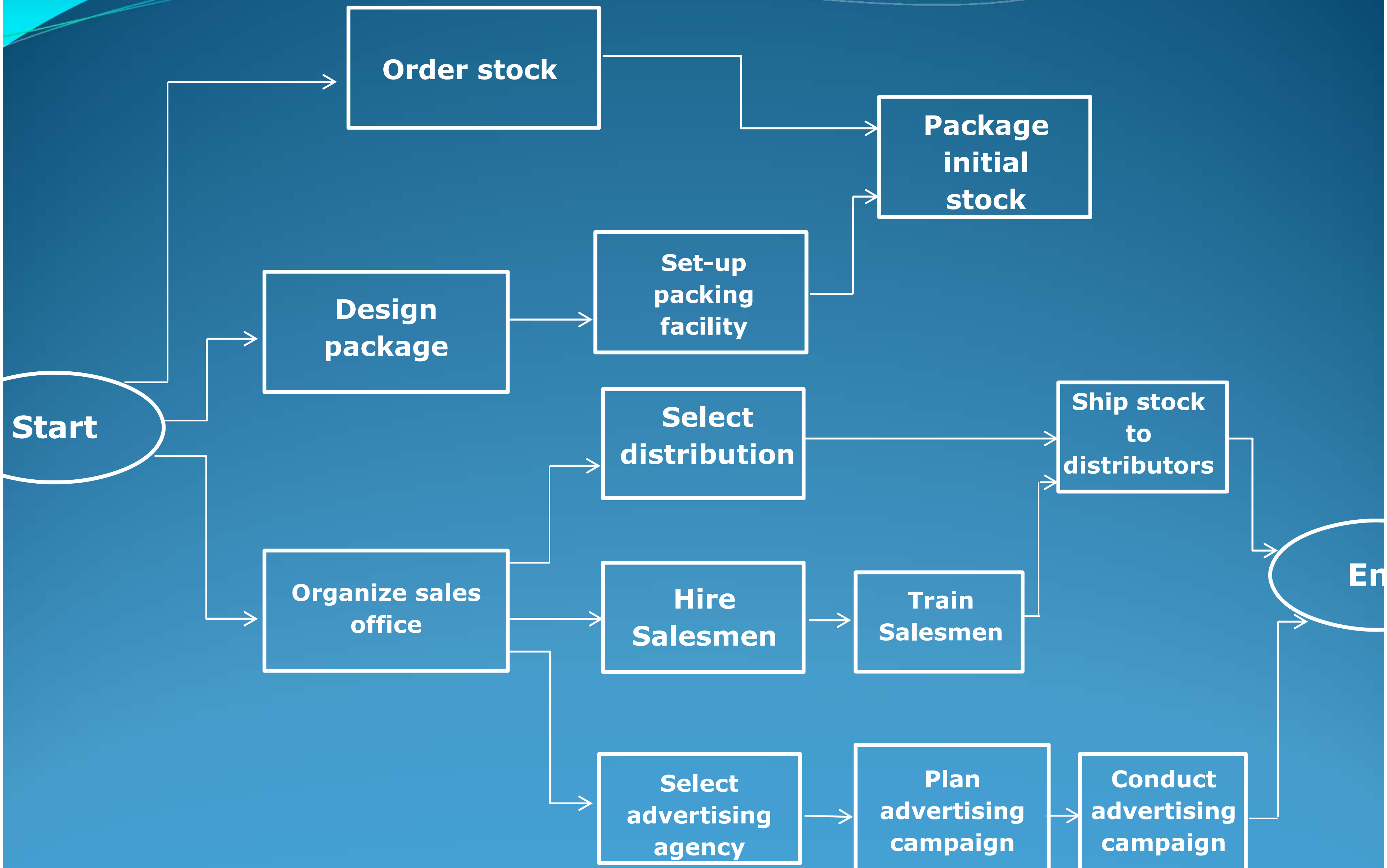
It will buy the product from a manufacturing concern, package it, and sell it to a number of distributors that have been selected on a geographical basis.

Market research has already indicated the volume expected and the size of sales force required.

Critical Path - Example

Activity	DESCRIPTION	PREDECESSORS	DURATION (DAYS)
A	ORGANISE THE SALES OFFICE	-	6
B	HIRE SALESMAN	A	4
C	TRAIN SALESMAN	B	7
D	SELECT ADVERTISING AGENCY	A	2
E	PLAN ADVERTISING CAMPAIGN	D	4
F	CONDUCT ADVERTISING CAMPAIGN	E	10
G	DESIGN PACKAGE	-	2
H	SETUP PACKAGING FACILITIES	G	10
I	PACKAGE INITIAL STOCK	J, H	6
J	ORDER STOCK FROM MANUFACTURER	-	13
K	SELECT DISTRIBUTOR	A	9
L	SELL TO DISTRIBUTOR	C, K	3
M	SHIP STOCKS TO DISTRIBUTOR	I, L	5

Critical Path - Example



Critical Path - Example

The company can begin to organize the sales office, design the package, and order the stock immediately.

Also the stock must be ordered and the packing facility must be set up before the initial stocks are packaged.

i. Draw an arrow diagram for this project
ii. Indicate the critical path
iii. For each non-critical activity, find the total and free float

Critical Path - Solution

The arrow diagram for the given project, along with the E – values and L- values, is shown in the figure.

Let us determine the earliest start time (E_i) and the latest finish time (L_j) for each event by proceeding as follows:

Critical Path - Solution

Forward pass method:

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 6 = 6$

$E_3 = E_1 + t_{1,3} = 0 + 2 = 2$

$E_4 = \text{MAX}\ \{E_i + t_{i,4}\} = \text{MAX}\ \{E_1 + t_{1\ 4}; E_3 + t_{3\ 4}\}$ where, I = 1, 3 which = MAX { 0 + 13, 2 +10} = 13

$E_5 = E_2 + t_{2,\ 5} = 6 + 4 = 10$

Critical Path - Solution

Forward pass method:

E_6 = MAX $\{E_i + t_{i,\,6}\}$ where I = 2, 5 = MAX $\{E_2 + t_{2,\,6};\ E_5 + t_{5,\,6}\}$ = MAX {6 + 9; 10 + 7} = 17

$E_7 = E_2 + t_{2,\,7} = 6 + 2 = 8$

$E_8 = E_7 + t_{7,\,8} = 8 + 4 = 12$

E_9 = MAX $\{E_i + t_{i,\,9}\}$ where I = 4, 6 = MAX $\{E_4 + t_{4,\,9};\ E_6 + t_{6,\,9}\}$ = MAX{13 + 6; 17 + 3} = 20

E_{10} = MAX $\{E_i + t_{i,\,10}\}$ where I = 8, 9 = MAX $\{E_8 + t_{8,\,10};\ E_9 + t_{9,\,10}\}$ = MAX { 12 + 10; 20 + 5} = 25

Critical Path - Solution

Backward Pass method:

$L_{10} = E_{10} = 25$

$L_9 = L_{10} - t_{9,\,10} = 25 - 5 = 20$

$L_8 = L_{10} - t_{8,\,10} = 25 - 10 = 15$

$L_7 = L_8 - t_{7,\,8} = 15 - 4 = 11$

$L_6 = L_9 - t_{6,\,9} = 20 - 3 = 17$

$L_5 = L_6 - t_{5,\,6} = 17 - 7 = 10$

$L_4 = L_9 - t_{4,\,9} = 20 - 6 = 14$

Critical Path - Solution

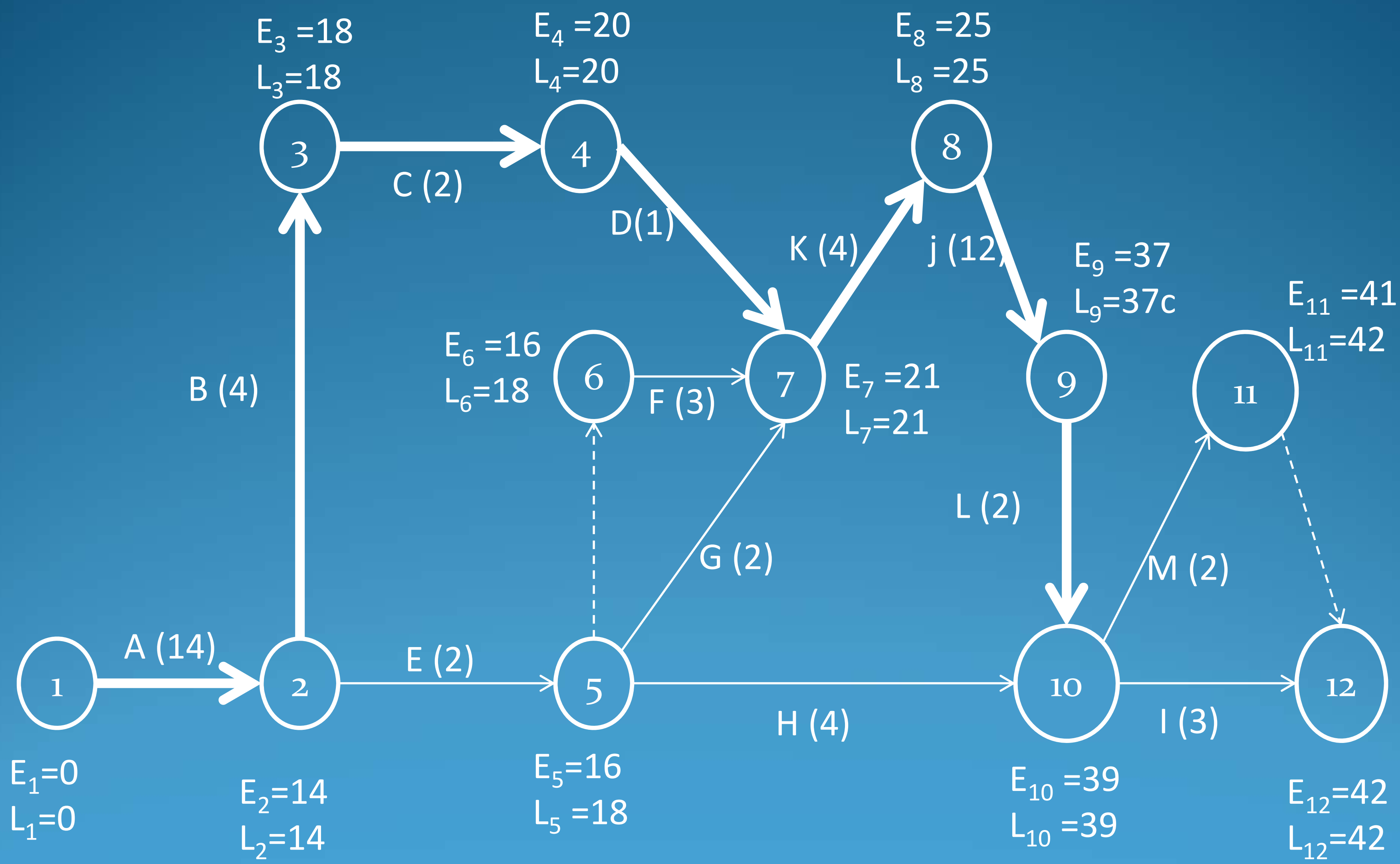
Backward Pass method:

$L_3 = L_4 - t_{3,4} = 14 - 10 = 4;$

$L_2 = \text{MIN} \{ L_j - t_{2,j} \}$ where j =5, 6, 7 = MIN{ $L_5 - t_{2,5}$; $L_6 - t_{2,6}$; L7 - $t_{2,7}$} =MIN{ 10 - 4: 17 - 9; 11 - 2} =6;

$L_1 = \text{MIN} \{ L_j - t_{1,j} \}$ where j =2, 3, 4 =MIN{ $L_2 - t_{1,2}$; $L_3 - t_{1,3}$; $L_4 - t_{1,4}$} =MIN {6 - 6; 4 - 2; 14 - 13} =0

Critical Path - Solution



Critical Path - Solution

Activity	Duration	Earliest time		Latest time		Float	
		Start (E_i)	Finish (E_i+t_{ij})	Start ($L_j - t_{ij}$)	Finish (L_j)	Total ($l_j - t_{ij}$)- E_i	Free ($E_j - E_i$) $-t_{ij}$
1-3	2	0	2	2	4	2	0
1-4	13	0	13	1	14	1	0
2-6	9	6	15	8	17	2	2
2-7	2	6	8	9	11	3	0
3-4	10	2	12	4	14	2	1
4-9	6	13	19	14	20	1	1
7-8	4	8	12	11	15	3	0
8-10	10	12	22	15	25	3	3