1. Introduction

Welcome to the series of E-learning modules on Economic lot size models for the case of known uniform demand and instantaneous.

By the end of this session, you will be able to:

- Explain the single item inventory control models without shortage

Let us start with an introduction:
The understanding of the nature of demand, that is its size and pattern, for the given inventory item is essential to determine an optimal inventory policy for that item.
The size of demand refers to the number of units of the items required in each period (cycle or season).

The size is not measured in terms of the number of units sold because the demand may remain unfulfilled due to shortage of sufficient stock or due to delay in delivery.
The size of demand may be either deterministic or probabilistic.

In the deterministic case, the demand over a period of time is known with certainty. This can be fixed or static or can vary (dynamic) from period to period.
But in the probabilistic case, the demand over a period of time is not known with certainty but the nature of such demand can be described by a known probability distribution.

The known probability distribution may be either stationary or non-stationary from period to period.
These terms are equivalent to static and dynamic demand.
The pattern of demand is the manner in which inventory items are required by the customers.

The demand for a given period of time may be satisfied instantaneously at the beginning of the period, or uniformly during that period.
The effect of instantaneous and uniform demand reflects directly on the total inventory cost.

2. Replenishment Lead Time

Replenishment lead time:
Order cycle: The order cycle is the time period between two successive replenishments and is determined in the following ways:

- **Continuous review:** This is also called perpetual inventory record, because the number of units of an item on hand is always known. In this case an order of fixed size is placed every time the inventory level reaches at an ore-specified level called order point or reorder level. This decision rule is also referred to as the two bin system, fixed order system or Q-system

- **Periodic review:** In this case the orders are placed at equal intervals of time, but the size of the order may vary depending on the inventory on hand as well as an order at the time of the review. This decision rule is also referred to as the fixed order interval system or P-system

Decisions on the size and timing of replenishment orders for an item can be made by adopting the following basic inventory control policies or systems:
Following are the policy under Continuous review system:

- **(s, Q) policy:** whenever the inventory position (items on hand plus item on order) drops to a given level ‘s’ or below, an order is placed for a fixed quantity ‘Q’. This policy is also known as a fixed order quantity policy or re-order point policy. Here ‘s’ is also termed as reorder point or level, ‘I’ is denoted by ‘R’

- **(s, S) policy:** whenever the inventory position (items on hand plus item on order) drops to a given level ‘s’ or below, an order is placed for a sufficient quantity to bring the inventory position up to a given level ‘S’ (a pre-determined maximum level)

Following are the policy under Periodic review systems:

- **(T, S) policy:** inventory position (items on hand plus item on order) is reviewed at regular intervals of time, spaced a time intervals of length ‘T’. At each review, an order is placed for a sufficient quantity to bring the inventory position up to a predetermined maximum level

- **(T, s, S) policy:** Inventory position (items on hand plus item on order)is reviewed at regular intervals of time, spaced at time intervals of length ‘T’. At each review, if the inventory position is at level ‘s’ or below, an order is placed for sufficient quantity to bring inventory position up to a given level, S; if the inventory position is above ‘s’, no order is placed. This policy is also known as periodic review policy or fixed interval policy

Lead time or delivery lag:

- When an order is placed, it may require some time before the delivery of the items ordered is reached
- The time delay between placing an order and receipt of delivery is called delivery lag or lead time
- In general, the lead time may be deterministic or probabilistic

Stock replenishment:
Although an inventory may operate with lead time, the actual replenishments of stock may occur instantaneously or gradually.
Instantaneous replenishment is possible when the stock is purchased from outside sources, while gradual replenishment is possible due to a finite production within the firm.

Length of planning period:

- The length of planning period defines period over which a particular inventory level will be maintained
- This period may be finite or infinite depending on the nature of the demand

3. Replenishment Order Size Decision & Concept of EOQ

Let us discuss about Replenishment order size decision and concept of EOQ:
The size of replenishment orders affects inventory level to be maintained at various stocking points.
Large order quantities may reduce the frequency of orders to be placed to procure inventory items and reduce the total ordering cost.
This decision will however increase the cycle stock inventories and the cost of carrying inventories.

Any decision on replenishment order size or batch size for production, should provide economical trade-off between relevant inventory costs, that is ordering, carrying and shortage costs and is stated in terms of economic order (or lot size) quantity (EOQ).

This concept was first developed by Ford W Harris in 1913 for finding the optimum order quantity to balance costs of holding excess stock against that of ordering small quantities too frequently.
This model has become the basic economic order quantity formula and it serves as the basis for many of the inventory policies that are currently in practice.

Economic order quantity is:

- The optimal replenishment order size or lot size of inventory item or items that achieves the optimum total or variable inventory cost during the given period of time
- The order quantity that minimizes total inventory holding costs and ordering costs

It is one of the oldest classical production scheduling models.
The framework used to determine this order quantity is also known as Barabas EOQ Model or Barabas Formula.

Despite its highly restrictive assumptions that:

- Demand is relatively constant and is known or predictable
- The item is purchased in lots or batches and not continuously
- The order and preparation costs (acquisition or purchase cost per order) and the inventory carrying costs are constant and known and
- Replacement of inventory occurs all at once

And knowing how to apply EOQ practically is just as important as being able to use the formula calculation itself.
Advantageously, EOQ is very insensitive to parameter errors because those errors are muted by the presence of the square root function in the EOQ formula.

Such insensitivity is advantageous whenever EOQs are computed with imprecise estimates, forecasts or costs.

EOQ involves determining the optimal quantity to purchase when orders are placed. For example, small orders result in low inventory levels and inventory carrying costs, frequent orders and higher ordering costs; while large orders result in higher inventory levels and inventory carrying costs and infrequent orders and lower ordering costs.

Classification of EOQ models:
A broad classification of EOQ models into three categories are:

1. **Models with no shortage -** Demand rate constant in all cycles, supply is instantaneous; different rates of demand in different cycle but total demand is known over the entire planning period and demand rate constant, supply is non-instantaneous
2. **Models with shortage -** Variable order cycle time, supply is instantaneous; constant order cycle time, supply is gradual; demand rate gradual, supply is non – instantaneous
3. **Resource constraints models** – Floor space constraints, investment constraints, inventory size constraints

List of symbols used for the development of various inventory models discussed in this table and the brackets indicate the unit of measurement of each of them.
'C' denotes Purchase (or manufacturing) cost of an item (Rs. Per unit).
'C' naught denotes Ordering (or set-up) cost per order (Rs per order).
'r' denotes Cost of carrying one rupees worth of inventory expressed in terms of percent of rupee value of inventory (per cent per unit time).
'C' of 'h' denotes C into r equal to cost of carrying one unit of an item in the inventory for a given length of time (Rs per item per unit time).
'C' of 's' denotes Shortage cost per unit per time (Rs per unit time).
'D' denotes Annual requirement (demand) of an item (units per unit time).
'Q' denotes Order quantity, that is number of units ordered per order in units.
ROL denotes Reorder level (or point) that is the level of inventory at which an order is placed (units).
LT denotes Replenishment lead time (time period).
'n' denotes Number of orders per time period (orders per unit time).
't' denotes Reorder cycle time, that is the time interval between successive orders to replenish (time period)
't' of 'p' denotes Production period (time period).
'r' of 'p' denotes Production rate, that is the rate at which quantity Q is added to inventory (quantity per unit time).
TC denotes Total inventory cost in Rs.
TVC denotes Total variable inventory cost in Rs.

4. EOQ Model with Constant Rate of Demand

EOQ model with constant rate of demand:
In this model, demand is assumed to occur at a constant rate for an infinite time into the future.
The objective is to select an inventory policy, that is to choose an economic order quantity Q star EOQ, the ordering frequency (time when an order must be placed) in such a way that the total yearly inventory cost is minimized.

For this model, the following characteristics or inputs are assumed:

- The inventory system involves one type of item or product
- The demand is known and constant and is resupplied instantaneously
- The inventory is replenished in single delivery for each order
- Lead time (LT) is constant and known, that is replenishment is instantaneous, so that inventory increases by Q units as soon as an order is placed

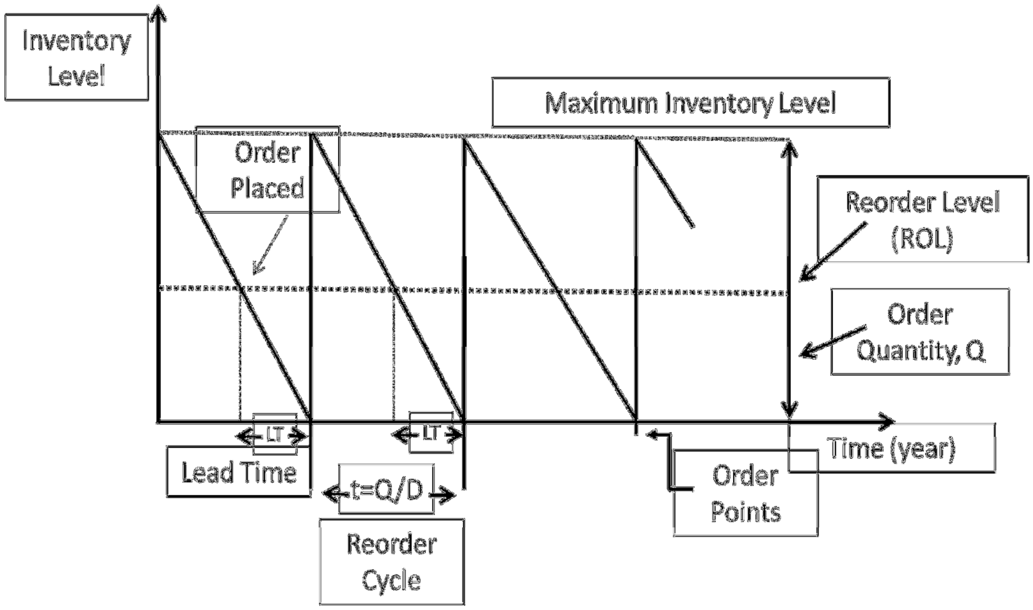
- Shortages are not allowed. That is, there is always enough inventory on hand to meet the demand
- Purchase price and reorder costs do not vary with the quantity ordered. That is quantity discount is not available
- Carrying cost per year (as a fraction of product cost) and ordering cost per order are known and constant
- Each item is independent and money cannot be saved by substituting by other items or grouping cost several items into a single order

Though in practice these assumptions seem unrealistic, however, we should remember the following two points:

1. The main purpose of this simplified model is to derive useful results rather than representing real life problems. The result so obtained may not provide an optimal answer to real life problems but they are good approximations and provide useful guidelines

2. It is a basic model and can be extended in many ways. The few assumptions made in this model will be removed in the subsequent models to bring them close to realistic problems of inventory control

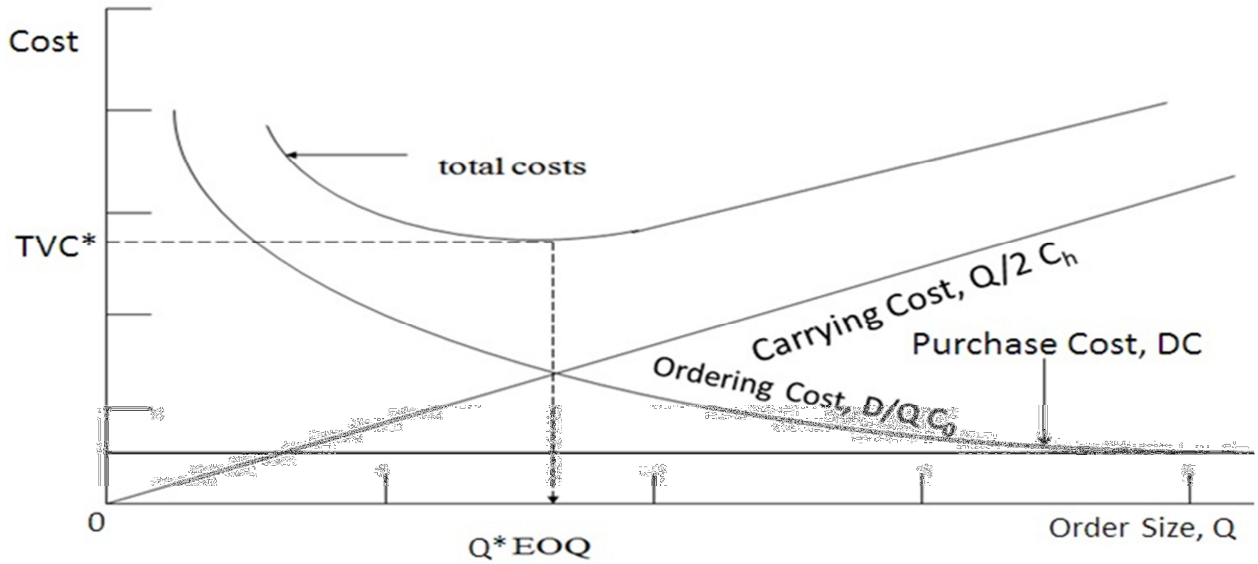
Figure 1



Look at this figure, it shows the behavior of an inventory system that operates on the assumptions listed above. At the beginning of the inventory cycle time we start with a maximum amount of inventory equal to the order quantity Q, as this amount is consumed, the level of inventory drops at a constant rate equal to the demand rate D. When it reaches a specific level called reorder level (ROL), enough inventory is available to cover expected demand during the lead time LT. At this, an order is placed equal to Q, which arrives at the end of lead time, when the inventory level reaches zero. This amount is placed in stock all at once and the inventory level goes up to maximum value.

Order quantity replenished in one inventory cycle is equal to consumption of stock in one inventory cycle that is Q equal to D into t.
In order to determine the optimal order size (Q), we need to calculate the total variable inventory cost for each order cycle. This cost is given by Total variable annual cost, TVC is equal to annual carrying cost plus annual ordering cost which is equal to average inventory level into carrying cost per unit per year plus number of orders placed per year into ordering cost per order.
Is equal to Imax plus Imin by 2 into ‘Ch’ into D by Q into C naught is equal to Q by 2 into ‘Ch’ plus D by Q into C naught.

Figure 2



The figure here represents the trade-off between inventory carrying cost and ordering cost.

The total variable inventory cost is minimum at a value of Q, which appears to be at the point where inventory carrying and ordering costs are equal.

That is, D by Q into C naught is equal to Q by 2 into Ch or Q square is equal to 2 into D into C naught by Ch or Q star EOQ is equal to square root of 2 into D into C naught by Ch which is equal to square root of 2 into annual demand into ordering cost divided by carrying cost.

This formula or Q star is also known as the Wilson or Harris lot size formula.

5. Important Formulae, Remarks & Example

Other important formulae are as follows:

1. Optimal length of inventory replenishment cycle time, optimal interval between the successive orders.
Q star is equal to annual demand into reorder cycle time is equal to D into t or t star is equal to Q star by D is equal to 1 by D into square root of 2 into D into C naught by Ch is equal to square root of 2 into C naught by D into Ch

2. Optimal number of orders to be placed in the given time period (assumed as one year).
N star is equal to annual demand by optimal order quantity is equal to D by Q star is equal to D into 1 by square root of 2 into D into C naught by Ch is equal to square root of D into Ch by 2 into C naught.

3. Optimal (minimum) total variable inventory cost.
TVC star is equal to D by Q star into C naught plus Q star by 2 into Ch is equal to D into C naught into 1 by square root of 2 into D into C naught by Ch plus Ch by 2 into square root of 2 into D into C naught by Ch is equal to square root of 2 into D into C naught into Ch.

4. Optimal total inventory cost is the sum of variable cost and fixed cost, thus, Total cost is equal to demand into purchase plus total variable inventory cost.

Remarks:

1. Other ways of calculating the TVC come from the observation that, at Q star , the reorder cost component equals the holding cost component, so total variable cost is equal to 2 into reorder cost component is equal to 2 into holding cost component. Then, TVC star is equal to 2 into D into C naught by Q star is equal to Q star into Ch.
It does not matter which formula for TVC star is used, it always leads to the same results

2. Often the carrying cost is expressed as a percentage of the monetary value of inventory items. In such cases, the total annual carrying cost is calculated as: carrying cost is equal to inventory carrying rate into unit into unit cost of item is equal to r into C

3. Generally, the annual demand for inventory item is expressed in rupee value rather than in units. In such cases, the demand may be expressed in units as follows, provided unit cost of the item is known. Demand in units is equal to rupee value in demand by unit cost of the item.

But if unit cost of the item is not known, then EOQ in rupee terms is expressed as follows: Q star (EOQ) is equal to square root of 2 into annual demand in rupee into ordering cost by inventory carrying cost is equal to square root of 2 into (C into D) into C naught by R into c

4. The optimal value of Q that minimizes the TVC can also be obtained by using differential calculus (concepts of maxima and minima) as follows: TVC is equal to D by Q into C naught plus Q by 2 into Ch.

Differentiating TVC with respect to Q, we have the differential of Q with respect to TVC (d by dQ) (TVC) is equal to minus D by Q square into C naught plus half Ch.

Since for maximum or minimum value of TVC, its first derivative should de zero, differential of Q (d by dQ) of TVC is equal to zero or D by Q square into Ch is equal to zero,
On simplification, we get Q star is equal to square root of 2 into D into C naught by Ch, economic order quantity.
To ensure the global minimum of Q, verify that the second derivative of TVC with respect to Q that is d square by dQ square of TVC is equal to 2D by Q cube into C naught is positive for any finite value of Q greater than 0.

5. The change (increase or decrease) in the total variable inventory cost due to change in the order quantity Q star is expressed as: let new order size Q be k times the Q star(EOQ), that is Q is equal to kQ star.
Then k is equal to Q by Q star and the ratio of TVC's associated with Q and Q star will be TVC(Q) by TVC(Q star) is equal to half of (1 by K plus K) or half of (Q star by Q plus Q by Q star); k is equal to Q by Q star.

Let us take an example to understand the model.
The production department of a company requires three thousand six hundred kg of raw material for manufacturing a particular item per year.
It has been estimated that the cost of placing an order is Rs 36 and the cost of carrying the inventory is 25 per cent of the investment in the inventories.
The price is Rs 10 per Kg. help the purchase manager to determine an ordering policy for raw material.

Solution:
From the data of the problem we know that D is equal to three thousand six hundred kg per year; C naught is equal to Rs 36 per order and Ch is equal to 25 percent of the investment is equal to Rs 10 into 0 point 25 is equal to 2 rupees 50 paisa per kg per year

a) The optimal lot size is given by Q star is equal to square root of 2 into D into C naught by Ch is equal to square root of 2 into three thousand six hundred into 36 by 2 rupee 50 paise is equal to three twenty one point 99 kg per order.

b) The optimal order cycle time is t star is equal to Q star by D is equal to three twenty one point 99 by thee thousand six hundred is equal to 0 point 894 year

c) The minimum yearly variable inventory cost TVC is equal to square root of 2 into D into C naught into Ch is equal to 2 into three thousand six hundred into 36 into 2 point 5 is equal to Rs eight hundred four point 98 per year.

d) The minimum yearly total inventory cost TC is equal to TVC plus DC is equal to Rs eight hundred four point 98 plus three thousand six hundred kg into Rs 10 per kg is equal to Rs thirty six thousand eight hundred four point 98 per year.

Here’s a summary of our learning in this session, where we have understood:

• Implementation of a single item inventory control models without shortage