Summary

- An assignment problem is a particular case of a transportation problem where the sources are assigned and the destinations are tasks. Furthermore, every source has a supply of 1 (since each assignee is to be assigned to exactly one task) and every destination has a demand of 1 (since each task is to be performed by exactly one assignee). Also, the objective is to minimize the total cost or to maximize the total profit of allocation

- The problem of assignment arises because the resources that is available such as men, machine etc has varying degree of efficiency for performing different activities. Therefore, the cost, profit or time of performing different activities is also different. Thus, the problem is 'how the assignments should be made so as to optimize the given objective

- Some of the problem where the assignment technique may be useful are assignments of workers to machine, salesman to different sales areas, clerks to various check counters, classes to rooms, vehicle to routes, contracts to bidders, etc

- **Mathematical model of assignment problem:**
  Given n resources (or facilities) n activities (or jobs), and effectiveness (in terms of cost, profit, time, etc) of each resource (facility) for each activity (job), the problem lies in assigning each resource to one and only one activity (job) so that the given measures of effectiveness is optimized. The data matrix for this problem is shown in the table. It may be noted that this data matrix is the same as the transportation cost matrix except that the supply (or availability) of each of the resources and the demand at each of the destination is taken to be one. It is due to this fact that the assignments are made one-to-one basis.
  Remark: In an assignment problem if a constant is added to or subtracted from every element of any row or column of the given cost matrix, then an assignment that minimizes the total cost in one matrix also minimizes the total cost in the other matrix

- **Solution methods of assignment problem:**
  An assignment problem can be solved by the following four methods:
  1. Enumeration method
  2. Transportation method
  3. Simplex method
  4. Hungarian method

- **Enumeration method:**
  In this method, a list of all possible assignments among the given resources (men, machines, etc) and activities (jobs, sales areas, etc) is prepared. Then an assignment that involves the minimum cost (or maximum profit), time or distance is selected. If two or more assignments have the same minimum cost (or maximum profit), time or distance, the problem has multiple optimal solutions.
  In general, if an assignment problem involves n workers/jobs, then in total there are n! possible assignments. For example, for an n is equal to 5 workers/job problem, we have to evaluate a total of 5 factoral or 120 assignments. However, when n is large, the method is unsuitable for manual calculations. Hence, this method is suitable only when the value of n is small

- **Transportation method:**
  Since an assignment problem is a special case of the transportation problem, it can also be solved by transportation methods already discussed. However, every basic feasible solution of a general assignment problem that has a square payoff matrix of order n should have m plus n minus 1 is equal to n plus n minus 1 is equal to 2n minus 1 assignments. But due to the special structure of this problem, none of this solution can have more than n assignments. Thus, the assignment problem is inherently degenerate. In order to remove degeneracy, (n minus 1) number of dummy allocations (deltas or epsilons) will be required in order to proceed with the algorithm for solving a transportation problem. Thus, the problem of degeneracy at each solution makes the transportation method computationally inefficient for solving an assignment problem

- **Simplex method:**
  Since each assignment problem can be formulated as a 0 or 1 integer linear programming problem, such a problem can also be solved by the simplex method. As can be seen in the general mathematical formulation of the assignment problem, there are n into n decision variables and n plus n or 2n equalities. In particular, for a problem that involves 5 workers/jobs, there will be 25 decision variables and 10 equalities. This again, is difficult to solve manually

- **Hungarian Method:**
  The Hungarian method (developed by Hungarian mathematician D.konig) of assignment provides us with an efficient method of finding the optimal solution, without having to make a direct comparison of every solution. It works on the principle of reducing the given cost matrix to a matrix of opportunity costs. Opportunity costs show the relative penalties associated with assigning a resource to an activity as opposed to making the best or least cost assignment. If we can reduce the cost matrix to the extent of having at least one zero in each row and column, it will be possible to make optimal assignments (when the opportunity costs are all zero)