Summary

- An optimal solution is one in which there is no opportunity cost. That is there is no other set of transportation routes (allocation) that will reduce the total transportation cost. Thus, we have to evaluate each unoccupied cell (represent unused route) in the transportation table in terms of opportunity cost

- The unoccupied cell with the largest negative opportunity cost is selected to include in the new set of transportation routes. This is also known as incoming variable. The outgoing variable in the current solution is the occupied cell (basic variable) in the unique closed path (loop) whose allocation will become zero first as more units are allocated to the unoccupied cell with the largest negative opportunity cost. Such an exchange reduces total transportation cost

- The process is continued until there is no negative opportunity cost. That is the current solution can be improved further. This is the optimal solution.

- The modified distribution (MODI) method also called as the u-v method which helps in comparing the relative advantage of alternative allocations for all unoccupied cells simultaneously. The MODI method is based on the concept of duality

- **Dual of transportation model:**
  For a basic feasible solution if we associate numbers (also called dual variables or multipliers) u_i and v_j with row i (i equal to 1, 2, 3…..,m) and column j (j equal to 1, 2, 33, …., n) of the transportation table respectively, then u_i and v_j must satisfy the equation u_i plus v_j is equal to c_{ij} for each occupied cell (i,j). These equations yield m plus n minus 1 equations is m plus n unknown dual variables. The values of these variables can be determined from the above relationship by assigning arbitrarily zero value to any one of these variables and then the value is of the remaining m plus n minus 1 variable can be obtained algebraically. Once the values u_i and v_j have been determined, evaluation in terms of opportunity cost of each unoccupied cell called non basic variable or unused route is done using the equation: d_{rs} is equal to c_{rs} minus (u_r plus v_s) for each unoccupied cell (r,s)

- **Economic interpretation of u_i's and v_j's**
  The value of each variable u_i measure the comparative advantage of either the location or the value of a unit of capacity at the supply center i and, therefore, may be termed as location rent. Similarly, the value of each variable v_j measures the comparative advantage of an additional unit of commodity transported to demand centre j and, therefore, may be termed as market price

- **Steps of MODi method (transportation algorithm)**
  The steps to evaluate unoccupied cells are as follows:
  Step1: For an initial basic feasible solution with m plus n minus 1 occupied cell, calculate u_i and v_j for rows and columns. The initial solution can be obtained by any one of the three methods discussed earlier
  To start with, any one of u_i's or v_j's is assigned the value zero to a particular u_i or v_j where there are maximum number of allocation in a row or a column respectively, as this will reduce the considerably arithmetic work. Then complete the calculations of u_i's and v_j's for other rows and columns by using the relation c_{ij} is equal to u_i plus $v_{j,}$ for all occupied cells (i, j)

  Step 2: For unoccupied cells, calculate the opportunity cost (the difference that indicates the per unit cost reduction that can be achieved by an allocation in the unoccupied cell). Do this by using the relationship d_{ij} is equal to c_{ij} minus (u_i plus v_j), for all I and j

Step 3: Examine sign of each d_{ij},

i. If d_{ij} is greater than 0, then the current basic feasible solution is optimal
ii. If d_{ij} is equal to 0, hen the current basic feasible solution will remain unaffected but an alternative solution exisits
iii. If one or more d_{ij} is less than 0, then an improved solution can be obtained by entering unoccupied cell (i,j) in the basis. An unoccupied cell having the largest negative value of d_{ij} is chosen for entering into the solution mix (new transportation schedule)

Step 4: Construct a closed path (or loop) for the unoccupied cell with largest negative opportunity cost. Start the closed path with the selected unoccupied cell and mark a plus sign (+) in this cell. Trace a path along the rows (or columns) to an occupied cell, mark the corner with a minus sign (-) and continue down the column (or row) to an occupied cell. Then mark the corner with plus sign (+) and minus sign (-) alternatively. Close the path back to the selected unoccupied cell

Step 5: Select the smallest quantity amongst the cells marked with minus sign on the corners of closed loop. Allocate this value to the selected unoccupied cell and add it to other occupied cells marked with plus signs. Now subtract this from the occupied cells marked with minus sign

Step 6: Obtain a new improved solution by allocating units to the unoccupied cell according to step 5 and calculate the new total transportation cost

Step 7: Further test the revised solution for optimality. The procedure terminates when all d_{ij} is greater than equal to 0 for unoccupied cells