Summary

- **Degenerate transportation problem**
  While solving a transportation problem we often come across various variations like,
  - Unbalanced supply and demand
  - Degeneracy and its resolution
  - Alternative optimal solutions
  - Prohibited transportation routes
  - Trans-shipment problem

- **Unbalanced supply and demand:**
  For a feasible solution to exist, it is necessary that the total supply must equal the total demand. That is, for a total supply is equal to total demand. Which is expressed as summation a_1 is equal to summation b_j. But a situation may arise when the total available supply is not equal to the total demand. The following two cases may arise:
  a. If the total supply exceeds the total demand, then an additional column (called a dummy demand centre) can be added to the transportation table in order to absorb the excess supply. The unit transportation cost for the cells in this column is set equal to zero because these represent product items that are not being made and not being sent
  b. If the total demand exceeds the total supply, a dummy row (called a dummy supply centre) can be added to the transportation table to account for the excess demand quality. The unit transportation here also for the cells in the dummy row is set equal to zero

- **Degeneracy and its resolution:**
  A basic feasible solution for the general transportation problem must consist of exactly m plus n minus 1 positive allocation in independent positions in the transportation table. A solution will only be called degenerate if the number of occupied cells is less than the required number, m plus n minus 1. In such cases, the current solution cannot be improved upon because it is not possible to dram a closed path for every occupied cell. Also the values of dual variables u_i and V_j that are used to test the optimality cannot be computed. Thus, we need to remove the degeneracy in order to improve the given solution. The degeneracy in the transportation problems may occur at two stages:
  a. When obtaining an initial basic feasible solution we may have less than m plus n minus 1 allocations
  b. At any stage while moving towards optimal solution. This happens when two or more occupied cells with the same minimum allocation are simultaneously unoccupied.

- **Case 1: Degeneracy at the initial solution:**
  To resolve degeneracy at the initial solution, we proceed by allocating a very small quantity close to zero to one or more (if needed) unoccupied cells so as to get m plus n minus 1 number of occupied cells. This amount is denoted by a Greek letter ε (epsilon) or Δ (delta). This quantity would neither affect the total cost nor the supply and demand values. In a minimization transportation problem it is better to allocate delta to unoccupied cells that have lowest transportation costs, whereas in maximization problems it should be allocated to a cell that has a high payoff value. In some cases, delta must be added in one of those unoccupied cells that uniquely makes possible the determination of u_i and v_j
  The quantity delta is considered to be so small that if it is transferred to an occupied cell, it does not change the quantity of allocation. That is x_{ij} plus delta is equal to x_{ij}.

Delta minus delta is equal to 0, 0 plus delta is equal to delta. Minus delta is equal to x_{ij}, delta plus delta is equal to delta and k into delta is equal to delta.
It is also obvious then that delta does not affect the total transportation cost of the allocation. Hence the quantity delta is said to evaluate unoccupied cells and to reduce the number of improvement cycles necessary to reach an optimal solution. Once the purpose is over, delta can be removed from the transportation table

- **Case 2: Degeneracy at subsequent iterations:**
  To resolve degeneracy, which occurs during optimality test, the quantity may be allocated to one or more cells that have recently been unoccupied; to have m plus n minus 1 number of occupied cells in the new solution

- **Alternative Optimal solution:**
  The existence of alternative optimal solutions can be determined by an inspection of the opportunity costs, d_{ij} for the unoccupied cells. If an unoccupied cell in an optimal solution has an opportunity cost of zero, an alternative optimal solution can be formed with another set of allocations, without increasing the total transportation cost.
  Prohibited Transportation Routes
  Situations like road hazards (snow, flood, etc), traffic regulations, etc may arise because of which it is usually not possible to transport goods from certain sources to certain destinations. Such type of problems can be handled but by assigning a very large cost, say M (or ∞) to that route (or cell)

- **Maximization transportation problem:**
  In general the transportation model is used for cost minimization problems. However, if it is also used to solve the problems in which the objective is to maximize total value or benefit. That is instead of unit cost C_{ij}, the unit profit or payoff P_{ij} associated with each route (I, j) is given. The objective function in terms of total profit or payoff is then stated as follows: maximize Z is equal to summation of P_{ij} into X_{ij}. The algorithm for solving this problem is same as that for the minimization problem

- **Trans-shipment problem:**
  In a transportation problem the shipment of a commodity takes place among sources and destinations. But instead of direct shipments to destinations, the commodity can be transported to a particular destinations through one or more intermediate or trans-shipment points. Each of these points, in turn supply to other points. Thus when the shipments pass from destination to destination and from source to source, we have a trans-shipment problem. The solution to this problem can be obtained by using the transportation model. The solution procedure is as follows; if there are m sources and n destinations, we shall have a transportation table of size (m plus n) into (m plus n) instead of m into n as in the usual case. If the total number of units transported from all sources to all destinations is N, then the given supply at each source and demand at each destination are set to be equal to N. the problem can then be solved by the usual MODI method for transportation problems. In the final solution, ignore the units transported from a point to itself, that is diagonal cells, because they do not have any physical meaning (no transportation)