Summary

An optimal as well as feasible solution to an linear programming problem is obtained by choosing among several values of decision variables x1, x2, ….,xn the one set of values that satisfy the given set of constraints simultaneously and also provide the optimal (maximum or minimum) value of the given objective function.
For linear programming problems that have only two variables it is possible that the entire set of feasible solutions can be displayed graphically by plotting linear constraints to locate a best (optimal) solution. The technique used to identify the optimal solution is called the graphical solution technique for an linear programming problem with two variables.

Two graphical solution techniques or approaches: extreme point enumeration approach, and iso-profit (cost) function approach to find the optimum solution to an linear programming problem.
A convex set is a polygon and by ‘convex’ we mean that if any two points of the polygon are selected arbitrarily, then a straight line segment joining the two points lies completely within the polygon.
The extreme points of the convex set give the basic solutions to the linear programming problem.

Extreme point enumeration approach:
The solution method for an linear programming problem is divided into five steps:
Step 1: state the given problem in the mathematical form
Step 2: graph the constraints, by temporarily ignoring the inequality sign and decide about the area of feasible solutions according to the inequality sign of the constraints. Indicate the area of feasible solutions by a shaded area, which forms a convex polyhedron.
Step 3: determine the coordinates of the extreme points of the feasible solution space.
Step 4: evaluate the value of the objective function at each extreme point.
Step 5: determine the extreme point to obtain the optimum (best) value of the objective function.
Iso-Profit (cost) function approach:
The steps of iso-profit (cost) function approach are as follows;
Step 1: identify the feasible region and extreme points of feasible region
Step 2: draw an iso-profit (iso – cost) line for a particular value of the objective function. The word iso here implies that the iso-profit (iso-cost) function is a straight line on which every points has the same total profit (cost).
Step3: move iso-profit (iso-cost) lines parallel in the direction of increasing (decreasing) objective function values.
Step 4: the feasible extreme point for which the value of iso-profit (iso-cost) is largest (smallest) is the optimal solution.

The simplex is an important term in mathematics, one that represents an object in an n-dimensional space, connecting n+1 points. In one dimension, a simplex is a line segment connecting two points, in two dimensions, it is a triangle formed by joining three points; in three dimensions, it is a four sided pyramid, having four corners.

The concept of simplex method is similar to the graphical method. In the graphical method, extreme points of the feasible solution space are examined in order to search for the optimal solution that lies at one of these points. For linear programming problems with several variables, we may not be able to graph the feasible region but the optimal solution will still lie at an extreme point of the many sided, multi-dimensional figure called n-dimensional polyhedron that represents the feasible solution space.

The simplex method examines the extreme points in a systematic manner repeating the same set of steps of the algorithm until an optimal solution is found. It is for this reason that it is called the iterative method.
Since the number of extreme points (corners or vertices) of the feasible solution space are finite, the method assures an improvement in the value of the objective function as we move from one iteration (extreme point) to another and achieve the optimal solution in a finite number of steps. The method also indicates when an unbound solution is reached.

Standard form of linear programming problem:
The use of simplex method to solve an linear programming problem requires that the problem can be converted into its standard form. The standard form of the linear programming problem should have the following characteristics :

1. All the constraints should be expressed as equations by adding the slack or surplus and /or artificial variables.
2. The right hand side of each constraint should be made non-negative if it is not already, this should be done by multiplying both sides of the resulting constraint by minus 1.
3. The objective function should be of the maximization type.

A slack variable represents an unused resource, either in the form of time on a machine, labour hours, money, warehouse space or any number of such resources in various business problems, since these variables don't yield any profit, therefore such variables are added to the original objective function with zero coeffcients.

A surplus variable represents the amount by which solution values exceed a resource. These variables are also called negative slack variables. Surplus variables, like slack variables carry a zero coefficient in the objective function.

Simplex Algorithm (maximization case)
The steps of the simplex algorithm for obtaining an optimal solution (if it exists) to a linear programming problem are as follows:
Step 1 Formulation of the mathematical model:
Step 2: Set-up the initial solution:
Step3: Test for optimality
Step 4: Select the variable to enter the basis
Step 5: Test for feasibility (variable to leave the basis)
Step 6: Finding the new solution
Step 7: Repeat the procedure