E-Learning Module on Problems-formulation-solution by graphical method, simplex algorithm, Charne's M technique

Learning Objectives

By the end of this session, you will be able to:

Graphical Method-Problem 1

Use the graphical method to solve the following Linear programming problem

Maximize $Z=2x_1 + x_2$

subject to constraints

(i) $x_1 + 2x_2 \leq 10$;

(ii) $x_1 + x_2 \leq 6$;

(iii) $x_1 - x_2 \leq 2$;

(iv) $x_1 - 2x_2 \leq 1$ and where $x_1, x_2 \geq 0$.

Graphical Method-Solution 1

Mathematical form:

Maximize $Z=2x_1 + x_2$

subject to constraints

(i) $x_1 + 2x_2 \leq 10$;

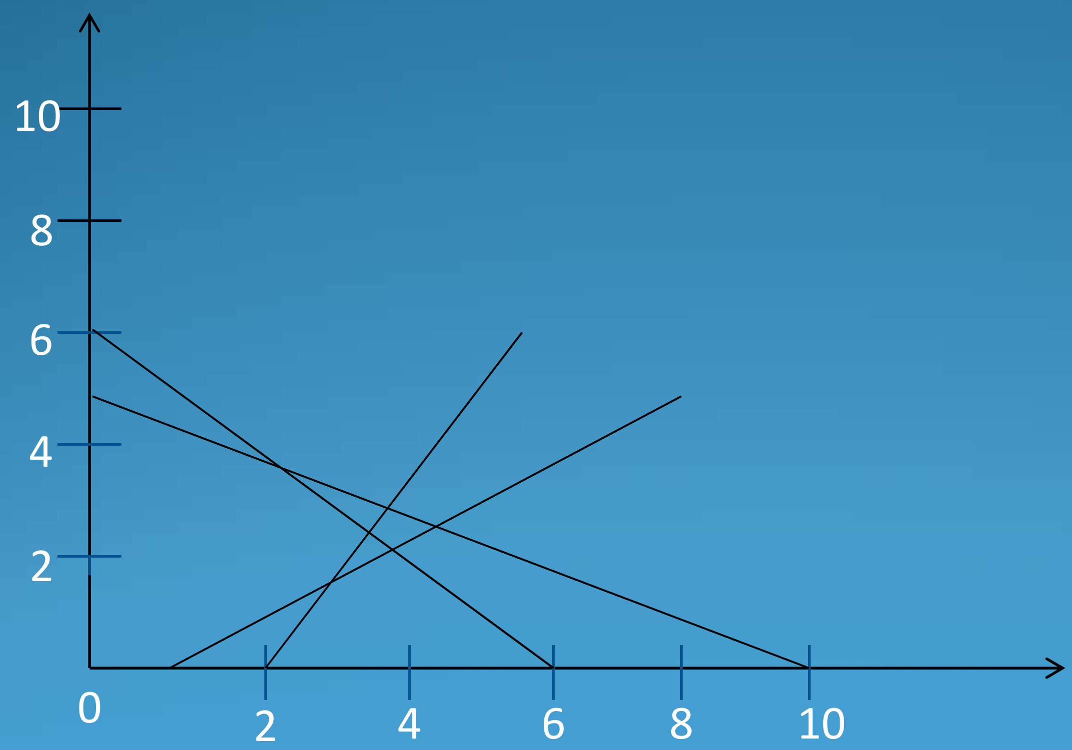
(ii) $x_1 + x_2 \leq 6$;

(iii) $x_1 - x_2 \leq 2$;

(iv) $x_1 - 2x_2 \leq 1$ and where $x_1, x_2 \geq 0$.

Graphical Method-Solution 1

Graphical Representation:



Graphical Method-Solution 1

Since the optimal value of the objective function occurs at one of the extreme points of the feasible region, it is necessary to determine their coordinates. The coordinates of extreme points of the feasible region are:

O = (0,0); A = (1,0); B = (3, 1); C = (4, 2); D is equal (2, 4) and E(0,5).

Graphical Method-Solution 1

Extreme point	Coordinates	Objective function value
O	(0, 0)	2(0) + 1(0) = 0
A	(1, 0)	2(1) + 1(0) = 2
B	(3, 1)	2(3) + 1(1) = 7
C	(4, 2)	2(4) + 1(2) = 10
D	(2, 4)	2(2) + 2(4) = 8
E	(0, 5)	2(0) + 1(5) = 5

Graphical Method-Solution 1

Since we desire Z to be maximum, from the table we can conclude that the maximum value of Z =10 is achieved at the point extreme C(4, 2). Hence the optimal solution to the given linear programming problem is $x1 = 4$, $x2 = 2$ and Max Z =10.

Graphical Method-Problem 2

G J Breweries Lt. have two bottling plants one located at 'G' and the other at 'J'. Each plant products three drinks whisky, beer and brandy named A, B and C respectively. The number of the bottles are shown in the produced per day are shown in the table:

Drink	Plant at	
	G	J
Whisky	1,500	1500
Beer	3,000	1000
Brandy	2,000	5000

Graphical Method-Problem 2

A market survey indicates that during the month of july, there will be a demand of 20,000 bottles of whisky, 40,000 bottles of beer and 44,000 bottles of brandy. The operating cost per day for plants at G and J are 600mand 400 monetary units. For how many days each plant has to run in july so as to minimize the production cost, while still meeting the market demand? Solve graphically

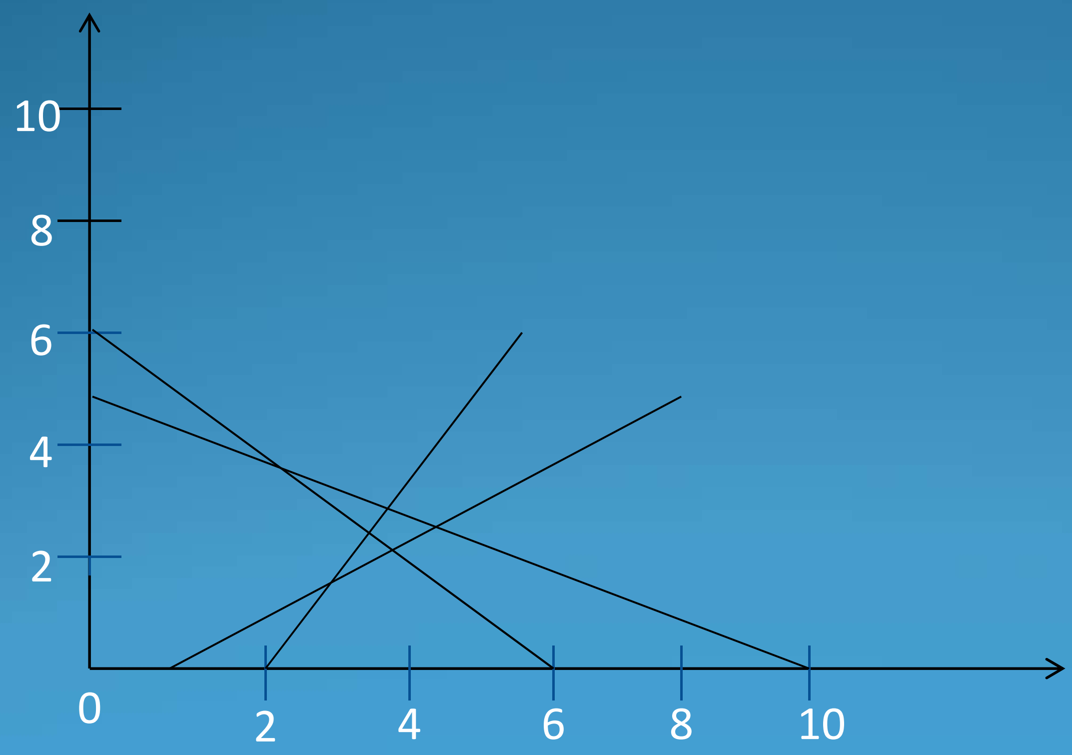
Graphical Method- Solution 2

Step 1: State the problem in the mathematical form: Let us define the following decision variables x_1 and x_2 =number of days at plant G and J respectively. Then the LP model of the given problem can be expressed as:

Minimize $Z = 600x_1 + 400x_2$ subject to the constraints (i) $3x_1 + 3x_2 \geq 40$, (ii) $3x_1 + x_2 \geq 40$ and (iii) $2x_1 + 5x_2 \geq 44$ and $x_1, x_2 \geq 0$.

Graphical Method-Solution 2

Graphical Representation:



Graphical Method- Solution 2

Since the optimal value of the objective function occurs at one of the extreme points of the feasible region, it is necessary to determine their coordinates. The coordinates of extreme points of the feasible region are:

A = (22,0); B = (12, 4); C = (0, 40).

Graphical Method-Solution 1

Extreme point	Coordinates	Objective function value
A	(22, 0)	600(22) + 400(0) = 13,200
B	(12, 4)	600(12) + 400(4) = 8,800
C	(0, 40)	600(0) + 400(40) = 16,000

Graphical Method- Solution 2

Since we desire Z to be minimum, from the table we can conclude that the minimum value of Z = 8800 is achieved at the point extreme B (12, 4). Hence the optimal solution to the given linear programming problem is $x1 = 12$ days, $x2 = 4$ days and Min Z = Rs. 8800.

The Big – M Method- Problem 3

Solve the following linear programming problem using the penalty (Big-M) method:

Minimize $Z = 5x1 + 3x2$

Subject to the constraints

$2x1 + 4x2 \leq 12$

$2X1 + 2x2 = 10$

$5X1 + 2x2 \geq 10$

And $x1, x2 \geq 0$

The Big – M Method- Solution 3

Introduce the slack variable s1, sur+ variable s2 and artificial variable A1 and A2 in the constraints of the given linear programming problem.

Maximize Z = 5x1 + 3x2 + 0s1 + 0s2 + MA1 + MA2

Subject to the constraints

2x1+4x2 + s1 = 12

2X1 + 2x2 + A1 = 10

5X1 + 2x2 –s2 + A2 = 10

And x1, x2, s1, s2, A1, A2 ≥ 0

The Big – M Method- Solution 3

An initial basic feasible solution is obtained by letting $x1=x2=s2=0$. Therefore, the initial basic feasible solution is $x1 = 12$, $A1 = 10$, $A2 = 10$ and Min $Z = 10M + 10M = 20M$.

Here it may be noted that the columns which correspond to the current basic variable and from the basis (identity) matrix are s1 (slack variable), A1 and A2 (both artificial variables).

The Big – M Method- Solution3

		Cj	5	3	0	0	M	M	
cB	B	b (=X_B)	x1	x2	s1	s2	A1	A2	**Min ratio xB / x1**
0	S1	12	2	4	1	0	0	0	12/2
M	A1	10	2	2	0	0	1	0	10/2
M	A2	10	5	2	0	-1	0	1	10/5
Z=20 M		Zj	7M	4M	0	-M	M	M	
		CJ-ZJ	5-7M	3-4M	0	M	0	0	

The Big – M Method- Solution3

Iteration 1:

Introduce variable x1 into the basis and remove A2 form the basis by applying the following row operations.

The row solution is shown in the table below:

R3(new)→R3 (old)/5;

R2(new) →R2(old) - 2R3 (new);

R1 (new) →R1 (old) - 2R3(new)

The Big – M Method- Solution 3

		Cj	5	3	0	0	M	
cB	B	b (=X_B)	x1	x2	s1	s2	A1	**Min ratio xB / x1**
0	S1	8	0	16/5	1	2/5	0	8/(16/5)
M	A1	6	0	6/5	0	2/5	1	6/(6/5)
5	A2	2	1	2/5	0	-1/5	0	2/(2/5)
Z=10 +6M		Zj	5	(6M/5) +2	0	(2m/5)-1	M	
		CJ-ZJ	0	(-6M/5) +1	0	(- 2M/5) +1	0	

The Big – M Method- Solution 3

Iteration 2:

Introduce variable x2 into the basis and remove s1 from the basis by applying the following elementary row operations. The new solution is shown in the table below:

R1(new)→R1 (old)x5/16;

R2(new) →R2(old) - 6/5R1 (new);

R1 (new) →R3 (new) - R3(old)- 2/5 R1(new).

The Big – M Method- Solution 3

		Cj	**5**	**3**	**0**	**0**	**M**	
cB	B	b (=X_B)	x1	x2	s1	s2	A1	**Min ratio xB / x1**
0	S1	5/2	0	1	5/16	1/8	0	(5/2)/(1/8)
M	A1	3	0	0	-3/8	¼	1	3/(1/4)
5	X1	1	1	0	-1/8	-1/4	0	
Z=5 +3M		Zj	5	3	-3M/8 + 5/16	M/4 – 7/8	M	
		CJ-ZJ	0	0	3M/8- 5/16	-M/4 + 7/8	0	

The Big – M Method- Solution 3

Iteration 3:

Introduce s2 into the basis and remove A1 from the basis by applying the following new operation

R2(new)→R2 (old) x 4;

R1(new) →R1(old) - 1/8R2 (new);

R3 (new) →R3 (old) + ¼ R2(new)

The Big – M Method-Solution 3

		Cj	5	3	0	0	M
cB	B	b (=X_B)	X1	x2	s1	s2	A1
3	X2	1	0	1	½	0	-1/2
0	S2	12	0	0	-3/2	1	4
5	X1	4	1	0	-1/2	0	-1
Z = 23		ZJ	5	3	-1	0	7/2
		CJ-ZJ	0	0	1	0	M-7/2

The Big – M Method- Solution 3

From the above table it is observed that all $c_j - Z_j \geq 0$. Thus an optimal solution has arrived at with value of variables as $x_1 = 4$, $x_2 = 1$, $x_1 = 0$, $s_2 = 12$ and Min $Z = 23$

The Big – M Method- Problem 4

An air force is experimenting with three types of bombs, P,Q and R in which three kinds of explosives, that is A, B and C will be used. Taking the various factors into account, it has been decided to use the maximum 600kg of explosive A, atleast 480kg of explosive B and exactly 540kg of explosive C.

The Big – M Method- Problem 4

Bomb P requires 3, 2, 2 kg, bomb Q requires 1, 4, 3 kg and bomb R requires 4, 2, 3 kg of explosive. Bomb P is estimated to give the equivalent of a 2 ton explosion, bomb Q, a 3 ton explosion and bomb R, a 4 ton explosion respectively. Under what production schedule can the Air force make the biggest bang?

The Big – M Method- Solution 4

Let $x1$, $x2$ and $x3$ be the number of bombe of type P, Q and R to be experimented, respectively. Then the LP model for the problem is written as:

Maximize $Z = 2x1 + 3x2 + 4x3$ subject to the constraints (i) $3x1 + x2 + 4x3 \leq 600$ (ii) $2x1 + 4x2 + 2x3 \geq 480$ and (iii) $2x1 + 3x2 + 3x3 = 540$ and $x1$, $x2$ and $x3 \geq 0$.

The Big – M Method- Solution 4

Introducing slack, sur+ and artificial variables, to convert the LP problem into its standard form as follows;

Maximize $Z = 2x1 + 3x2 + 4x3 + 0s1 + 0s2 - MA1 - MA2$ subject to the constraints (i) $3x2 + x2 + 4x3 + s1 = 600$, (ii) $2x1 + 4x2 + 2x3 - s2 + A1 = 480$, (iii) $2x1 + 3x2 + 3x3 + A2 = 540$ and $x1, x2, x3, s1, s2, A1, A2 \geq 0$

The Big – M Method-Solution4

		Cj	2	3	4	0	0	-M	-M	Min ratio
cB	B	b (=X_B)	X1	x2	X3	s1	s2	A1	A2	X_B/X 2
0	S1	600	3	1	4	1	0	0	0	600/ 1
-M	A1	480	2	4	2	0	-1	1	0	480/ 4
-M	A2	540	2	3	3	0	0	0	1	540/ 3
Z=10 20 M		ZJ	-4M	- 7M	- 5M	0	M	-M	-M	
		CJ - ZJ	2+ 4M	3+ 7M	4+ 5M	0	-M	0	0	

The Big – M Method- Solution 4

R2(new) = R2(old) * ¼(key element);

R1(new) = R1(old) - R2(new);

R3(new) = R3(old) - 3R2(new).

The Big – M Method-Solution4

		Cj	2	3	4	0	0	-M	Min ratio
cB	B	b (= X_B)	X1	x2	X3	s1	s2	A1	X_B/X_2
0	S1	480	5/2	0	7/2	1	¼	0	960/7
3	X2	120	½	1	1/2	0	-1/4	0	240
-M	A2	180	1/3	0	3/2	0	3/4	1	120
Z=360 -180M		ZJ	3/2-M/2	3	3/2 – M/2	0	-3/4 - 3M/4	-M	
		CJ - ZJ	½+ M/2	0	5/2 + 3M/2	0	¾ + 3M/4	0	

The Big – M Method- Solution 4

For this apply the following row operations:

R3(new) = R3(old) * 2/3(key element);

R1(new) = R1(old) - (7/2) R3(new);

R2(new) = R2(old) - (1/2)R3(new) to get the new solution

The Big – M Method-Solution4

		Cj	2	3	4	0	0
cB	B	b (=X_B)	X1	x2	X3	s1	s2
0	S1	60	4/3	0	0	1	-3/2
3	X2	60	1/3	1	0	0	-1/2
4	X3	120	1/3	0	1	0	½
Z = 660		Zj	7/3	3	4	0	½
		Cj - Zj	-1/3	0	0	0	-1/2