Sampling plans by attributes

In this session we are going to discuss some basic sampling procedures by attributes. The topic is subdivided in to following subdivisions.

1. Definition of Single sampling plan by attributes and derivation of OC curve
2. Effect of Changes in the OC Curve of single sampling plan
3. ATI and AOQ of Single sampling plan
4. Construction of Single sampling plan
5. Definition of Double sampling plan and derivation of OC curve
6. ASN, ATI and AOQ of Double sampling plan
7. Comparison of Single Sampling and Double Sampling Plans
8. Conclusion

1. Definition of Single sampling plan by attributes (SSP)and derivation of OC curve**:**

Let the units are submitted for inspection in lots of size N each. A sample of n units is taken at random from each lot and is inspected. The number of nonconforming units (defectives) in such samples, say 'd', is identified. If the sample contained more than c (a stated number of allowable nonconforming units) defectives (d>c) the whole lot is rejected. If the number of defectives d ≤ c, the whole lot is accepted. The number 'c' is called the acceptance number; that is, the maximum tolerable number of defectives in a sample of size n.

When the number of defectives is more than c (resulting in rejection of the whole lot), the remainder of the lot is inspected 100% and the defectives found are replaced with the non defectives. This is called Rejection-Rectification procedure.

The parameters of SSP are N, n , c and generally denoted by $\begin{bmatrix} N \\ n \\ c \end{bmatrix}$

If d is the number of defectives found in a sample of size n, then an estimate of the lot quality is p'=d/n. Then the number of defectives,'x' out of n items follow hypergeometric distribution with probability p(x) = $\dfrac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$, D being the total number of defectives present in the lot of size N.

If N is assumed to be very large, n is assumed to be small which is the usual practice in industrial manufacturing situations, the probability can be approximated by binomial model, with probability of getting x defectives is given by p(x) = $\binom{n}{x} p^{x} (1-p)^{(n-x)}$

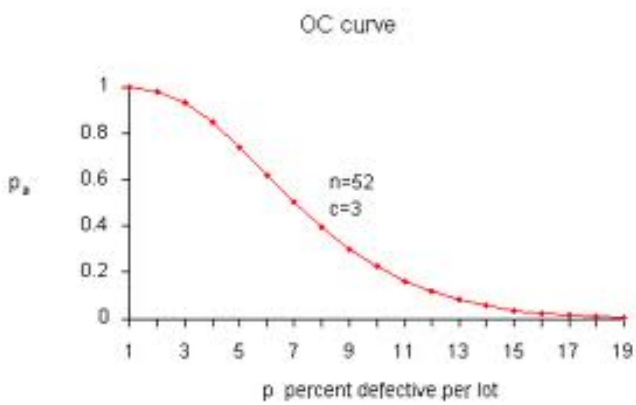
In addition to the above condition, if the lot size is sufficiently large, and if np<5, then the probability can be approximated by the Poisson model with probability

$$p(x) = \frac{e^{-np}(np)^x}{x!}$$

The probability of acceptance of the lot based on a SSP is

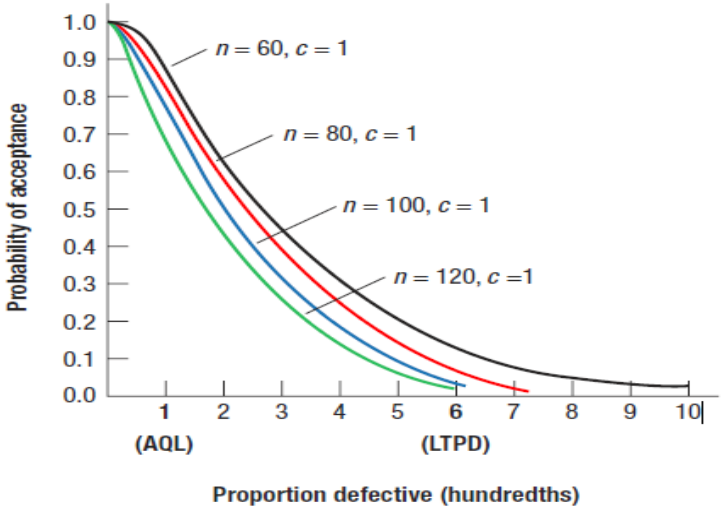
$$P_a = P(X \le c) = \sum_{x=0}^{c} \frac{e^{-np}(np)^x}{x!}$$

The image of OC curve of the single sampling plane is given below.

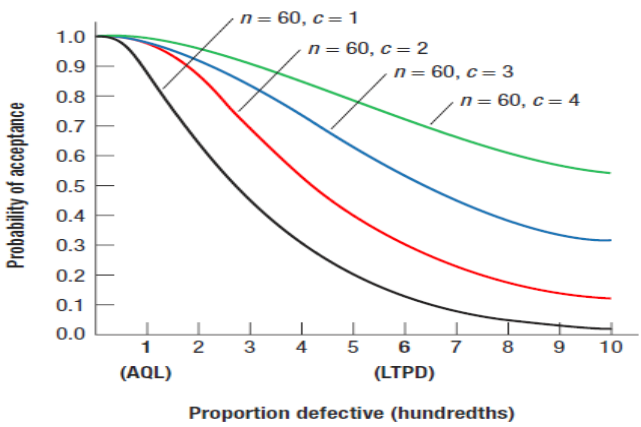


2. Effect of Changes in the OC Curve of single sampling plan:

Effect of Sample Size:The effect of keeping the acceptance number as constant and increasing sample size. Increasing the sample size while holding c constant increases the producer's risk and reduces the consumer's risk.



Effect of Change in Acceptance Number:The effect of keeping the sample size as constant and increasing acceptance number. Increasing acceptance number(c) while holding n constant decreases the producer's risk and increases the consumer's risk.



3. ATI and AOQ of Single sampling plan:

Average Total Inspection(ATI): The number of items inspected per lot is a random variable which is described as follows. A sample of size 'n' is always inspected. That is, for each accepted lots only n items are inspected. Where as in the case of rejected lots all the N items are inspected (sample size 'n' + remainder of the lot (N-n)). If P_a is the probability of acceptance of the lot we have the distribution of inspection is

$$P_I = \begin{cases} n & P_a \\ N & 1-P_a \end{cases}$$

Then, ATI = I = n P_a + N(1-P_a) = n + (N-n) (1-P_a)

Average Outgoing Quality (AOQ): The quality of the lot after inspection is referred to as the outgoing quality. For SSP, the average total number of items inspected per lot is I. Therefore (N-I) items are not be inspected. If 'p' is the quality of the lot then the average number of defectives passed out by inspection is (N-I) p.

Hence, AOQ = Average number of defectives passed out of inspection per lot of size N

= (N-I) p /N

= (N-n-(N-n)(1-P_a))p/N

= (N-n)pP_a/N

= (1-n/N)pP_a ~ p P_a, when N is sufficiently large.

AOQL: The maximum value of AOQ is the AOQ limit (AOQL) and is denoted by p_L.

4. Construction of Single sampling Plan: (AQL-LQL Plans)

Given p_1, p_2, α and β we construct a SSP such that its OC curve passes through (p_1, 1- α) and (p_2,β). The parameters n and c are obtained such that it satisfy the following conditions.

P (accepting a lot of quality p_1)= $\sum_{x=0}^{c} \frac{e^{-np_1}(np_1)^x}{x!}$ = 1-α . . . (1)

P(accepting a lot of quality p_2) = $\sum_{x=0}^{c} \frac{e^{-np_2}(np_2)^x}{x!}$ = β . . . (2)

The above two equations can be solved by trial and error method. Take an arbitrary value of c. From equation (2) and using cumulative Poisson table obtain the value of np_2 (say m) from which n can be determined. Thus we have a trial plan given by n and c. Using these values of n and c in equation (1) obtain the value of α. If the calculated value of α is different from the givenα, change the value of 'c' and repeat the procedure until a closer value of α is obtained of that of given α.

5. Double sampling plan and derivation of OC curve:

If a single sampling plan does not lead to the acceptance or rejection of the lot, one has to restore to some other sampling plans to take a decision to accept or reject the lot. In other words a single sample is not bad enough to reject a lot straight away or good enough to accept the lot. In this situation a second sample is necessary and a decision is taken on the basis of the results of the first and second samples combined. One method of selecting a second sample to arrive at a more reliable decision is known as Double sampling plan. In this scheme a second sample is drawn from the same lot and the decision is taken according to the procedure given below.

Double sampling plan is designated by $\begin{bmatrix} N \\ n_1 \\ n_2 \\ c_1 \\ c_2 \end{bmatrix}$

Operating procedure:

Step 1: First inspect a sample of size n_1 from a lot of size N

Step 2: If the number of defectives found in the sample (d_1) is less than or equal to c_1 accept the lot.

Step 3: If the number of defectives (d_1) exceeds c_2 reject the lot and inspect the remainder of the lot 100% inspection.

Step 4: If the number of defectives found is between c_1 and c_2, that is, if $c_1 < d_1 \leq c_2$ inspect the second sample of size n_2 items.

Step 5: If the number of defectives observed in the second sample (d_2) plus the first sample combined (d_1+d_2) combined is $\leq c_2$, accept the lot; otherwise reject.

Step 6: Correct or replace defectives observed by good items.

OC function: Let $P_{x,n,p} = e^{-np}(np)^x /x!$ is the probability of getting x defectives in a sample of size n from the lot of quality p under Poisson distribution.

If the number of defectives found in the first sample of size n_1 is $\leq c_1$, we accept the lot and probability of accepting the lot based on this sample is

$$P_1 = \sum_{x=0}^{c_1} \frac{e^{-n_1 p}(n_1 p)^x}{x!} = \sum_{x=0}^{c_1} P_{x,n1\,p}$$

A second sample is taken when $c_1 < d_1 \leq c_2$. If the number of defectives found (d_2) in the second sample of size n_2 is such that ($d_1+d_2 \leq c_2$) we accept the lot. This happens in the following mutually exclusive ways.

Sl.No.	d_1	d_2	P
1	c_1+1	$\leq c_2-c_1-1$	$P_{c1+1,\,n1,p} \sum_{x=0}^{c2-c_1-1} P_{x,n2,\,p}$
2	c_1+2	$\leq c_2-c_1-2$	$P_{c1+2,\,n1,p} \sum_{x=0}^{c2-c_1-2} P_{x,n2,\,p}$
.	.	.	.
.	.	.	.
.	.	.	.
c_2-c_1	c_2	0	$P_{c2,\,n1,p}\ P_{0,n2.\,p}$
Total			$\sum_{y=c_1+1}^{c_2} P_{y,n_1,p} \sum_{x=0}^{c_2-y} P_{x,n_2,p}$

Hence probability of accepting the lot based on the second sample is $P_2 = \sum_{y=c_1+1}^{c_2} P_{y,n_1,p} \sum_{x=0}^{c_2-y} P_{x,n_2,p}$.

And probability of accepting the lot based on the Double sampling plan is

$$P_a = \sum_{x=0}^{c_1} P_{x,n1,\,p} + \sum_{y=c_1+1}^{c_2} P_{y,n_1,p} \sum_{x=0}^{c_2-y} P_{x,n_2,p}$$

6. ASN, ATI and AOQ of Double sampling plan

Average Sample Number (ASN): In the Double sampling plan, the first sample is always inspected on sampling basis. Thus n_1 items are always inspected and n_2 items are taken if the number of defectives in the first sample exceeds c_1 but $\leq c_2$. Therefore, in the long run the number of units inspected on sampling basis is a random variable. The expected value of the number of units inspected on sampling basis is given by,

$$ASN = n_1 + n_2 \sum_{x=c_1+1}^{c_2} P_{x,n_1,p}$$

Average Total Inspection (ATI):

(a) Only n_1 items are inspected per lot if the lot is accepted based on the first sample and this happens with probability P_1.

(b) Only $n_1 + n_2$ items are inspected if the lot is accepted based on the first and second sample combined and this occurs with probability P_2.

(c) All the N items are inspected if the lot is rejected. This happens with probability (1-P_a).

Hence, ATI = I = $n_1P_1 + (n_1 + n_2)P_2 + N(1 - P_a)$

Average Outgoing Quality (AOQ): The average outgoing quality for Double sampling plan is

AOQ = $\frac{(N - I)p}{N}$ = $\frac{[(N - n_1)P_1 + (N - n_1 - n_2)P_2]p}{N}$

7. Comparison of Single Sampling and Double Sampling Plans:

Single Sampling plan	**Double Sampling Plan**
Simple, easy to design and administer.	Not simple and not easy as compared with single sampling plan.
Calls for decision on acceptance or rejection on the basis of the evidence of only one single sample from the lot.	It appears more convincing to say that the lot is accepted or rejected after inspecting two samples.
Generally Involves more number of inspection than Double sampling plan.	Involves less number of inspection than single sampling plan
Discriminatory power between good and bad lots by single sampling procedure is lower than that of double sampling plan	Discriminatory power between good and bad lots by double sampling procedure is higher than that of single sampling plan

8. Conclusion: In this lecture we studied the Single sampling plan for attributes, its OC function and properties. We learnt the change in the shape of the OC curve when sample size changes fixing the acceptance number and also the changes in the OC curve when acceptance number changes fixing the sample number.We discussed the construction of Single sampling plan given two points on the OC curve; Double sampling plan and its OC function, the comparison between Single and Double sampling plan,