Frequently asked questions:

1. **Describe Single sampling plan.**
   Let the units are submitted for inspection in lots of size N each. A sample of n units is taken at random from each lot and is inspected. The number of nonconforming units (defectives) in such samples, say 'd', is identified. If the sample contained more than c (a stated number of allowable nonconforming units) defectives (d>c) the whole lot is rejected. If the number of defectives d ≤ c, the whole lot is accepted. The number 'c' is called the acceptance number; that is, the maximum tolerable number of defectives in a sample of size n.
2. **Derive OC function of Single sampling plan**
   If d is the number of defectives found in a sample of size n, then an estimate of the lot quality is p'=d/n. if the lot size is sufficiently large, and if np<5, then the probability can be approximated by the Poisson model with probability of observing x defective unit is given by

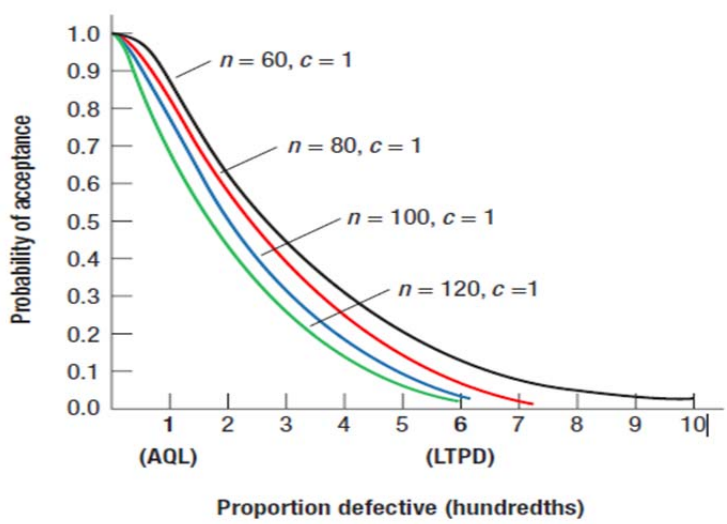
   $$p(x) = \frac{e^{-np}(np)^x}{x!}$$

   The probability of acceptance of the lot based on a SSP is

   $$P_a = P(X \le c) = \sum_{x=0}^{c} \frac{e^{-np}(np)^x}{x!}$$

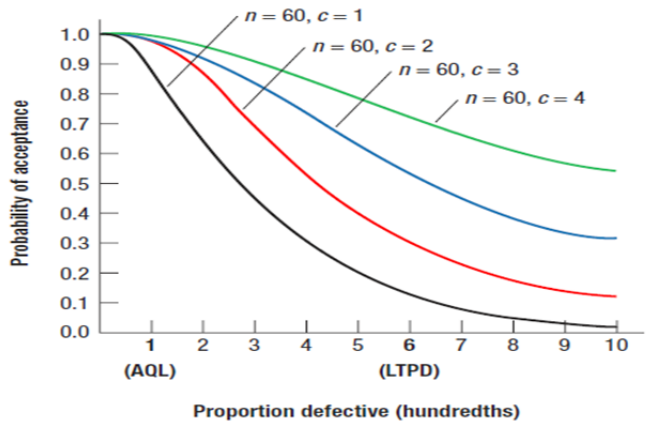
3. **With suitable graph explain the Effect of Changes in the OC Curve of single sampling plan when the sample size increases for a fixed acceptance number.**

Increasing the sample size while holding c constant increases the producer's risk and reduces the consumer's risk. As explained the following graph the OC curve is more steeply as sample size increases.



4. **With suitable graph explain the Effect of Changes in the OC Curve of single sampling plan when the acceptance number increases for a fixed sample size.**

Increasing acceptance number(c) while holding n constant decreases the producer's risk and increases the consumer's risk. As seen in the graph as c increases there is more chance of accepting the bad



5. **Derive the expression for ATI function of SSP**

Expression for ATI: The number of items inspected per lot is a random variable which is described as follows. A sample of size 'n' is always inspected. That is, for each accepted lots only n items are inspected. Where as in the case of rejected lots all the N items are inspected (sample size 'n' + remainder of the lot (N-n)). If P_a is the probability of acceptance of the lot we have the distribution of inspection is

$$P_I = \begin{cases} n & P_a \\ N & 1-P_a \end{cases}$$

Then, ATI = I = n P_a + N(1-P_a) = n + (N-n) (1-P_a)

6. **Derive the expression for AOQ function for SSP**

Expression for AOQ: The quality of the lot after inspection is referred to as the outgoing quality. For SSP, the average total number of items inspected per lot is I. Therefore (N-I) items are not be inspected. If 'p' is the quality of the lot then the average number of defectives passed out by inspection is (N-I) p.

Hence, AOQ = Average number of defectives passed out of inspection per lot of size N

= (N-I) p /N

= (N-n-(N-n)(1-P_a))p/N

= (N-n)pP_a/N

= (1-n/N)pP_a ~ p P_a, when N is sufficiently large

7. **Explain the Construction of Single sampling Plan when p_1, p_2, α and βare given**

Given p_1, p_2, α and β we construct a SSP such that its OC curve passes through (p_1, 1- α) and (p_2,β). The parameters n and c are obtained such that it satisfy the following conditions.

$$P \text{ (accepting a lot of quality } p_1) = \sum_{x=0}^{c} \frac{e^{-np_1}(np_1)^x}{x!} = 1-\alpha \qquad \ldots (1)$$

P(accepting a lot of quality p_2) = $\sum_{x=0}^{c} \frac{e^{-np_2}(np_2)^x}{x!} = \beta$ . . . (2)

The above two equations can be solved by trial and error method. Take an arbitrary value of c. From equation (2) and using cumulative Poisson table obtain the value of np_2 (say m) from which n can be determined. Thus we have a trial plan given by n and c. Using these values of n and c in equation (1) obtain the value of α. If the calculated value of α is different from the givenα, change the value of 'c' and repeat the procedure until a closer value of α is obtained of that of given α.

8. **Write the Operating procedure of double sampling plan**

Step 1: First inspect a sample of size n_1 from a lot of size N
Step 2: If the number of defectives found in the sample (d_1) is less than or equal to c_1 accept the lot.
Step 3: If the number of defectives (d_1) exceeds c_2 reject the lot and inspect the remainder of the lot 100% inspection.
Step 4: If the number of defectives found is between c_1 and c_2, that is, if $c_1 < d_1 \leq c_2$ inspect the second sample of size n_2 items.
Step 5: If the number of defectives observed in the second sample (d_2) plus the first sample combined (d_1+d_2) combined is $\leq c_2$, accept the lot; otherwise reject.
Step 6: Correct or replace defectives observed by good items.

9. **Derive the OC function of double sampling plan**

:Let $P_{x,n,p} = e^{-np}(np)^x / x!$ is the probability of getting x defectives in a sample of size n from the lot of quality p under Poisson distribution.

If the number of defectives found in the first sample of size n_1 is $\leq c_1$, we accept the lot and probability of accepting the lot based on this sample is

$$P_1 = \sum_{x=0}^{c_1} \frac{e^{-n_1 p}(n_1 p)^x}{x!} = \sum_{x=0}^{c_1} P_{x,n1\,p}$$

A second sample is taken when $c_1 < d_1 \leq c_2$. If the number of defectives found (d_2) in the second sample of size n_2 is such that ($d_1+d_2 \leq c_2$) we accept the lot. This happens in the following mutually exclusive ways.

Sl.No.	d_1	d_2	P
1	c_1+1	$\leq c_2-c_1-1$	$P_{c1+1,\,n1,p} \sum_{x=0}^{c2-c_1-1} P_{x,n2,\,p}$
2	c_1+2	$\leq c_2-c_1-2$	$P_{c1+2,\,n1,p} \sum_{x=0}^{c2-c_1-2} P_{x,n2,\,p}$
.	.	.	.
.	.	.	.
.	.	.	.
c_2-c_1	c_2	0	$P_{c2,\,n1,p}\ P_{0,n2.\,p}$

Total			$\sum_{y=c_1+1}^{c_2} P_{y,n_1,p} \sum_{x=0}^{c_2-y} P_{x,n_2,p}$

Hence probability of accepting the lot based on the second sample is $P_2 = \sum_{y=c_1+1}^{c_2} P_{y,n_1,p} \sum_{x=0}^{c_2-y} P_{x,n_2,p}$.

And probability of accepting the lot based on the Double sampling plan is

$$P_a = \sum_{x=0}^{c_1} P_{x,n1,p} + \sum_{y=c_1+1}^{c_2} P_{y,n_1,p} \sum_{x=0}^{c_2-y} P_{x,n_2,p}$$

10. Derive the expression for ASN for double sampling plan

Average Sample Number : In the Double sampling plan, the first sample is always inspected on sampling basis. Thus n_1 items are always inspected and n_2 items are taken if the number of defectives in the first sample exceeds c_1 but $\leq c_2$. Therefore, in the long run the number of units inspected on sampling basis is a random variable. The expected value of the number of units inspected on sampling basis is given by,

$$ASN = n_1 + n_2 \sum_{x=c_1+1}^{c_2} P_{x,n_1,p}$$

11. Derive the expression for ATI of double sampling plan

Average Total Inspection (ATI):

(a) Only n_1 items are inspected per lot if the lot is accepted based on the first sample and this happens with probability P_1.

(b) Only $n_1 + n_2$ items are inspected if the lot is accepted based on the first and second sample combined and this occurs with probability P_2.

(c) All the N items are inspected if the lot is rejected. This happens with probability $(1-P_a)$.

Hence, ATI = I = $n_1P_1 + (n_1 + n_2)P_2 + N(1 - P_a)$

12. Derive the expression for AOQ of double sampling plan

Average Outgoing Quality**:** The average outgoing quality for Double sampling plan is

$$AOQ = \frac{(N - I)p}{N} = \frac{[(N - n_1)P_1 + (N - n_1 - n_2)P_2]p}{N}$$