Basics of Statistical Quality Control

Introduction

Quality is a relative term and it is generally used with reference to the end product. The quality of a product can be evaluated in many ways referred to as dimensions of quality.

Performance - Will the product do the intended job?
Reliability - How often does the product fail?
Durability - How long does the product last?
Serviceability - How easy is it to repair the product?
Aesthetics - What does the product look like?
Features - What does the product do?
Perceived quality - What is the reputation of the company or its product?
Conformance to standards - Is the product made exactly as the designer intended?

The traditional definition of quality is based on the view point that products and services must meet the requirements of those who use them. Hence quality takes the definition ***fitness for use***. Quality is also understood to be inversely proportional to variability implying that the quality of a product increases if the variability of the product decreases.

Quality control (QC) is a systematic process of maintaining and managing various factors that affect the quality of the product. QC can also be defined as the tools, device(s) or skills through which quality activities are carried out. QC majorly depends on - material, machine, types of labour, working conditions, measuring instruments, etc. Thus the process, through which we establish and meet standards to achieve Quality of Conformance, is commonly termed as Quality Control.

Quality improvement (QI) is the reduction of variability in processes and products. One goal of this course is to learn how to implement statistical tools that can be used to improve quality.

Meaning of Statistical Quality Control (SQC) - Application of Statistical Techniques to measure and evaluate the quality of a product, service or process etc. Two major areas are:

1. Statistical Process Control (SPC) – Control Charts
2. Product control - Acceptance Sampling

Statistical Process Control (SPC)

SPC is application of Statistical techniques to determine whether a process is functioning as desired. Control chart is one of the primary techniques of SPC which helps to discover the unusual sources of variation present in the process.

Product control - Acceptance Sampling

Product Control is to check if the manufactured products meet the specifications for various quality parameters. This leads to quality guarantee. This is done by sampling inspection of manufactured products.

Variations in Quality:

In any production process, regardless of how well designed or carefully maintained a certain amount of inherent or natural variability will always exist. There are several reasons we should be concerned with variation. Variations may change the shape, dispersion, and central tendency of the distribution of the product characteristic being measured, resulting in manufacturing undesirable products. Variations are usually characterized in two generic forms, viz., *assignable variation* (usually correctable) and *chance variation* (usually cannot be corrected or stabilized economically).

Chance Variation:

Variation that is random in nature. This type of variation cannot be completely eliminated unless there is a major change in the equipment or material used in the process. Internal machine friction, slight variations in material or process conditions (such as the temperature of the mold being used to make glass bottles), atmospheric conditions (such as temperature, humidity, and the dust content of the air), and vibrations transmitted to a machine are a few examples of sources of chance variation.

a) Consist of many individual causes.
b) Any one chance causes results in only a minute amount of variation. (However, many of the chance causes act simultaneously so that the total amount of chance variation is substantial).
c) Some typical chance causes of variation are :
    i. Slight variations in raw material (though within the specifications)
    ii. Slight vibration of machine
    iii. Lack of human perfection in reading instruments and setting controls.
d) As a practical matter, chance variation cannot economically be eliminated from a process.

Assignable Variation: Variation that is not random. It can be eliminated or reduced by investigating the problem and finding the cause(s).

a) Consists of one or just a few individual causes.
b) Any one assignable cause can result in a large amount of variation.
c) Some typical assignable causes of variation are: Batch of defective raw material Faulty set up Untrained operator
d) The presence of assignable variation can be detected and action to eliminate the causes is usually justified.

Advantages of Statistical Quality Control:

1. Identification and elimination of assignable causes of variation help in
   (i) Detection and correction of many product troubles.
   (ii) Bringing about a substantial improvement in the product quality and reduction of spoilage and rework.
2. Prevents frequent and unwarranted adjustment in the process by indicating when to leave the process alone and when to take corrective actions.
3. It helps in making more or less radical change in the process when the process in control is not good enough.
4. If the process is in good control quality guarantee can be given.
5. If testing is destructive. A process in control gives confidence in the quality or untested product.
6. Provides better quality assurance at lower inspection cost.
7. Reduces wastages of time and materials to the absolute minimum by giving an early warning about the occurrence of defects.

Quality measurements:

Attributes:

Characteristics that are measured as either "acceptable" or "not acceptable"; “conforms to specification” or “do not conform to specifications”; “go” or “no go” - thus have only discrete, binary, or integer values.

Variables:

Characteristics those are measurable in numerical measurements on a continuous scale. e.g., Height, weight, diameter, kilogram, liter, temperature, volume…etc

Control Charts for process control

Random variability of a process with respect to a certain quality characteristic is expected to conform to a consistent pattern, which can be approximated by some known probability distribution such as Binomial, Poisson, Normal etc. Using the appropriate probability distribution, it is possible to calculate the upper and lower limits of natural variation of the quality characteristic. Any variability beyond these limits can be attributed to assignable causes.

Walter A. Shewhart, called the father of statistical quality control developed the concepts of statistical quality control. Shewhart proposed 3σ limits due to certain probabilistic considerations. The control limits were located on the control chart so that the probability that a subgroup mean will fall outside of either limit when the process mean is at the central line is less than 1%. This is accomplished by placing the control limits at a distance of 3 standard errors from the central line. If it is assumed that the sample mean have normal distribution then the probability that any observed mean will fall outside of either control limit is approximately 0.0026 when 3σ limits are used.

Measurements on any quality characteristic from a process working under a system of chance causes are expected to follow the Normal pattern generally. It can be seen that

99.7% of the observations from a Normal population (with average or mean μ and standard deviation σ) fall within $\mu \pm 3\sigma$). This knowledge is used to **establish central limits at ± 3σ from the centralline of the process**. Successive observations from this process can be plotted on a chart containing a control line μ and control limits $\mu + 3\sigma$ and $\mu - 3\sigma$. Any observation falling outside the limits is very unlikely when only chance causes are operating on the process; hence this indicates existence of assignable causes.

The 3σ limits are called natural tolerance limits. The width 6σ is generally known as **Natural Tolerance.**

Specification Limits:

The specification limits are generally set for individual measurements. These limits are set by a design engineer in accordance with the needs of the consumer. Generally these specification limits are denoted by upper specification limits (USL) and Lower specification limits (LSL). For example, Specification limits for

Resistance of a coil are 800+ 5R

Carbon in steel is 20 to 30 %

Compression of springs is 12+ 0.5 lbs etc.

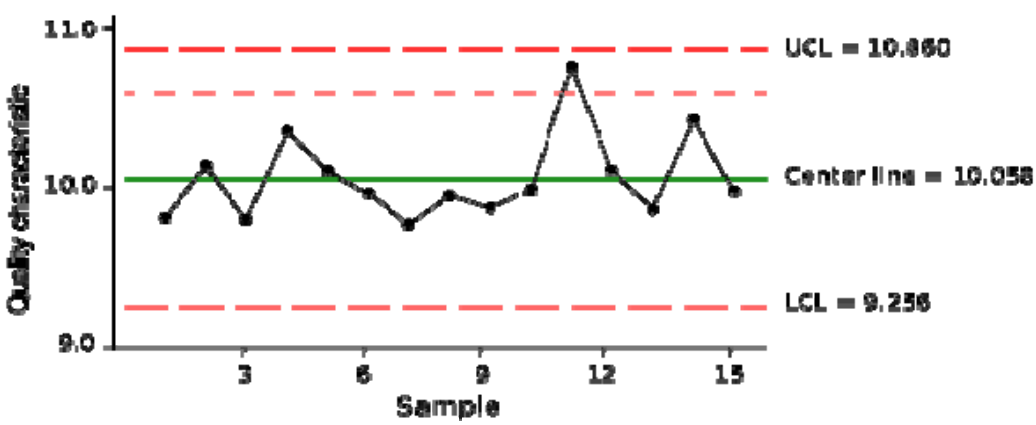
Probability Limits:

If the control limits are such that the probability of getting a point beyond the control limits (when random causes are operating) is 0.001 each, then they are called probability limits. These are widely used in England where as the 3σ limits are common in USA. It can be checked that the control limits in $\overline{X}$ - chart will be at a distance of 3.09 from the central line when 0.001 probability limits are used. For R chart the control limits are calculated such that P(R>UCL) = 0.0005 and P(R<LCL) = 0.0005

Warning Limits:

If 2σ limits are also drawn in a control chart, then these 2σ limits are called Warning Limits.

A typical control chart:



The plots are the average of measurements of a quality characteristic in samples taken from the process versus time. The chart consists of a central line (CL) the upper and lower control limits (UCL and LCL). The central line represents whether this process characteristics should fall if there is no unusual source of variation present. The control limits are determined from some simple statistical considerations.

A control chart exhibits a state of control when:

1. 2/3 of the points are near the central line
2. A few of the points are on or near the central line
3. The points appear to float back and forth across the line
4. The points are balanced (in roughly equal numbers) on both sides of the centerline
5. There are no points beyond the control limits
6. There are no patterns or trends on the chart

A process that is not under control or is unstable displays patterns of variation. The process needs to be investigated for presence of assignable causes.

Some of the signs of out of control situation are:

1. Any point beyond the 3-sigma control limits.
2. Seven (or more) consecutive points above or below the central line
3. Seven consecutive increasing or decreasing points
4. Two out of three beyond two sigma limit on the same side of the central line
5. Four out of five beyond one sigma limit on the same side of the central line
6. Ten out of 11 consecutive points on the same side of the center line
7. Twelve out of 14 consecutive points on the same side of the central line
8. A series of points "hugging" on the central line
9. A series of points "hugging" on the control limits
10. Fourteen consecutive points alternating ups and down (saw tooth pattern)
11. Any nonrandom pattern (such as a cycle)

Subgroup and rational subgroup:
A subgroup is a small set of observations on process output closely together in time. A rational subgroup is one in which assignable causes of variation are not likely to be present but which contains all the effects of random causes. Generally subgroups should be selected in such a way that there is a minimum chance for the measurements in each subgroup to differ from each other.

Size and method of drawing subgroups:
Generally subgroups should be selected in such a way that there is a minimum chance for measurements in each subgroup to differ and a maximum chance for the subgroups to differ from each other. The most common basis for sub grouping is the order of production. These subgroups should be as like as possible being subject to all the random variation that

affects the process and they should represent population or process at a particular point of time. The within the subgroup variation, as measured by the subgroup ranges or standard deviation shows how the process variability is varying. The subgroup means the frequency, larger the number of samples, the more closely the distribution of sample means comes to a normal distribution. This factor tends to work in favour of large samples. Another factor in favour of large samples relates to sensitivity of the central chart in detecting assignable causes. Therefore usually a small subgroup size with large number of samples is preferred. Say for example, subgroup size of 4 with k=25 or subgroup size of 5 with k=20 may be used owing to the cost and destructiveness of inspection.