FAQ

1. Explain the general model of control limits:

Let T be a sample statistic that measures some quality characteristic of interest. Suppose that the mean of T is μ_T and the standard deviation of T is σ_T. Then the general model of control limits are given by,

Upper Control Limit (UCL) $= \mu_T + 3\sigma_T$

Center Line (CL) $= \mu_T$

Lower Control Limit (LCL) $= \mu_T - 3\sigma_T$

2. Derive the control limits for X bar chart when the process average is known

Let T denote the process average observed by sample means ($\overline{X}_i$) at regular intervals of time.

Upper Control Limit (UCL) $= \mu_T + 3\sigma_T = \mu + \frac{3\sigma}{\sqrt{n}}$, since $E(\overline{X}) = \mu$ and $\sigma_T = \frac{\sigma}{\sqrt{n}}$

Center Line $= E(\overline{X}) = \mu$

Lower Control Limit (LCL) $= \mu_T - 3\sigma_T = \mu - \frac{3\sigma}{\sqrt{n}}$, since $E(\overline{X}) = \mu$ and $\sigma_T = \frac{\sigma}{\sqrt{n}}$

Upper Control Limit (UCL) $= \mu + \frac{3\sigma}{\sqrt{n}}$

Center Line $= \mu$

Lower Control Limit (LCL) $= \mu - \frac{3\sigma}{\sqrt{n}}$

OR

Upper Control Limit (UCL) $= \mu + A\sigma$

Center Line $= \mu$

Lower Control Limit (LCL) $= \mu - A\sigma$ where the constant A is $\frac{3}{\sqrt{n}}$ obtained directly from the table of constants.

3. Derive the control limits for X bar chart when the process average is unknown and estimated using sample range.

When μ and σ are unknown. In this case μ is estimated by $\overline{\overline{X}}$ and σ is estimated using the sample ranges as $\hat{\sigma} = \frac{\overline{R}}{d_2}$. Then for the $\overline{X}$ -Chart, the control limits are:

Central line = $CL_{\overline{X}} = \overline{\overline{X}}$

Upper control limit $UCL_{\overline{X}} = \overline{\overline{X}} + \frac{3}{\sqrt{n}}\frac{\overline{R}}{d_2} = \overline{\overline{X}} + A_2\overline{R}$ and,

Lower control limit $LCL_{\overline{X}} = \overline{\overline{X}} - \frac{3}{\sqrt{n}}\frac{\overline{R}}{d_2} = \overline{\overline{X}} - A_2\overline{R}$

where $A_2 = \frac{3}{d_2\sqrt{n}}$ is a constant tabulated for different values of 'n' obtained directly from the table.

4. Derive the control limits for X bar chart when the process average is unknown and estimated using sample standard deviation.

When μ and σ are unknown. In this case μ is estimated by $\overline{\overline{X}}$ and σ is estimated using the sample standard deviation as $\hat{\sigma} = \frac{\overline{s}}{c_2}$. Then for the $\overline{X}$ -Chart, the control limits are:

Central line = $CL_{\overline{X}} = \overline{\overline{X}}$

Upper control limit $UCL_{\overline{X}} = \overline{\overline{X}} + \frac{3}{\sqrt{n}}\frac{\overline{s}}{c_2} = \overline{\overline{X}} + A_1\overline{s}$ and,

Lower control limit $LCL_{\overline{X}} = \overline{\overline{X}} - \frac{3}{\sqrt{n}}\frac{\overline{s}}{c_2} = \overline{\overline{X}} - A_1\overline{s}$

where $A_1 = \frac{3}{c_2\sqrt{n}}$ is a constant tabulated for different values of 'n' ontained directly from the table .

5. Derive the control limits for R - chart when the process standard deviation is (i) known (ii)unknown If sample Range is used to measure the process variability the control limits are:

Upper Control Limit (UCL) = E(R) +3SE(R)
Center Line = E(R)
Lower Control Limit (LCL) = E(R) -3SE(R)

From the sampling distribution of R it is known that E(R) = $d_2\sigma$ and SE(R) = $d_3\sigma$. Accordingly the control limits are simplified as:

***Case 1*: σ known**

Upper Control Limit UCL_R = $(d_2 + 3d_3)\,\sigma = D_2\sigma$ where $D_2 = d_2 + 3d_3$
Central Line CL_R = $d_2\sigma$
Lower Control Limit LCL_R = $(d2 - 3d_3)\,\sigma = D_1\,\sigma$ where $D_1 = d_2 - 3d_3$
Where the constants D_1 and D_2 directly obtained from the table

***Case 2*: σ unknown**

When σ is unknown, it is replaced by its estimate $\frac{\overline{R}}{d_2}$. Then the control limits are

Upper Control Limit UCL_R = $(d_2 + 3d_3)\frac{\overline{R}}{d_2} = D_4\overline{R}$

Central Line CL_R = $d2\sigma = d_2\frac{\overline{R}}{d_2} = \overline{R}$

Lower Control Limit LCL_R = $(d_2 - 3d_3)\frac{\overline{R}}{d_2} = D_3\overline{R}$

where, the constants $D_4 = (d_2 + 3d_3)/d_2$ and $D_3 = (d_2 - 3d_3)/d_2$ are directly obtained from the table of constants

6. Derive the control limits for S - chart when the process standard deviation is i) unknown ii) known

If sample standard deviation is used to measure the process variability the control limits are:

Upper Control Limit (UCL) = E(s) +3SE(s)
Center Line = E(s)
Lower Control Limit (LCL) = E(s) -3SE(s)

From the sampling distribution of 's' it is known that $E(s) = c_2\sigma$ and $SE(s) = c_3\sigma$. Accordingly the control limits are simplified as:

Case 1: **σ known**

Upper Control Limit $UCL_s = (c_2 + 3c_3)\sigma = B_2\sigma$ where $B_2 = c_2 + 3c_3$
Control Limit $CL_s = c_2\sigma$
Lower Control Limit $LCL_s = (c_2 - 3c_3)\sigma = B_1\sigma$ where $B_1 = c_2 - 3c_3$

Where the constants B_1 and B_2 directly obtained from the table

Case 2: **σ unknown**

When σ is unknown, it is replaced by its estimate s/c_2. The control limits are

Upper Control Limit $UCL_R = (c_2+3c_3)\sigma = (c_2 + 3c_3)\dfrac{\bar{s}}{c_2} = B_4\,\bar{s}$

Central line $CL_R = c2\sigma = c_2\dfrac{\bar{s}}{c_2} = \bar{s}$

Lower Control Limit $LCL_R = (c_2-3c_3)\sigma = (c_2 - 3c_3)\dfrac{\bar{s}}{c_2} = B_3\,\bar{s}$

Where, $B_4 = (c_2 + 3c_3)/c_2$ and $B_3 = (c_2 - 3c_3)/c_2$
Where the constants B_3 and B_4 directly obtained from the table

7. Write the **Steps in constructing $\overline{X}$ Chart and R-Chart**

Step 1. *Gather the data.*
Step 2. Derive the control limits
Step 2. *Plot the data.*
Step 3: *Interpret the Charts*

8. Write the out of control situation for X bar chart

1. One or more points beyond the UCL and LCL
2. Seven (or more) consecutive points above or below the center line
3. Seven consecutive increasing or decreasing points
4. Two out of three beyond two sigma limit on the same side of the center line
5. Four out of five beyond one sigma limit on the same side of the center line
6. Ten out of 11 consecutive points on the same side of the center line
7. Twelve out of 14 consecutive points on the same side of the center line

8. A series of points "hugging" the centerline
9. A series of points "hugging" the control limits
10. Fourteen consecutive points alternating up and down (saw tooth pattern)
11. Any nonrandom pattern (such as a cycle)

9.Write the out of control situation for R chart

1. One or more points beyond the UCL
2. Seven (or more) consecutive points above Central Line
3. Seven consecutive increasing points
4. Two out of three beyond two sigma limit above the center line
5. Four out of five beyond one sigma limit above the center line

6. Ten out of 11 consecutive points above the center line
7. Ten out of 11 consecutive points above the center line
8. A series of points "hugging" the centerline
9. A series of points "hugging" the upper control limits
10. Fourteen consecutive points alternating up and down (saw tooth pattern)
11. Any nonrandom pattern (such as a cycle)

9. What are Trial control limits .

The trial limits are control limits for analyzing the past data or at the initial stage of establishing control chart, calculated following the procedure explained above. They may require modification before extending them for application for future production and maintenance of stable process. If the data points are within the control limits exhibiting a stable process, we assume the process is in control. These limits are used for future use. Otherwise, we revise the limits by looking at the out-of-control points. Samples for which the plotted values go outside control limits are eliminated for calculation of revised limits. This procedure is continued until all the points are within the control limits or the procedure is free from any assignable cause in obtaining the final control limits.

10. **Why two control charts** simultaneously for varibales?

$\overline{X}$ -Chart chart monitors the *between* sample variability and σ-Chart or R-Chart monitors the *within* sample variability. Hence two charts are necessary to study the process control. Range Chart (R-Chart) indicates a change in the process variability and that of the average chart ($\overline{X}$ -Chart) indicates a change or shift in the level of the process. Hence two charts are constructed simultaneously to see whether the process is working in control or going out of control

11. Samples of size n=5 are taken from a manufacturing process every hour. A quality characteristic is measured, and $\overline{X}$ and R are computed for each sample. After 25 samples have been analyzed, we have $\sum_{i=1}^{25}\overline{x}_i$ =662.50 and $\sum_{i=1}^{25}R_i$ =9.00 The quality characteristic are normally distributed.Find the control limits for the $\overline{X}$ and R charts.

Control limits for X bar chart:

Calculate $\overline{\overline{X}}$ = 26.5 and $\overline{R}$ = 9/25=0.36 from table A_2 =0.577, D_3=0, D_4=2.115 (refer for n=5)

Control limits for X bar chart:

Central line = $CL_{\overline{X}} = \overline{\overline{X}}$ = 662.50/25 = 26.5

Upper control limit $UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$ = 26.5 + (0.577 X0.36)=26.71

Lower control limit $LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$ = 6.29

Control limits for R chart:

Upper Control Limit	UCL_R	= $D_4\overline{R}$ = 0.76
Central Line	CL_R	= $\overline{R}$ = 0.36
Lower Control Limit	LCL_R	= $D_3\overline{R}$ = 0

12. A process is to be monitored with standard values μ = 10, and σ = 2.5. The sample size is 3.(i) Find the central line and control limits for $\overline{X}$ chart.(ii)) Find the central line and control limits for R chart (ii) Find warning limits in above cases.

From table the value of A for n=3 is 1.732, D_2 = 4.358 and D_1 = 0 d_2 = 1.693

Control limits for X bar chart:

Upper Control Limit (UCL)= $\mu + A\sigma$ = 10+(1.732 X 2.5) = 10+4.33 = 14.33

Center Line = μ = 10

Lower Control Limit (LCL) = $\mu - A\sigma$ = 5.67

Control limits for R chart:

Upper Control Limit	UCL_R	= $(d_2 + 3d_3)\,\sigma = D_2\sigma$ =4.358 X 2.5 = 10.895
Central Line	CL_R	= $d_2\sigma$ = 1.693 X 2.5 = 4.23
Lower Control Limit	LCL_R	= $(d2 - 3d_3)\,\sigma = D_1\,\sigma$ = 0