

Attribute control charts

In this session we discuss the construction of Attribute control charts. The entire topic is divided in the following subdivisions:

1. Attribute control charts
2. Control chart for nonconforming units (np –chart)
3. Control chart for fraction nonconforming units(p-Chart)
4. Control chart for number of nonconformities(C and U chart)
5. Advantages of Attribute control charts
6. Worked out examples
7. Conclusion

Attribute Control Charts:

Generally control charts for attributes are grouped into the following four general groups.

1. CONTROL CHART FOR NONCONFORMING UNITS

2. The p – chart: Control chart for fraction nonconforming units (defectives).
3. The np – chart: Control chart for number of nonconforming units (defectives).

Control Chart for Number of Nonconformities

4. The c– chart: Control chart for number of nonconformities (defects).
5. The u – chart: Control chart for number of nonconformities per unit (defects).

CONTROL CHART FOR NONCONFORMING UNITS

Control chart for nonconforming units is used for attribute types of data whenever the quality characteristics observed result in the classification of a product as nonconforming or conforming units.

Consider a production process where the items are inspected for nonconforming units. If the records of inspection over a period are scrutinized, the fraction nonconforming will show variation. Under conditions of control, the fraction nonconforming readings tend to approach a fixed value provided the number of items inspected is very large.

This value to which the fraction nonconforming readings tend to approach is the standard fraction nonconforming or probability of a nonconforming unit from the process.

If P represents the probability of a nonconforming unit from the process, the probability of obtaining r fraction nonconforming from a sample of size n is given by the Binomial Distribution

$$\binom{n}{r} (P)^r (1-P)^{n-r}$$

For example, if the probability of a fraction nonconforming unit is 0.01 and if samples of size 50 are taken, it will be seen that the probability of obtaining 6 fraction nonconforming units are noted it is clear indication that some external cause has influenced the process.

The mean of a Binomial distribution is given by nP and standard deviation by $\sqrt{nP(1-P)}$

The Binomial distribution approximates to the Normal distribution in many practical situations (particularly when $nP \geq 5$ and $n(1-P) \geq 5$) with the mean and standard deviation as that of binomial. This property is utilized in constructing charts for fraction nonconforming to know whether the process is in control or not.

Thus, when the values of P and n are known, the standard deviations are easily calculated and $\pm 3\sigma$ limits from the average provide the basis for judging the process for control. This is the basis of control chart for nonconforming units.

When the sample size n is constant among subgroups samples it is simpler to plot the actual number of non-conforming units rather than fraction nonconforming. Such charts are known as **number nonconforming units chart** or simply **np-chart**. When the sample size is not constant among subgroups samples we plot the fraction nonconforming. In such cases we have chart for **fraction nonconforming** or simply the **p-chart**.

Control limits - Number Nonconforming Units Chart - np Chart

Case 1. When the mean of the population (nP) from which samples are taken is known.

$$UCL = nP + 3\sqrt{nP(1-P)}$$

$$CL = nP$$

$$LCL = nP - 3\sqrt{nP(1-P)}$$

Case 2. When the mean of the population (nP) from which samples are taken is unknown known and P is estimated using $\bar{p} = \frac{\text{Number of nonconforming units in all the samples combined}}{\text{Number of items in all the samples combined}}$.

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$CL = n\bar{p}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

Construction of the control charts for of nonconforming units:

1. Preliminary data for construction of number of non-conforming units chart are generally obtained from past records. Collect k (about 25) samples of size n , n being the same for all samples and record them.

Table 1

Sl. No.	Sample size	No. of defectives
1	n	d_1
2	n	d_2

3	n	d_3
.	n	
.		
.		
k	n	d_k
total	n x k	$\sum_{i=1}^k d_i$

2. Obtain an estimate of the fraction nonconforming as

$$\bar{p} = \frac{\text{Number of nonconforming units in all the samples combined}}{\text{Number of items in all the samples combined}} = \frac{\sum_{i=1}^k d_i}{n \times k}$$

3. Compute trial upper and lower control limits for number nonconforming chart as follows:

$$UCL = n \bar{p} + 3 \sqrt{n \bar{p} (1 - \bar{p})}$$

$$CL = n \bar{p}$$

$$LCL = n \bar{p} - 3 \sqrt{n \bar{p} (1 - \bar{p})}$$

4. If no sample has number of nonconforming units violating the limits, the samples may be considered homogeneous.
5. If condition (4) is satisfied accept $\bar{p}$ as the standard fraction nonconforming.
6. If the condition (4) is not satisfied, remove those samples with number of non-conforming units more than UCL and compute $\bar{p}$ from the remaining samples
7. Repeat step (3) with new $\bar{p}$.
8. Compare the number of non-conforming units in the remaining samples with the revised limit. Consider the samples as homogeneous if none exceeds the limit.
9. If (8) is satisfied, accept the revised value of $\bar{p}$ as the standard for process.
10. If (8) is not satisfied, repeat (6) to (9) and repeat as often as it is necessary for it to be satisfied. If more than 25% of the samples get rejected, collect fresh data after exercising technical controls on the process.
11. Accept the final value of $\bar{p}$ as the standard fraction nonconforming for the process.
12. Compute $n \bar{p}$, $n \bar{p} + 3 \sqrt{n \bar{p} (1 - \bar{p})}$ and $n \bar{p} - 3 \sqrt{n \bar{p} (1 - \bar{p})}$ using the value of $\bar{p}$ obtained in (11).

13. Prepare a chart for nonconforming units with the vertical scale at the left show the number of nonconforming units and the horizontal scale indicating the subgroup number of samples.
14. Draw a horizontal line across the chart at $n\bar{p}$ (central line) together with two broken lines at the control limits $n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ and $n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$.
15. Continue selection and inspection of samples and plot the number of nonconforming units on this chart in the order of production. Mark on the chart any point at which any serious change in the process occurred.
16. The interpretation of this chart is similar to that of charts for measurements.
17. If any of the plotted points falls above UCL or if a run of points is noticed above the control line or any upward trend is noticed, it should lead to proper action to set right the situation. It may also indicate where charts for Averages and Range should be installed.
18. If any point falls below the LCL or if any run of points noted below the central line or if any trend is noticed, it indicates improvement in the process provided the standard of inspection has been maintained. Investigations are to be made in such cases to locate reasons for improvement and maintain such conditions. Estimate of the improved standard is to be obtained and control limits are to be revised.
19. The principle of rational subgroups mentioned in the case of $\bar{X} - R$ charts hold good in this case also.

2. Control Chart for Fraction Nonconforming: p-Chart

We know that the fraction nonconforming 'p', follows Binomial distribution with mean P and standard deviation by $\sqrt{P(1-P)/n}$

Sl.No.	Sample size	No. of defectives	Fraction number of defectives(p_i)
1	n_1	d_1	d_1/n_1
2	n_2	d_2	d_2/n_2
3	n_3	d_3	d_3/n_3
.			
.			
.			
k	n_k	d_k	d_k/n_k
total	$\sum_{i=1}^k n_i$	$\sum_{i=1}^k d_i$	

1. Collect k (about 25) samples. Say the i-th sample has a size n_i ; $i = 1, 2, \dots, 25$, n_i need not be same.
2. Calculate fraction nonconforming for each sample as

$$p_i = \frac{\text{Number of nonconforming units in the sample}}{\text{Number of items in the sample}}$$

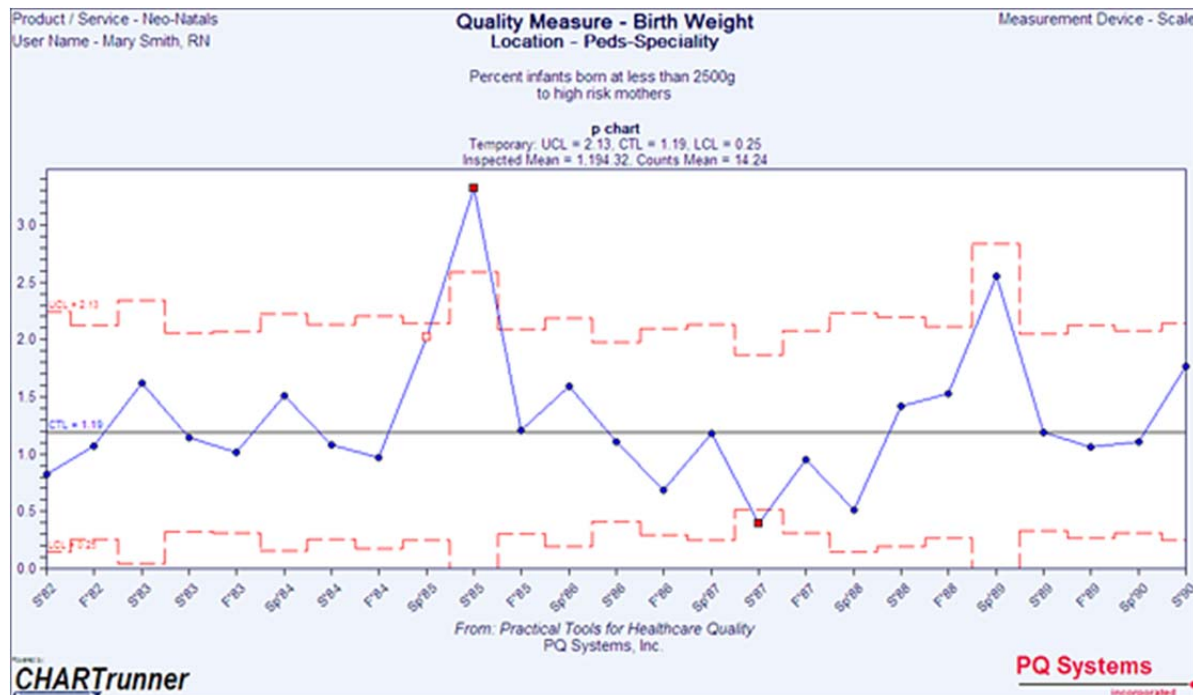
3. Obtain an estimate of the standard fraction nonconforming as

$$\bar{p} = \frac{\text{Number of nonconforming units in all the samples combined}}{\text{Number of items in all the samples combined}} = \frac{\sum_{i=1}^k d_i}{\sum_{i=1}^k n_i}$$

4. Compute control limits as $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i}$. It will be different for different samples (where, n_i = size of the sample)
5. Compare each p_i with its limits. If no sample point falls outside the limits, samples may be considered homogeneous.
6. Otherwise, homogenize the samples following the same procedure as in the previous case.
7. Accept the final value of $\bar{p}$ as the standard fraction nonconforming.
8. Compute $\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n_i}$ and $\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n_i}$ using the value of $\bar{p}$ obtained in (7)
9. Prepare a chart for fraction nonconforming with the vertical scale at the left showing fraction nonconforming and the horizontal scale indicating the sub group number of samples.
10. Draw a horizontal line across the chart at $\bar{p}$ (central line) together with two broken lines at the control limits at $\bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n_i}$ and $\bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n_i}$
11. Plot the number of nonconforming units on this chart in the order of production

12. The interpretation of this chart is similar to that of charts for measurements

A typical p chart as given below



Ref: <http://www.pqsystems.com/>

3. Control Chart For Number of Nonconformities: c-Chart

Data obtained in control chart for nonconforming units do not consider the magnitude of nonconformities contained in a time. There are many cases in industry where control of nonconformities is considered important. Examples are nonconformities in casting, imperfections in certain area of paper, fabric etc.

Control charts for numbers on nonconformities are useful in these cases.

The sample size for the control chart for number of nonconformities is usually fixed on duration of time, length, area, single unit or group of units.

The Poisson distribution generally explains the pattern of variation for the number of nonconformities. If the average number of nonconformities per unit is m , then the probability of getting r nonconformities in any sample units is $e^{-m} m^r / r!$

Let m be the average number of nonconformities per unit for a process. It is possible to calculate the value of r (nonconformities per unit in samples) for which the probability is nearly zero. If any sample unit has more nonconformities than this value, it is a clear indication that external cause has influenced the process.

The standard deviation for the Poisson distribution is given by $\sqrt{m}$.

The normal approximation to the Poisson distribution may be used provided $m > 5$. In many situations it is possible to arrange collection of data in such a way that the normal approximation could be used.

Using above information, limits for judging the process for control could be calculated.

Detailed procedure for constructing the c-chart is as follows:

Tabulate the data as below:

Table 3

Sl.No.	No. of nonconformities
1	C_1
2	C_2
3	C_3
.	
.	
.	
k	C_k
total	$\sum_{i=1}^k C_i$

Case 1: Sample size (n) is Constant:

1. Collect about 25 samples of size n and count the number of nonconformities 'c' in each sample (Table 3).
2. Compute average number of nonconformities per sample as

$$m = \bar{c} = \frac{\text{number of nonconformities in all samples}}{\text{total number of samples}} = \frac{\sum_{i=1}^k c_i}{k}$$

3. Compute trial $UCL = \bar{c} + 3\sqrt{\bar{c}}$ and $LCL = \bar{c} - 3\sqrt{\bar{c}}$.
4. Compare the number of nonconformities in each sample (c) with UCL and LCL. If no sample has more nonconformities than the UCL or less than the LCL, the samples may be considered homogeneous. Otherwise homogenize the samples.
5. Accept the final value of m as the standard number of nonconformities per unit.
6. Compute control limits $\bar{c} + 3\sqrt{\bar{c}}$ and $\bar{c} - 3\sqrt{\bar{c}}$ using the values of m obtained in (5).
7. Prepare a chart for number of nonconformities per sample (c-chart) with the vertical scale left indicating the number of nonconformities (c) and horizontal scale indicating subgroup of samples.
8. Draw a horizontal line across the chart at m (Central Line) and two broken lines at the control limits $\bar{c} + 3\sqrt{\bar{c}}$ and $\bar{c} - 3\sqrt{\bar{c}}$.

9. Continue collection and inspection of samples and plot the number of nonconformities in each sample on this chart. Mark on the chart any point at which any change in the process occurred.
10. Interpretation of this chart is similar to that of np-chart.

Case (ii): Sample size is not constant (U - chart)-

Tabulate the data as below

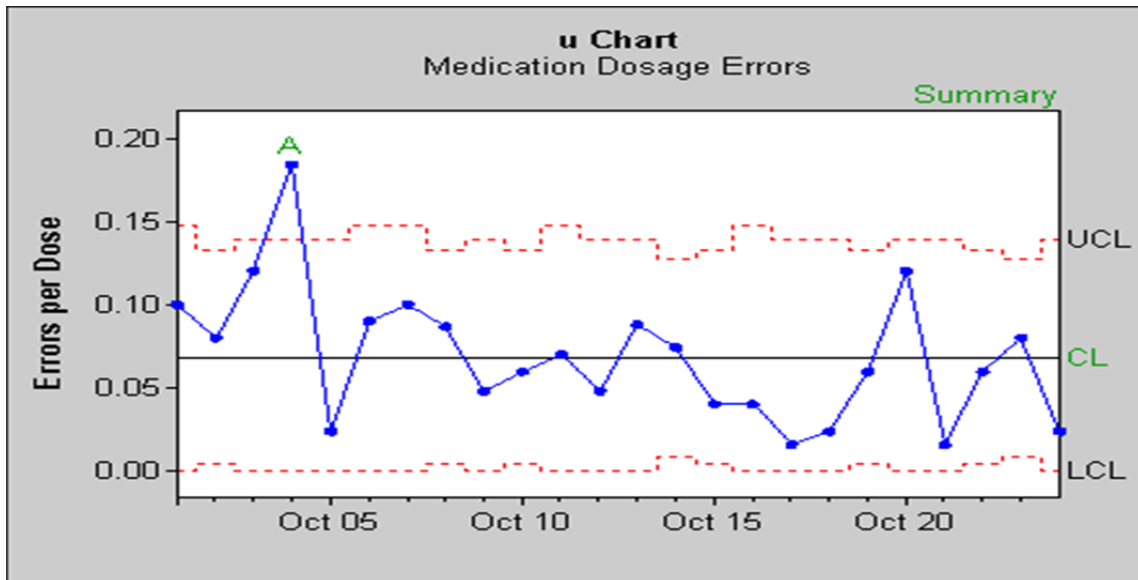
Sl. No.	Sample size	No. of nonconformities	Fraction nonconformities(u_i)
1	n_1	c_1	c_1/n_1
2	n_2	c_2	c_2/n_2
3	n_3	c_3	c_3/n_3
.			
.			
.			
k	n_k	c_k	c_k/n_k
total	$\sum_{i=1}^k n_i$	$\sum_{i=1}^k c_i$	$\frac{\sum_{i=1}^k c_i}{\sum_{i=1}^k n_i}$

1. Construction of U chart is similar to C chart. In this case, the number of nonconformities in the samples is not plotted. Instead $u_i = c_i/n_i$ is calculated for each sample and u_i values are plotted. Central line will be at $\bar{u}$ and $UCL = \bar{u} + 3\sqrt{\bar{u}/n_i}$ and $LCL = \bar{u} - 3\sqrt{\bar{u}/n_i}$

where $\bar{u} = \frac{\sum_{i=1}^k c_i}{\sum_{i=1}^k n_i}$. The control limits will be varying from sample to sample

2. Procedure for identifying out of control situations and interpretation are similar to that of p- chart.

A typical C and U chart are given below



Defect(nonconformity) and defective(nonconforming unit): A defective is an article that in some way fails to confirm the one or more given specifications. Each instance of the article, lack of conformity to specification is a defect. For eg., a television set is a defective one as its picture tube is not functioning or in other words non functioning picture tube is a defect in the television set. Every defective contains one or more defects. E.g. number of defects in a piece of cloth like improper colouring, improper printing etc., number of surface defects observed in a roll of coated paper etc.

4. Advantages of Attribute Control charts over $\bar{X} - R$ Charts:

The following are the advantages of attribute control charts over $\bar{X} - R$ charts:

- Attribute control charts do not require measurements and hence, the skill and effort of measurement required to maintain $\bar{X} - R$ charts is involved.
- By virtue of not requiring measurements attributes control charts are more economical and less demanding on inspection time.
- An attribute control chart can be applied to a number of Quality characteristic checked at a work station; whereas, $\bar{X} - R$ charts require one chart for each characteristic.
- Attribute control charts are less demanding on skills of the personnel collecting data and maintaining charts.
- The chart can be easily read and understood by on-line production personnel and hence can act as self evaluation by operators.
- Attribute control chart analysis is easier and simpler in comparison to $\bar{X} - R$ chart analysis.
- Most often data collection effort on attribute control charts is very minimal, since data collected for other purposes can be used for the chart.

- Attribute control chart information is also useful as quality history and is useful for management Information of Operating Efficiency, and of control of operating Efficiency.

5. Worked out examples

Data were collected on stub joints in different indents. The following table shows the number of joints welded and number of nonconforming joints. Analyze the data using p-chart.

Sample Number	Joints Welded	Number of nonconforming joints	Sample Number	Joints Welded	Number of nonconforming joints
1	160	49	11	24	4
2	80	28	12	28	3
3	60	20	13	55	4
4	160	41	14	114	11
5	80	21	15	56	1
6	156	40	16	32	8
7	80	20	17	60	0
8	56	2	18	44	18
9	98	7	19	98	8
10	400	83	20	108	11
			21	102	10

Solution:

Sp. No.	ni	di	pi=ni/di	UCL	LCL	
1	160	49	0.3063	0.2827	0.0967	ni=Joints Welded
2	80	28	0.3500	0.3212	0.0582	di=Number of nonconforming joints
3	60	20	0.3333	0.3415	0.0379	$\bar{p}$ is 0.1897
4	160	41	0.2563	0.2827	0.0967	
5	80	21	0.2625	0.3212	0.0582	
6	156	40	0.2564	0.2839	0.0955	
7	80	20	0.2500	0.3212	0.0582	
8	56	2	0.0357	0.3469	0.0325	
9	98	7	0.0714	0.3085	0.0709	
10	400	83	0.2075	0.2485	0.1309	
11	24	4	0.1667	0.4298	0.0000	
12	28	3	0.1071	0.4120	0.0000	
13	55	4	0.0727	0.3483	0.0311	
14	114	11	0.0965	0.2999	0.0795	
15	56	1	0.0179	0.3469	0.0325	
16	32	8	0.2500	0.3976	0.0000	
17	60	0	0.0000	0.3415	0.0379	
18	44	18	0.4091	0.3670	0.0124	
19	98	8	0.0816	0.3085	0.0709	
20	108	11	0.1019	0.3029	0.0765	
21	102	10	0.0980	0.3061	0.0732	
Total	2051	389				

6. **Conclusion:** In this lecture we discussed how to construct control charts for attributes in different situations and how to interpret them. We discussed application of binomial and Poisson distribution in setting the control limits. We discussed the difference between the charts for Nonconforming units and nonconformities. We also discussed the use of attribute control charts over variable control charts.