Frequently asked questions

1. Give the layout of a 5x5 la==Latin square design

   Let A,B,C,D and E are the 5 treatments . The layout is

Row	column				
	1	2	3	4	5
1	**A**	**B**	**C**	**D**	**E**
2	**B**	**C**	**D**	**E**	**A**
3	**C**	**D**	**E**	**A**	**B**
4	**D**	**E**	**A**	**B**	**C**
5	**E**	**A**	**B**	**C**	**D**

2. Define Latin square design

A Latin square of order p is an arrangement of p symbols in p^2 cells arranged in p rows and p columns such that each symbol occurs once and only once in each row and in each column.

Thus in LSD the treatments are grouped into replications in two ways
   Once in rows' and in columns

3. Write the model assumed in Latin square design

Let $y_{i\,jk}$ represent observation from k^{th} row, j^{th}column and receiving i^{th} treatment, I,j,k= 1….p. The model assumed is
$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk}$
   Where μ general effect
   α_i – i^{th} treatment effect
   βj – j^{th} block effect;
   γk – k^{th} row effect;
   ϵijk- error term , which are identically and independently distributed normal random variables with mean 0 and variance σ^2.

4. State the models to be tested in Latin square designn
   We test the following model
   1. Related to the treatment effects

H_A: $\alpha_1 = \alpha_2 = \ldots \alpha_p = 0$
And the alternative hypothesis is
H_{A1}: at least one $\alpha_i \neq \alpha_j$ for all I,j

2. Related to the block effects

H_B: $\beta_1 = \beta_2 = \ldots \beta_p = 0$

And the alternative hypothesis is

H_{B1}: at least one $\beta_i \neq \beta_j$ for all I,j

3. Related to the row effects

H_C: $\gamma_1 = \gamma_2 = \ldots \gamma_p = 0$

And the alternative hypothesis is

H_{C1}: at least one $\gamma_i \neq \gamma_j$ for all I,j

5. Derive the Least square estimates of the parameters of Latin square design:

The parameters μ, α_i, β_j and γ_k are estimated by the method of least squares. i.e. by minimising error sum of squares. L = $\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p} \varepsilon_{ijk}^{2}$ =

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p} (y_{ijk} - \mu - \alpha_i - \beta_j - \gamma_k)^2$$

And solving the normal equations $\frac{\partial L}{\partial \mu} = 0$, , $\frac{\partial L}{\partial \alpha_i} = 0$ and $\frac{\partial L}{\partial \beta_j} = 0$, $\frac{\partial L}{\partial \gamma_k} = 0$, I,j,k=1,2…p

we obtain 3p+1 normal equations as ,

$$p^2\mu + \sum_{i=1}^{p} p\alpha_i + \sum_{j=1}^{p} p\beta_j + \sum_{k=1}^{p} p\gamma_k = \sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p} y_{ijk}$$

$$p\mu + p\alpha_i + \sum_{j=1}^{k}\beta_j + \sum_{k=1}^{k}\gamma_k = \sum_{j=1}^{p}\sum_{k-1}^{p} y_{ijk}$$, i= 1,2 …p

And

,

$$p\mu + \sum_{i=1}^{p}\alpha_i + p\beta_j + \sum_{k=1}^{p}\gamma_k = \sum_{i=1}^{p}\sum_{k-1}^{p} y_{ijk}$$, j=1.2,…k

$$p\mu + \sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{p}\beta_j + p\gamma_k = \sum_{i=1}^{p}\sum_{j-1}^{p} y_{ijk}$$

Adding independent constraint, $\sum_{i=1}^{p} \alpha i = 0$, $\sum_{j=1}^{p} \beta_j = 0$ $\sum_{k=1}^{p} \gamma_k = 0$ and Solving we get the solutions as

$\hat{\mu} = \bar{y}..$ and $\hat{\alpha}_i = \bar{y}_{i.} - \bar{y}..$ i=1,2..p βj = $\hat{\beta}_j = \bar{y}._j. - \bar{y}..$ and $\hat{\gamma}_k = \bar{y}.._k - \bar{y}..$

6. Obtain an expression for the error sum of squares in Latin square design.

Substituting the values of the estimators μ, α_i, β_j in the model we get

Y_{ijk}= $\hat{\mu}$ + $\hat{\alpha}_i$ + $\hat{\beta}_j$ + $\hat{\gamma}_k$ + ϵijk

Or

Y_{ijk-} $\bar{y}...$ =($\bar{y}_{i..} - \bar{y}...$) + ($\bar{y}._j. - \bar{y}...$)+($\bar{y}.._K - \bar{y}...$)+ $(yijk - \overline{yi..} - \overline{y.j.} - \overline{y..k} + 2\overline{y...})$,

Squaring both sides and summing over all the observations we get

$$\sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{k=1}^{p} (y_{ijk} - \overline{y...})^2 = \sum_{i=1}^{p} p(\overline{yi..} - \overline{y_{...}})^2 + \sum_{j=1}^{p} p(\overline{y.j.} - \overline{y_{...}})^2 + \sum_{j=1}^{p} p(\overline{y._k.} - \overline{y_{...}})^2 + \sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{k=1}^{p} (yijk - \overline{yi..} - \overline{y.j.} - \overline{y..k} + 2\overline{y...})^2$$

as all the cross product vanishes.

OR

SST=SSTR+SSB+SSR+SSE where

$$SST = \sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{k=1}^{p} (y_{ijk} - \overline{y_{..}.})^2 = \sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{k=1}^{p} y_{ijk}^2 - \frac{y_{...}^2}{p^2}$$

SSTR = $\sum_{i=1}^{p} p(\overline{yi.} - \overline{y}_{..})^2$ = $\sum_{i=1}^{p} \frac{y_{i...}^2}{p} - \frac{y_{..}^2}{p^2}$

SSB = $\sum_{j=1}^{p} p(\overline{y.j} - \overline{y}_{..})^2$ = $\sum_{i=1}^{k} \frac{y_{.j.}^2}{p} - \frac{y_{...}^2}{p^2}$

SSR = $\sum_{j=1}^{p} p(\overline{y.}_k - \overline{y}_{..})^2$ = $\sum_{i=1}^{k} \frac{y_{..k.}^2}{p} - \frac{y_{...}^2}{p^2}$

And hence SSE = $\sum_{i=1}^{p} \sum_{j=1}^{p} \sum_{k=1}^{p} (yijk - \overline{yi..} - \overline{y.j.} - \overline{y..k} + 2\overline{y...})^2$

7. Write the degrees of freedom of different sum of squares.

The TSS is computed with p^2 quantities, which will carry p^2-1 degree of freedom, SSTR will have p-1 degree of freedom, SSB will have p-1 degree of freedom and SSR will have p-1 degree of freedom and SSE will carry (p^2-1) – (p-1) –(p-1) –(p-1)= (p-1) (p-2) degree of freedom.

8. Write the test statistics to test the different hypothesis in the analysis of Latin square design .

   To test the hypothesis H_A: $\alpha_1 = \alpha_2 = \dots \alpha_p = 0$ against the alternative the test statistics is

$F_A = \frac{SSTR/(p-1)}{SSE/(p-1)(p-2)} = \frac{MSSTR}{MSSE}$ and is distributed as F with p-1 and (p-1)(p-2) degrees of freedom.

To test the hypothesis H_B: $\beta_1 = \beta_2 = \dots \beta_p = 0$ against the alternative the test statistics is

$F_B = \frac{SSB/(p-1)}{SSE/(p-1)(p-2)} = \frac{MSSB}{MSSE}$ is distributed as F with p-1 and (p-1)(p-2) degrees of freedom.

To test the hypothesis H_C: $\gamma_1 = \gamma_2 = \dots \gamma_p = 0$ against the alternative the test statistics is

$$F_C = \frac{SSR/(p-1)}{SSE/(p-1)(p-2)} = \frac{MSSR}{MSSE}$$ is distributed as F with p-1 and (p-1)(p-1) degrees of freedom.

9. Write Analysis of Variance Table for the analysis in Latin square design :

Sources of variation	Degree of freedom	Sum of squares	Mean sum of squares	F-Value
Treatments	p-1	SSTR	MSSTR = SSTR/(p-1)	FA = MSSTR/ MSSE
Blocks	p-1	SSB	MSSB = SSB/(k-1)	FB = MSS**B**/ MSSE
Rows	p-1	SSR	MSSR = SSR/(P-1)	FB = MSS**R**/ MSSE
Error	(p-1) (p-2)	SSE	MSSE = SSE/(p-1) (p-2)	
total	P^2-1	TSS		

10. Write the Advantages of LSD:

1. Controls more variation than CRD or RCBD designs because of 2-way stratification. Results in a smaller mean square for error.
2. Simple analysis of data
3. Analysis is simple even with missing plots.
4. LSD is an incomplete 3 way layout. Its advantages over the 3 way layout is that instead of m^3 experimental units only m^2 units are needed.
5. More than one factor can be investigates simultaneously and with fewer trials than more complicated designs

11. Write the Disadvantages of LSD

1. The number of treatments, rows and columns must be the same
2. Number of treatments is limited to the number of replicates which in normal situations difficult to arrive at (exceeds 10 generally).
3. If have less than 5 treatments, the df for controlling random variation is relatively large and the df for error is small.
4. The effect of each treatment must be approximately the same across rows and columns.
5. In an field layout, RBD is much easy to manage the LSD as the former can perform a square or rectangular field easily or field of any shape where as for the later approximately a square field is necessary.

12. **Complete the following Anova Table for a LSD**:

Sources of variation	Degree of freedom	Sum of squares	Mean sum of squares	F-Value
Treatments	3	1372.1225	457.3742	-
Blocks	-	-	51.7575	-
Rows	-	-	86.4375	-
Error	-	-	-	
total	-	1943.0775		