Frequently Asked Questions

1. What do you mean by MP test?
Answer:
Among the critical regions of the same size α that which renders the minimum Type II error is called the most powerful critical region. The test based on the most powerful critical region is called the most powerful test. Therefore among all tests possessing the same size of Type I error, choose one for which the size of the Type II error is small as possible. This test is called the Most Powerful test.

2. What are the initial steps of testing problem?
Answer:
In any testing problem the first few steps involve the form of the population distribution, the parameters of interest and the framing of a null hypothesis and alternative hypothesis should be obvious from the description of the problem. The most crucial step is the choice of the best test that is the best statistic and the critical region C whereby best we mean one which addition to controlling alpha at any desired low level has the minimum type II error beta, or maximum power 1- β compared to β of all other tests having this alpha.

3. What are the contributions of Neyman Pearson to testing of Hypothesis?
Answer:
Neymann Pearson Lemma based on the magnitude of the ration of the two probability density functions provides the best test for testing a null hypothesis against an alternative hypothesis. The best test in any given situation depends upon the nature of the population distribution and the form of the alternative hypothesis being considered.
Neymann Pearson designed a Most Powerful Test which is the test procedure used to test the simple null hypothesis against a simple alternative hypothesis. But when we need to test a simple or composite null hypothesis against a simple or composite alternative hypothesis the test procedure adopted given by Neyman Pearson is known as a Uniformly Most Powerful test.
A test introduced by Neyman and Pearson for testing a hypothesis simple or composite against a simple or composite alternative hypothesis. This test is related to the maximum likelihood estimates.

4. Define a Parameter space.
Answer:
Let us consider a random variable X with p.d.f f(x, θ). In most common applications , though not always the functional form of population distribution is assumed to be known except for the value of some unknown parameters θ which may take any value on a set Ω .This is expressed by writing a p.d.f in the form f(x,θ), θ ε Ω
The set Ω which is the set of all possible values of θ is called the parameter space.

5. Give examples for a parameter space
Answer:
For example: 1) X~N (μ, σ^2), then the parameter space

$$\Omega = \{(\mu, \sigma^2) : -\infty < \mu < \infty; 0 < \sigma < \infty\}$$

2) X~B (n, p), then the parameter space

$$\Omega = \{(n, p) : n > 0, 0 < p < 1\}$$

6. How do we classify the parameter space Ω?
Answer:

We shall consider a general family of distributions
{f(x, θ1, θ2,….., θk), θi ε Ω, i=1,2,……,k}
Let x1,x2,…,xn be a random sample of size n>1 from a population with p.d.f f(x, θ1, θ2,….., θk), where Ω ¡s the parameter space ,is the totality of all the points that (θ1, θ2,….., θk) can assume. Then we are interested to test the null hypothesis Ho:(θ1, θ2,….., θk) ε θ0 against alternative hypothesis of the type H1:(θ1, θ2,….., θk) ε θ1 where θ1 = Ω – θ0
Hence entire parameter space Ω can be thought to be composed of two sub parameter spaces, namely θ0 and θ1, That is Ω = θ0 U θ1
Where θ0 is the parameter space under the null hypothesis H_0 which belongs to the parameter space Ω and θ1 is the parameter space under the alternative hypothesis H_1 which is also a part of the parameter space Ω

7. Briefly explain Maximum Likelihood Estimates
Answer:
A likelihood function of a number of sample observations is defined to be their joint density function. To obtain a of maximum likelihood estimate, one first specifies the joint density function for all observations. For an i.i.d. sample, this joint density function is

$F(x_1, x_2 \ldots x_n, \theta) = f(x1, \theta)\, f(x2, \theta)\, f(x3, \theta) \quad \ldots\ldots\ldots\, f(xn, \theta)$

In case the sample observations are independent the likelihood function happens to be the product of the density functions of the random observations, then the likelihood function of the random observations denoted by $L = L(x_1, x_2, \ldots, x_n, \theta)$ is given by

$\prod_{i=1}^{n} f(x_{i,}\theta)$ where f (xi, θ) is the p.d.f of the population.

The method of maximum likelihood is the one in which for a given set of values $(x_1, x_2 \ldots x_n)$ an estimator of θ is found that maximizes L. Thus if there exists $\hat{\theta}$, a function of $(x_1, x_2, \ldots, x_n)$ for which L is maximum for variations in θ . Then $\hat{\theta}$ is the m.l.e of θ then

$\frac{\partial L}{\partial \theta} = 0$ and $\frac{\partial^2 L}{\partial \theta^2} < 0$

8. What do you mean by likelihood ratio test?
Answer:
A test introduced by Neymann and Pearson for testing a hypothesis simple or composite against a simple or composite alternative hypothesis is known as a likelihood ratio test.
In statistics, a likelihood ratio test is a statistical test used to compare the fit of two models, one of which (the null model) is a special case of the other (the alternative model). The test is based on the likelihood ratio, which expresses how many times more likely the data are under one model than the other. This likelihood ratio, or equivalently its logarithm, can then be used to compute a p-value, or compared to a critical value to decide whether to reject the null model in favour of the alternative model.
Substituting maximum likelihood estimates we obtain the maximum values of the likelihood function for the variation of the parameters in the parameter space. Then the criteria for the likelihood ratio tests are defined as the quotient of these two maxima that is maximum under parameter space under the null hypothesis and a total parameter space.

9. Briefly explain a Likelihood ratio test procedure
Answer:
Suppose a composite null hypothesis $H_0 : \theta \in \Theta_0$ is to be tested against a composite alternative hypothesis $H_1 : \theta \in \Theta_1$

For testing the above null hypothesis a test procedure called Likelihood ratio test procedure is followed which is explained below.
Let a random sample x1, x2…xn of size n be drawn from from the given population with p.d.f f(x, θ)

$$\text{Let } \lambda = \frac{\underset{\theta \in \Theta_0}{Sup}\ L(\theta, x1, x2, ..., xn)}{\underset{\theta \in \Theta}{Sup}\ L(\theta, x1, x2, ..., xn)}$$

Since the supremum in the denominator is over a larger set of numbers. $\lambda \leq 1$. Also likelihood functions are nonnegative and hence $\lambda \geq 0$. Thus $0 \leq \lambda \leq 1$
That is, the test procedure is as follows:
If λ is very large that is $\lambda \geq \lambda_\alpha$ the null hypothesis is accepted and if λ is very small that is $\lambda < \lambda_\alpha$ the null hypothesis is to be rejected. The constant λ_α t is so chosen that the size of the test is α

10. What is the principle behind Likelihood ratio test procedure?
Answer:
If the null hypothesis is true the sample drawn must have a larger likelihood under null hypothesis so that the numerator of λ may be larger and consequently λ may be larger. So for higher values of λ the null hypothesis may be accepted and if the value of λ is small, the hypothesis may be rejected.

11. How do you interpret LRTP?
Answer:
Being a function of the data xi, , the Likelihood ratio λ is therefore a statistic. The likelihood-ratio test rejects the null hypothesis if the value of this statistic is too small. How small is too small depends on the significance level of the test, *i.e.*, on what probability of Type I error is considered tolerable ("Type I" errors consist of the rejection of a null hypothesis that is true).

The numerator corresponds to the maximum likelihood of an observed outcome under the null hypothesis. The denominator corresponds to the maximum likelihood of an observed outcome varying parameters over the whole parameter space. The numerator of this ratio is less than the denominator. The likelihood ratio hence is between 0 and 1. Lower values of the likelihood ratio mean that the observed result was much less likely to occur under the null hypothesis as compared to the alternative. Higher values of the statistic mean that the observed outcome was more than or equally likely or nearly as likely to occur under the null hypothesis as compared to the alternative, and the null hypothesis cannot be rejected.
Hence the statistic λ is bound to be between 0 (likelihoods are non-negative) and 1 (the likelihood of the smaller model can't exceed that of the larger model because it is *nested* on it). Values close to 0 indicate that the smaller model is not acceptable, compared to the larger model, because it would make the observed data very unlikely. Values close to 1 indicate that the smaller model is almost as good as the large model, making the data just as likely.

12. How the assumptions of the model can be tested using LRTP
Answer:

1. Calculate the maximum likelihood of the sample data based on an assumed distribution model (the maximum occurs when unknown parameters are replaced by their maximum likelihood estimates). Repeat this calculation for other candidate distribution models that also appear to fit the data (based on probability plots). If all the models have the same number of unknown parameters, and there is no convincing reason to choose one particular model over another based on the failure mechanism or previous successful analyses, then pick the model with the largest likelihood value.

2. Many model assumptions can be viewed as putting restrictions on the parameters in a likelihood expression that effectively reduce the total number of unknown parameters. For example:
   It is suspected that a type of data, typically modelled by a Weibull distribution, can be fit adequately by an exponential model. The exponential distribution is a special case of the Weibull, with the shape parameter γ set to 1. If we write the Weibull likelihood function for the data, the exponential model likelihood function is obtained by setting γ to 1, and the number of unknown parameters has been reduced from two to one.

13. What is an alternative procedure for LRTP?

Answer:

This can be quantified at a given confidence level as follows:

1. Calculate $\chi^2 = -2\ln\lambda$. The smaller λ is, the larger χ^2 will be
2. We can tell when χ^2 is significantly large by comparing it to the 100 × (1-α) percentile point of a Chi Square distribution with *k* degrees of freedom. χ^2 has an approximate Chi-Square distribution with *k* degrees of freedom and the approximation is usually good, even for small sample sizes
3. The likelihood ratio test computes χ^2 and rejects the assumption if χ^2 is larger than a Chi-Square percentile with *k* degrees of freedom, where the percentile corresponds to the confidence level chosen by the analyst

14 . State the applications of LRTP

Answer:

- Likelihood-ratio test, a statistical test for comparing two models
- Likelihood ratios in diagnostic testing, ratios based on sensitivity and specificity, used to assess diagnostic tests
- Bayes factor, ratio of likelihoods used to update prior probabilities to posterior in Bayes' theorem and Bayesian inference

15. What are the properties of LRTP?

Answer:

The important properties of LR test are

- Under certain conditions -2 lnλ has an asymptotic Chi- square distribution that is -2 lnλ is distributed as central Chi-square when null hypothesis holds
- Under certain conditions LR test is consistent
- LR test procedure is atleast asymptotically unbiased

This is because as $n \to \infty, P_T(\theta) \to 1 \forall \theta \in \Theta_1$ by consistency and therefore

$$\inf_{\theta \in \Theta_1} P_T(\theta) = 1 \geq \sup_{\theta \in \Theta_0} P_T(\theta)$$