Frequently Asked Questions

1. What is a Best Critical Region?
Answer:
Let a simple null hypothesis H_0:$\theta=\theta_0$ be tested against the simple alternative hypothesis H_1:$\theta=\theta_1$. Let C be a critical region for testing H_0 of size α against H_1. C is called the best critical region of size α for testing H_0 against H_1 if C has at least the same power as any other critical region of size α that is if C is a critical region of size α and C' is any other critical region of size α then Power of C greater than or equal to Power of C' .
In testing of hypothesis we keep α the level of significance at a fixed level (say 0.05 or 0.01) and try to minimize the Type II error. The sample space may be partitioned into several ways so that each critical region, w has the same size α. Of all these critical regions choose that which has least type II error. This is called the best critical region of size α.

2. What do you mean by MP test?
Answer:
Among the critical regions of the same size α that which renders the minimum Type II error is called the most powerful critical region. The test based on the most powerful critical region is called the most powerful test. Therefore among all tests possessing the same size of Type I error, choose one for which the size of the Type II error is small as possible. This test is called the Most Powerful test.

3. State the Neyman- Pearson Lemma for finding out the best test.
Answer:
Let the probability density function of the population be f(x, θ) where θ represents the parameters. Draw a sample (x1, x2…xn) from this population. Let L be the likelihood function then L=f(x1, x2… xn).
Let L1 stands for the likelihood function when H1 is true and L0 stands for the likelihood function when H0 is true (H0 and H1 are simple hypothesis)
Let α be the level of significance. Let k be any constant such that the size of the critical region defined as below is α

$$C=\left\{(x1,x2,....,xn):\frac{L_1(x_{i,}\theta)}{L_0(x_{i,}\theta)}>k\right\}$$

4. Briefly explain the Procedure to obtain a Best Critical Region.
Answer:
- Obtain likelihood functions under null hypothesis and alternative hypothesis and call them as L0 and L1
- Obtain the ratio L1/L0
- Obtain the critical region as a region satisfying the condition L1 / L0 > k, where k is determined such that Probability of x belongs to the above mentioned Critical region under null hypothesis is equal to the size of the test

5. Find the BCR of size α for testing H0: μ=μ0 against H1: μ=μ1(>μ0)when a sample of size n is drawn from the Normal population with an known variance σ^2

Answer:

Let x1, x2…xn be a random sample of size n from a Normal population with parameters μ and σ^2

By NP Lemma a size α BCR is given is given by

$$C = \left\{ (x1, x2, ..., xn) : \frac{L_1(xi,\mu)}{L_0(xi,\mu)} > k \right\}$$

$$P[x \in C / H_0] = \alpha -----(*)$$

Where k is such that

$$\frac{L_1(xi,\mu)}{L_0(xi,\mu)} = \frac{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{\frac{-1}{2\sigma^2}\sum_i (xi-\mu_1)^2}}{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{\frac{-1}{2\sigma^2}\sum_i (xi-\mu_0)^2}} = e^{\frac{-1}{2\sigma^2}[\sum_i (xi-\mu_1)^2 - \sum_i (xi-\mu_0)^2]}$$

$$\Rightarrow e^{\frac{-1}{2\sigma^2}[\sum_i xi^2 + n\mu_1^2 - 2\mu_1 \sum_i xi \quad -(\sum_i xi^2 + n\mu_0^2 - 2\mu_0 \sum_i xi \quad)]}$$

$$\Rightarrow e^{\frac{-1}{2\sigma^2}[n(\mu_1^2-\mu_0^2)-2\sum_i xi(\mu_1-\mu_0)]}$$

Now $\frac{L_1(xi,\mu)}{L_0(xi,\mu)} > k \Rightarrow e^{\frac{-1}{2\sigma^2}[n(\mu_1^2-\mu_0^2)-2\sum_i xi(\mu_1-\mu_0)]}$ >k

$$\Rightarrow \frac{-1}{2\sigma^2}[n(\mu_1^2-\mu_0^2)-2\sum_i xi(\mu_1-\mu_0)] > \ln k$$

$$\Rightarrow \frac{-1}{2\sigma^2}[n(\mu_1^2-\mu_0^2)-2n\bar{x}(\mu_1-\mu_0)] > \ln k$$

$$\Rightarrow \bar{x}(\mu_1-\mu_0) > [\ln k + \frac{n}{2\sigma^2}(\mu_1^2-\mu_0^2)]\frac{\sigma^2}{n} = \lambda_1(say)$$

When $\mu_1 > \mu_0$

$$\frac{L_1(xi,\mu)}{L_0(xi,\mu)} > k \Rightarrow \bar{x} > \frac{\lambda_1}{(\mu_1-\mu_0)} = \lambda_2$$

$$\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} > \frac{\frac{\lambda_1}{(\mu_1-\mu_0)}-\mu_0}{\sigma/\sqrt{n}} = \lambda_2 ----(1)$$

When xi follows Normal with parameters

μ and σ^2 , a sample mean $\bar{x} \sim N(\mu, \sigma^2/n)$. Then $\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$ under Null hypothesis

By substituting equation (1) in equation (*) we get

$$P[x \in C / H_0] = \alpha \Rightarrow P[\frac{L_1(xi,\mu)}{L_0(xi,\mu)} > k / H_0] = \alpha$$

$$\Rightarrow P[\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} > \lambda_2] = \alpha \Rightarrow P[Z > \lambda_2] = \alpha$$

Now from the table of Normal Probabilities λ_2 can be read . Using the relation among λ_1, λ_2 and k, k can be determined. Hence the BCR can be found

The BCR when $\mu_1 > \mu_0$ is given by $C = \{(x1, x2, ..., xn) : Z > \lambda_2 = Z_{\alpha/2}\}$

6. Find the BCR of size α for testing H0: μ=μ0 against H1: μ=μ1(<μ0)when a sample of size n is drawn from the Normal population with an known variance σ^2

Answer:

Let x1, x2… xn be a random sample of size n from a Normal population with parameters μ and σ^2

By NP Lemma a size α BCR is given is given by

$$C = \left\{(x1, x2, ..., xn) : \frac{L_1(xi,\mu)}{L_0(xi,\mu)} > k\right\}$$

$$P[x \in C / H_0] = \alpha -----(*)$$

Where k is such that

$$\frac{L_1(xi,\mu)}{L_0(xi,\mu)} = \frac{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{\frac{-1}{2\sigma^2}\sum_i (xi-\mu_1)^2}}{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{\frac{-1}{2\sigma^2}\sum_i (xi-\mu_0)^2}} = e^{\frac{-1}{2\sigma^2}[\sum_i (xi-\mu_1)^2 - \sum_i (xi-\mu_0)^2]}$$

$$\Rightarrow e^{\frac{-1}{2\sigma^2}[\sum_i xi^2 + n\mu_1^2 - 2\mu_1 \sum_i xi \quad -(\sum_i xi^2 + n\mu_0^2 - 2\mu_0 \sum_i xi \quad)]}$$

$$\Rightarrow e^{\frac{-1}{2\sigma^2}[n(\mu_1^2 - \mu_0^2) - 2\sum_i xi(\mu_1 - \mu_0)]}$$

$$\frac{L_1(xi,\mu)}{L_0(xi,\mu)} > k \Rightarrow e^{\frac{-1}{2\sigma^2}[n(\mu_1^2 - \mu_0^2) - 2\sum_i xi(\mu_1 - \mu_0)]} > k$$

Now

$$\Rightarrow \frac{-1}{2\sigma^2}[n(\mu_1^2 - \mu_0^2) - 2\sum_i xi(\mu_1 - \mu_0)] > \ln k$$

$$\Rightarrow \frac{-1}{2\sigma^2}[n(\mu_1^2 - \mu_0^2) - 2n\bar{x}(\mu_1 - \mu_0)] > \ln k$$

$$\Rightarrow \bar{x}(\mu_1 - \mu_0) > [\ln k + \frac{n}{2\sigma^2}(\mu_1^2 - \mu_0^2)]\frac{\sigma^2}{n} = \lambda_1 (say)$$

When $\mu_1 < \mu_0$

$$\frac{L_1(xi,\mu)}{L_0(xi,\mu)} < k \Rightarrow \bar{x} < \frac{\lambda_1}{(\mu_1 - \mu_0)} = \lambda_2$$

$$\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} < \frac{\frac{\lambda_1}{(\mu_1-\mu_0)}-\mu_0}{\sigma/\sqrt{n}} = \lambda_2 \; ----(2)$$

By substituting equation (2) in equation (*) we get

$$P[x\in C/H_0]=\alpha \Rightarrow P[\frac{L_1(xi,\mu)}{L_0(xi,\mu)}<k/H_0]=\alpha$$

$$\Rightarrow P[\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} < \lambda_2] = \alpha \Rightarrow P[Z < \lambda_2] = \alpha$$

When $\mu_1 < \mu_0$ the BCR of size α is given by $C = \{(x1, x2,..., xn) : Z < \lambda_2 = -Z_{\alpha/2}\}$

7. Find the BCR of size α for testing H0: $\sigma=\sigma_0$ against H1: $\sigma=\sigma_1$ ($>\sigma_0$)when a sample of size n is drawn from the Normal population with a known mean μ

Answer:

Let x1, x2…xn be a random sample of size n from a Normal population with parameters μ and σ^2

By NP Lemma a size α BCR is given is given by

$$C = \left\{(x1, x2,..., xn) : \frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)} > k\right\}$$

$$P[x\in C/H_0]=\alpha -----(*)$$

Where k is such that

$$\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)} = \frac{(\frac{1}{\sigma_1^2})^{n/2}(\frac{1}{\sqrt{2\pi}})^n e^{\frac{-1}{2\sigma_1^2}\sum_i (xi-\mu)^2}}{(\frac{1}{\sigma_0^2})^{n/2}(\frac{1}{\sqrt{2\pi}})^n e^{\frac{-1}{2\sigma_0^2}\sum_i (xi-\mu)^2}}$$

$$\Rightarrow (\frac{\sigma_0^2}{\sigma_1^2})^{n/2} e^{\frac{-1}{2}(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2})\sum_i (xi-\mu)^2}$$

Now $\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)} > k \Rightarrow (\frac{\sigma_0^2}{\sigma_1^2})^{n/2} e^{\frac{-1}{2}(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2})\sum_i (xi-\mu)^2} > k$

$$\Rightarrow \frac{n}{2}\ln(\frac{\sigma_0^2}{\sigma_1^2}) - \frac{1}{2}(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2})\sum_i (xi-\mu)^2 > \ln k$$

$$\Rightarrow \frac{n}{2}\ln(\frac{\sigma_0^2}{\sigma_1^2}) - \frac{1}{2}(\frac{\sigma_0^2-\sigma_1^2}{\sigma_1^2\sigma_0^2})\sum_i (xi-\mu)^2 > \ln k$$

$$\Rightarrow \frac{n}{2}\ln(\frac{\sigma_0^2}{\sigma_1^2}) + \frac{1}{2}(\frac{\sigma_1^2-\sigma_0^2}{\sigma_1^2\sigma_0^2})\sum_i (xi-\mu)^2 > \ln k$$

$$\Rightarrow \frac{1}{2}\left(\frac{\sigma_1^2-\sigma_0^2}{\sigma_1^2\sigma_0^2}\right)\sum_i (xi-\mu)^2 \quad > \ln k - \frac{n}{2}\ln\left(\frac{\sigma_0^2}{\sigma_1^2}\right)$$

$$\Rightarrow (\sigma_1^2-\sigma_0^2)\sum_i (xi-\mu)^2 \quad > [\ln k - \frac{n}{2}\ln\left(\frac{\sigma_0^2}{\sigma_1^2}\right)]2\sigma_1^2\sigma_0^2 = \lambda_1 (say)$$

When $\sigma_1 > \sigma_0$

$$\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)} > k \Rightarrow \sum_i (xi-\mu)^2 \quad > \lambda_1 /(\sigma_1^2-\sigma_0^2)$$

$$\Rightarrow \frac{\sum_i (xi-\mu)^2}{\sigma_0^2} \quad > \frac{\lambda_1 /(\sigma_1^2-\sigma_0^2)}{\sigma_0^2} = \lambda_2 (say)$$

Now size of the test =α implies $P[\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)} > k / \sigma = \sigma_0] = \alpha ------(*)$

When xi follows Normal with parameters

μ and σ^2 , and when $\sigma=\sigma_0$, $\frac{\sum_i (xi-\mu)^2}{\sigma_0^2} \sim \chi^2(n)$. Then from the table of probabilities of Chi square variable with n degrees of freedom we have $\lambda_2 = \chi^2{}_\alpha(n)$

By substituting equation (1) in equation (*) we get

$$P[x \in C/H_0] = \alpha \Rightarrow P[\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)} > k / \sigma = \sigma_0] = \alpha$$

$$\Rightarrow P[\frac{\sum_i (xi-\mu)^2}{\sigma_0^2} > \lambda_2] \Rightarrow P[\frac{\sum_i (xi-\mu)^2}{\sigma_0^2} > \lambda_2 = \chi^2{}_\alpha(n)] = \alpha$$

Now from the table of Chi square Probabilities λ_2 can be read . Using the relation among λ_1, λ_2 and k, k can be determined. Hence the BCR can be found

The BCR when $\sigma_1 > \sigma_0$ is given by $C = \left\{ (x1, x2, ..., xn) : \frac{\sum_i (xi-\mu)^2}{\sigma_0^2} > \chi^2{}_\alpha(n) \right\}$

8. Find the BCR of size α for testing H0: $\sigma=\sigma_0$ against H1: $\sigma=\sigma_1$ ($<\sigma_0$) when a sample of size n is drawn from the Normal population with a known mean μ

Answer:

Let x1,x2,…,xn be a random sample of size n from a Normal population with parameters μ and σ^2

By NP Lemma a size α BCR is given is given by

$$C=\left\{(x1,x2,...,xn):\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)}>k\right\}$$

$$P[x\in C/H_0]=\alpha -----(*)$$

Where k is such that

$$\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)}=\frac{(\frac{1}{\sigma_1^2})^{n/2}(\frac{1}{\sqrt{2\pi}})^n e^{\frac{-1}{2\sigma_1^2}\sum_i(xi-\mu)^2}}{(\frac{1}{\sigma_0^2})^{n/2}(\frac{1}{\sqrt{2\pi}})^n e^{\frac{-1}{2\sigma_0^2}\sum_i(xi-\mu)^2}}$$

$$\Rightarrow(\frac{\sigma_0^2}{\sigma_1^2})^{n/2}e^{\frac{-1}{2}(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2})\sum_i(xi-\mu)^2}$$

Now

$$\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)}>k\Rightarrow(\frac{\sigma_0^2}{\sigma_1^2})^{n/2}e^{\frac{-1}{2}(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2})\sum_i(xi-\mu)^2}>k$$

$$\Rightarrow\frac{n}{2}\ln(\frac{\sigma_0^2}{\sigma_1^2})-\frac{1}{2}(\frac{1}{\sigma_1^2}-\frac{1}{\sigma_0^2})\sum_i(xi-\mu)^2 \quad >\ln k$$

$$\Rightarrow\frac{n}{2}\ln(\frac{\sigma_0^2}{\sigma_1^2})-\frac{1}{2}(\frac{\sigma_0^2-\sigma_1^2}{\sigma_1^2\sigma_0^2})\sum_i(xi-\mu)^2 \quad >\ln k$$

$$\Rightarrow\frac{n}{2}\ln(\frac{\sigma_0^2}{\sigma_1^2})+\frac{1}{2}(\frac{\sigma_1^2-\sigma_0^2}{\sigma_1^2\sigma_0^2})\sum_i(xi-\mu)^2 \quad >\ln k$$

$$\Rightarrow\frac{1}{2}(\frac{\sigma_1^2-\sigma_0^2}{\sigma_1^2\sigma_0^2})\sum_i(xi-\mu)^2 \quad >\ln k-\frac{n}{2}\ln(\frac{\sigma_0^2}{\sigma_1^2})$$

$$\Rightarrow(\sigma_1^2-\sigma_0^2)\sum_i(xi-\mu)^2 \quad >[\ln k-\frac{n}{2}\ln(\frac{\sigma_0^2}{\sigma_1^2})]2\sigma_1^2\sigma_0^2=\lambda_1(say)$$

When $\sigma_1<\sigma_0$

$$\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)}<k\Rightarrow\sum_i(xi-\mu)^2 \quad <\lambda_1/(\sigma_1^2-\sigma_0^2)$$

$$\Rightarrow\frac{\sum_i(xi-\mu)^2}{\sigma_0^2} \quad <\frac{\lambda_1/(\sigma_1^2-\sigma_0^2)}{\sigma_0^2}=\lambda_2(say)$$

Now size of the test =α implies $P[\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)} < k / \sigma = \sigma_0] = \alpha - - - - - (*)$

When xi follows Normal with parameters

μ and σ^2 , and when $\sigma=\sigma_0$, $\frac{\sum_i (xi-\mu)^2}{\sigma_0{}^2} \sim \chi^2(n)$. Then from the table of probabilities of Chi square variable with n degrees of freedom we have $\lambda_2 = \chi^2{}_{1-\alpha}(n)$

By substituting equation (1) in equation (*) we get

$$P[x \in C / H_0] = \alpha \Rightarrow P[\frac{L_1(xi,\sigma^2)}{L_0(xi,\sigma^2)} < k / \sigma = \sigma_0] = \alpha$$

$$\Rightarrow P[\frac{\sum_i (xi-\mu)^2}{\sigma_0{}^2} < \lambda_2] \Rightarrow P[\frac{\sum_i (xi-\mu)^2}{\sigma_0{}^2} < \lambda_2 = \chi^2{}_{1-\alpha}(n)] = \alpha$$

Now from the table of Chi square Probabilities λ_2 can be read . Using the relation among λ_1, λ_2 and k, k can be determined. Hence the BCR can be found

The BCR when $\sigma_1 < \sigma_0$ is given by

$$C = \left\{ (x1, x2, ..., xn) : \frac{\sum_i (xi-\mu)^2}{\sigma_0{}^2} < \chi^2{}_{1-\alpha}(n) \right\}$$

9. Obtain a BCR of size α for testing H0: λ=λ0 against H1: λ=λ1(<λ0)when a sample of size n is drawn from the Poisson population with unknown parameter λ?

Answer:

Let x1,x2,…,xn be a random sample of size n from Poisson population with parameter λ

By NP Lemma a size α BCR is given is given by

$$C = \left\{ (x1, x2, ..., xn) : \frac{L_1(xi,\lambda)}{L_0(xi,\lambda)} > k \right\}$$

$$P[x \in C / H_0] = \alpha$$

Where k is such that

$$\frac{L_1(xi,\lambda)}{L_0(xi,\lambda)} = \frac{\dfrac{e^{-n\lambda_1}\lambda_1^{\sum_i x_i}}{\prod_{i=}^{n} x_i!}}{\dfrac{e^{-n\lambda_0}\lambda_0^{\sum_i x_i}}{\prod_{i=}^{n} x_i!}} = e^{-n(\lambda_1-\lambda_0)}\left(\frac{\lambda_1}{\lambda_0}\right)^{\sum_i x_i}$$

Now $\frac{L_1(xi,\lambda)}{L_0(xi,\lambda)} > k \Rightarrow e^{-n(\lambda_1-\lambda_0)}\left(\frac{\lambda_1}{\lambda_0}\right)^{\sum_i x_i} > k$

$\Rightarrow -n(\lambda_1-\lambda_0) + \sum_i xi \ln\left(\frac{\lambda_1}{\lambda_0}\right)$>ln k

$\Rightarrow \sum_i xi \ln\left(\frac{\lambda_1}{\lambda_0}\right)$>ln k $+n(\lambda_1-\lambda_0) = k_1 (say)$

When $\lambda_1 < \lambda_0$

$\frac{L_1(xi,\lambda)}{L_0(xi,\lambda)} < k \Rightarrow \sum_i xi < \frac{k_1}{\ln\left(\frac{\lambda_1}{\lambda_0}\right)} = k_2$

The BCR when $\lambda_1 < \lambda_0$ is given by $C = \left\{(x1, x2,..., xn) : \sum_i x_i < k_2\right\}$

10. If x1,x2,…,xn is a random sample of size n from a distribution having probability density function of the form f(x, θ) = θx $^{\theta-1}$. Obtain a BCR for testing H0:θ=1 against H1:θ=2

Answer:

Let x1,x2,…,xn be a random sample of size n from population with probability density function of the form f(x, θ) = θx $^{\theta-1}$

Required to test H0:θ=1 against H1:θ=2

By NP Lemma a size α BCR is given is given by

$$C = \left\{(x1, x2,..., xn) : \frac{L_1(xi,\theta)}{L_0(xi,\theta)} \geq k\right\}$$

$$P[x \in C / H_0] = \alpha$$

Where k is such that

$$L(xj,\theta)=\prod_j f(xj,\theta)=\theta^n\prod_j x_j^{\theta-1}$$

$$\frac{L_1(xi,\theta)}{L_0(xi,\theta)}=\frac{\theta_1^{\,n}\prod_j x_j^{\theta_1-1}}{\theta_0^{\,n}\prod_j x_j^{\theta_0-1}}=\frac{2^n\prod_j x_j^{2-1}}{1^n\prod_j x_j^{1-1}}=2^n\prod_j x_j$$

$$\frac{L_1(xi,\theta)}{L_0(xi,\theta)}\geq k\Rightarrow 2^n\prod_j x_j\geq k\Rightarrow\prod_j x_j\geq\frac{k}{2^n}=c$$

A size α BCR is given by $C=\left\{(x1,x2,...,xn):\prod_j x_j\geq c\right\}$

11. Find BCR of size α for testing H0:p=p0 against H1: p=p1(>p0) when a sample of size m is drawn from a Binomial population with parameters n and p (n is known)

Answer:

Let x1,x2,…,xm be a random sample of size n from Binomial population with with parameters n and p

By NP Lemma a size α BCR is given is given by

$$C=\left\{(x1,x2,...,xm):\frac{L_1(xi,p)}{L_0(xi,p)}>k\right\}$$

$$P[x\in C/H_0]=\alpha$$

Where k is such that

$$\frac{L_1(xi,p)}{L_0(xi,p)}=\frac{(1-p_1)^{mn-\sum xi}\;p_1^{\sum xi}}{(1-p_0)^{mn-\sum xi}\;p_0^{\sum xi}}=\left(\frac{p_1}{p_0}\right)^{\sum xi}\left(\frac{(1-p_1)}{(1-p_0)}\right)^{mn-\sum xi}$$

$$=\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)^{\sum xi}\left(\frac{(1-p_1)}{(1-p_0)}\right)^{mn}$$

Now $\frac{L_1(xi,p)}{L_0(xi,p)}>k\Rightarrow\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)^{\sum xi}\left(\frac{(1-p_1)}{(1-p_0)}\right)^{mn}>k$

$$\sum xi\log\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)+mn\log\left(\frac{(1-p_1)}{(1-p_0)}\right)>\log k$$

$$\sum xi\log\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)>\log k-mn\log\left(\frac{(1-p_1)}{(1-p_0)}\right)=k1(say)$$

When $p_1>p_0$

$$\frac{L_1(xi,p)}{L_0(xi,p)} > k \Rightarrow \sum xi \;\; > \frac{k1}{\log(\frac{p_1(1-p_0)}{p_0(1-p_1)})} = k2$$

The BCR when $p_1 > p_0$ is given by $C = \left\{(x1, x2, ..., xm) : \sum_i x_i > k_2\right\}$

12. Find the BCR of size α for testing H0: λ=λ0 against H1: λ=λ1(>λ0)when a sample of size n is drawn from the Poisson population with unknown parameter λ?

Answer:

Let x1,x2,…,xn be a random sample of size n from Poisson population with parameter λ

By NP Lemma a size α BCR is given is given by

$$C = \left\{(x1, x2, ..., xn) : \frac{L_1(xi,\lambda)}{L_0(xi,\lambda)} > k\right\}$$

$$P[x \in C / H_0] = \alpha$$

Where k is such that

$$\frac{L_1(xi,\lambda)}{L_0(xi,\lambda)} = \frac{\dfrac{e^{-n\lambda_1}\lambda_1^{\sum_i x_i}}{\prod_{i=}^{n} x_i!}}{\dfrac{e^{-n\lambda_0}\lambda_0^{\sum_i x_i}}{\prod_{i=}^{n} x_i!}} = e^{-n(\lambda_1-\lambda_0)}\left(\frac{\lambda_1}{\lambda_0}\right)^{\sum_i x_i}$$

Now $\frac{L_1(xi,\lambda)}{L_0(xi,\lambda)} > k \Rightarrow e^{-n(\lambda_1-\lambda_0)}\left(\frac{\lambda_1}{\lambda_0}\right)^{\sum_i x_i} > k$

$\Rightarrow -n(\lambda_1-\lambda_0) + \sum_i xi \ln\left(\frac{\lambda_1}{\lambda_0}\right)$ >ln k

$\Rightarrow \sum_i xi \ln\left(\frac{\lambda_1}{\lambda_0}\right)$ >ln k $+ n(\lambda_1-\lambda_0) = k_1 (say)$

When $\lambda_1 > \lambda_0$

$$\frac{L_1(xi,\lambda)}{L_0(xi,\lambda)} > k \Rightarrow \sum_i xi > \frac{k_1}{\ln\left(\frac{\lambda_1}{\lambda_0}\right)} = k_2$$

The BCR when $\lambda_1 > \lambda_0$ is given by $C = \left\{(x1, x2, ..., xn) : \sum_i x_i > k_2\right\}$

13. Find BCR of size α for testing H0:p=p0 against H1: p=p1(<p0) when a sample of size m is drawn from a Binomial population with parameters n and p (n is known)

Answer:

Let x1,x2,…,xm be a random sample of size n from Binomial population with parameters n and p

By NP Lemma a size α BCR is given is given by

$$C = \left\{(x1, x2, ..., xm) : \frac{L_1(xi, p)}{L_0(xi, p)} > k\right\}$$

$$P[x \in C / H_0] = \alpha$$

Where k is such that

$$\frac{L_1(xi, p)}{L_0(xi, p)} = \frac{(1-p_1)^{mn-\sum xi} \; p_1^{\sum xi}}{(1-p_0)^{mn-\sum xi} \; p_0^{\sum xi}} = \left(\frac{p_1}{p_0}\right)^{\sum xi} \left(\frac{(1-p_1)}{(1-p_0)}\right)^{mn-\sum xi}$$

$$= \left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)^{\sum xi} \left(\frac{(1-p_1)}{(1-p_0)}\right)^{mn}$$

Now $\frac{L_1(xi, p)}{L_0(xi, p)} > k \Rightarrow \left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)^{\sum xi} \left(\frac{(1-p_1)}{(1-p_0)}\right)^{mn} > k$

$$\sum xi \log\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right) + mn \log\left(\frac{(1-p_1)}{(1-p_0)}\right) > \log k$$

$$\sum xi \log\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right) > \log k - mn \log\left(\frac{(1-p_1)}{(1-p_0)}\right) = k1(say)$$

When $p_1 < p_0$

$$\frac{L_1(xi, p)}{L_0(xi, p)} < k \Rightarrow \sum xi < \frac{k1}{\log\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)} = k2$$

The BCR when $p_1 > p_0$ is given by $C = \left\{(x1, x2, ..., xm) : \sum_i x_i < k_2\right\}$

14. If x1,x2,…,xn is a random sample of size n from an exponential distribution having probability density function of the form f(x, θ) = $\theta e^{\theta x}$, x>0. Derive BCR for testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1(>\theta_0)$. Obtain the null distribution of the test statistic

Answer:

Let x1,x2,…,xn be a random sample of size n from population with probability density function of the form f(x, θ) = θe $^{\theta x}$, x>0.

Required to test H0:θ=θ0 against H1:θ=θ1.

By NP Lemma a size α BCR is given is given by

$$C=\left\{(x1,x2,...,xn):\frac{L_1(xi,\theta)}{L_0(xi,\theta)}\geq k\right\}$$

$$P[x\in C/H_0]=\alpha$$

Where k is such that

$$L(xi,\theta)=\prod_i f(xi,\theta)=\theta^n e^{-\theta\sum_i xi}$$

$$\frac{L_1(xi,\theta)}{L_0(xi,\theta)}=\frac{\theta_1{}^n e^{-\theta_1\sum_i xi}}{\theta_0{}^n e^{-\theta_0\sum_i xi}}=\left(\frac{\theta_1}{\theta_0}\right)^n e^{-[\theta_1-\theta_0]\sum_i xi}$$

$$\frac{L_1(xi,\theta)}{L_0(xi,\theta)}>k\Rightarrow\left(\frac{\theta_1}{\theta_0}\right)^n e^{-[\theta_1-\theta_0]\sum_i xi}>k$$

$$\Rightarrow n\ln\left(\frac{\theta_1}{\theta_0}\right)-(\theta_1-\theta_0)\sum_i xi>\ln k$$

$$\Rightarrow(\theta_1-\theta_0)\sum_i xi\leq-\ln k+n\ln\left(\frac{\theta_1}{\theta_0}\right)=\lambda_1(say)$$

When $\theta_1>\theta_0$

$$\frac{L_1(xi,\theta)}{L_0(xi,\theta)}>k\Rightarrow\sum_i xi\leq\lambda_1/(\theta_1-\theta_0)=\lambda_2(say)$$

Now size of the test =α implies $P[\frac{L_1(xi,\theta)}{L_0(xi,\theta)}>k/\theta=\theta_0]=\alpha-----(*)$

When xi follows exponential with parameter θ and when $\theta=\theta_0$, $\sum_i xi\sim\gamma(n,\theta)$.

$$P[x\in C/H_0]=\alpha\Rightarrow P[\frac{L_1(xi,\theta)}{L_0(xi,\theta)}\geq k/\theta=\theta_0]=\alpha$$

$$\Rightarrow P[\sum_i xi\leq\lambda_2]=\alpha$$

Now from the table of Chi square Probabilities λ_2 can be read . Using the relation among λ_1, λ_2 and k, k can be determined. Hence the BCR can be found

The BCR when $\theta_1 > \theta_0$ is given by $C = \left\{ (x1, x2, ..., xn) : \sum_i xi \le \lambda_2) \right\}$

The null distribution of the test statistic is the distribution of $\sum_i xi$ when $\theta=\theta_0$ is nothing but the probability density function of Gamma distribution with parameters n and θ_0 and is given by $f(y) = \frac{\theta_0{}^n}{\overline{)n}} e^{-\theta_0 y} y^{n-1}, y \ge 0$,

15. If x1,x2,…,xn is a random sample of size n from an exponential distribution having probability density function of the form f(x, θ) = θe $^{\theta x}$, x>0. Derive BCR for testing H_0:θ=θ_0 against H_1:θ=θ_1(<θ_0). Obtain the null distribution of the test statistic

Answer:

Let x1, x2… xn be a random sample of size n from population with probability density function of the form f(x, θ) = θe $^{\theta x}$, x>0.

Required to test H0: θ=θ0 against H1: θ=θ1.

By NP Lemma a size α BCR is given is given by

$$C = \left\{ (x1, x2, ..., xn) : \frac{L_1(xi, \theta)}{L_0(xi, \theta)} \ge k \right\}$$

$$P[x \in C / H_0] = \alpha$$

Where k is such that

$$L(xi, \theta) = \prod_i f(xi, \theta) = \theta^n e^{-\theta \sum_i xi}$$

$$\frac{L_1(xi, \theta)}{L_0(xi, \theta)} = \frac{\theta_1{}^n e^{-\theta_1 \sum_i xi}}{\theta_0{}^n e^{-\theta_0 \sum_i xi}} = \left(\frac{\theta_1}{\theta_0} \right)^n e^{-[\theta_1 - \theta_0] \sum_i xi}$$

$$\frac{L_1(xi, \theta)}{L_0(xi, \theta)} > k \Rightarrow \left(\frac{\theta_1}{\theta_0} \right)^n e^{-[\theta_1 - \theta_0] \sum_i xi} > k$$

$$\Rightarrow n \ln \left(\frac{\theta_1}{\theta_0} \right) - (\theta_1 - \theta_0) \sum_i xi > \ln k$$

$$\Rightarrow (\theta_1 - \theta_0) \sum_i xi \le -\ln k + n \ln \left(\frac{\theta_1}{\theta_0} \right) = \lambda_1 (say)$$

When $\theta_1 < \theta_0$

$$\frac{L_1(xi, \theta)}{L_0(xi, \theta)} < k \Rightarrow \sum_i xi > \lambda_2 (say)$$

Now size of the test =α implies $P[\frac{L_1(xi, \theta)}{L_0(xi, \theta)} < k / \theta = \theta_0] = \alpha - - - - - - (*)$

$$P[x \in C / H_0] = \alpha \Rightarrow P[\frac{L_1(xi,\theta)}{L_0(xi,\theta)} < k / \theta = \theta_0] = \alpha$$

$$\Rightarrow P[\sum_i xi > \lambda_2] = \alpha$$

Hence the BCR can be found

The BCR when $\theta_1 < \theta_0$ is given by $C = \left\{ (x1, x2, ..., xn) : \sum_i xi > \lambda_2) \right\}$