Frequently Asked Questions

1. What do you mean by a Type I error?
Answer:
Type I error is the incorrect rejection of a true null hypothesis. Usually a type I error leads one to conclude that a thing or relationship exists when really it doesn't:
For example, that a patient has a disease being tested for when really the patient does not have the disease, or that a medical treatment cures a disease when really it doesn't.
Type I error is also called error of the first kind. Hence a wrong decision that we take by rejecting a null hypothesis when it is actually true is known as a Type I error

2. What are the Causes for Type I errors?
Answer:
Causes for Type I errors

- A Type I error occurs when the sample data appear to show a treatment effect when, in fact, there is none.
- In this case the researcher will reject the null hypothesis and falsely conclude that the treatment has an effect.
- Type I errors are caused by unusual, unrepresentative samples. Just by chance the researcher selects an extreme sample with the result that the sample falls in the critical region even though the treatment has no effect.
- The hypothesis test is structured so that Type I errors are very unlikely; specifically, the probability of a Type I error is equal to the alpha level

3. What are the possible incorrect decisions in a test of hypothesis?
Answer:
We make Incorrect Decision by:

- Retaining the null hypothesis when the null hypothesis is false (TYPE II or beta error)
- Rejecting the null hypothesis when the null hypothesis is true (TYPE 1 or alpha error)

4. Compare Type I and Type II errors in testing of hypothesis.
Answer:
In many practical applications type I errors are more delicate than type II errors. In these cases, care is usually focused on minimizing the occurrence of this statistical error.
Generally speaking, researchers are not willing to reject the H_0 hypothesis if the chance of rejecting it wrongly is greater than about 5%. We usually want to reject the null hypothesis but we only want to do this if the chance that we do it incorrectly is rather low. Unfortunately, setting some criterion about how much Type I error rate we are willing to tolerate does little to regulate or establish what level of Type II error we are willing to tolerate. The literature seems to be fixated only on the rate of Type I errors without much attention to Type II error. Because when we try to minimize the Type I error Type II error increases and vice versa. Hence we cannot minimize both of them simultaneously. So we fix the Type I error at a fixed level and try to minimize the Type II error.

5. How do we make correct decisions in a test of hypothesis?
Answer:
We make correct decisions by

- Retaining the null hypothesis when the null hypothesis is true, or
- Rejecting the null hypothesis when the null hypothesis is false

6. How do you accomplish the objectives of statistical tests?
Answer:

Now it must be understood that the objective of statistical test of null hypothesis is not to determine whether the null hypothesis is true but rather to determine if its truth is consistent with the resultant data. Therefore given this objective it is reasonable that a null hypothesis should be rejected only if the sample data are very unlikely when the null hypothesis is true. The classical way of accomplishing this is to specify a small value α and then require that the test have the property that whenever the null hypothesis is true its probability of being rejected is less than or equal to α. The value α is called the level of significance of the test and is usually set in advance with commonly chosen values being α = 0.10, 0.05 and 0.01

7. What do you mean by level of significance or size of the test?
Answer:
In decision theory, the probability of a Type I error is equal to the significance level α. Hence the probability with which we may reject the null hypothesis when it is true is called a level of significance and it is also called as a size of the test. Suppose, the probability for a type I error is 1% , then there is a 1% chance that the observed variation is not true. This is called the *level of significance*, denoted with the Greek letter α (alpha).
While 1% or 5% might be an acceptable level of significance for one application, a different application can require a very different level.

8. How do we use level of significance in testing of hypothesis?
Answer:
Deciding over suitable significance level is the third step in testing of hypothesis . In this step the validity of H0 against that H1 is decided to be ascertained at certain level of significance. The confidence with which the experimenter rejects or accepts a null hypothesis depends upon the significance level adopted. This significance level is expressed as a percentage.
For example: 5% is the probability of rejecting the null hypothesis, if it is true. When the hypothesis in question is accepted at 5% level the statistician is running a risk that n the long run he may be making a wrong decision about 5% of the time. Similarly by rejecting the null hypothesis at the same level he is running the risk of rejecting the true hypothesis 5 times out of every 100 occasions. Then the level of significance is 0.05 which implies in the long run a statistician is rejecting the null hypothesis 5 times out of every 100 times.

9. How do you compute the size of the test?
Answer:
Computation of the size of the test:
Size of the test = P (Type I error) = P [H_0 rejected/ H_0 is true]

$$= P[x \in C/ H_0 \text{ is true}] = \int_C L_0(x_1, x_{2,\ldots\ldots,} x_n)\, dx_1 dx_2 \ldots\ldots dx_n = \alpha$$

Where $L_0(x_1, x_{2,\ldots\ldots,} x_n)$ is the likelihood function under the null hypothesis. And C is the rejection region of the null hypothesis. In case of simple null hypothesis the probability of first kind of error is called the size of the test or level of significance

10. What do you mean by Operating Characteristic of a test (O.C)?
Answer:
If α is the size of the test or probability of Type I error then (1-α) is called a Operating Characteristic of a test
That is we have $\alpha = P[x \in C/ H_0 \text{ is true}] = 1 - P[x \in A/ H_0 \text{ is true}]$
which implies $1-\alpha = P[x \in A/ H_0 \text{ is true}]$
which implies 1-α=P[Accept H_0 / H_0 is true]
Where A is the acceptance region of the null hypothesis
Operating Characteristic can be defined as a probability of accepting H_0 when it is true.

11. Explain the pros and cons of setting up a significance level.
Answer:
Setting a significance level (before doing inference) has the advantage that the analyst is not tempted to choose a cut-off on the basis of what he or she hopes is true.
It has the disadvantage that it neglects that some significant-values might best be considered borderline. This is one reason why it is important to report significant -values when reporting results of hypothesis tests.
It is also good practice to include confidence intervals corresponding to the hypothesis test. (For example, if a hypothesis test for the difference of two means is performed, also give a confidence interval for the difference of those means. If the significance level for the hypothesis test is .05, then use confidence level 95% for the confidence interval.)

12. How do you decide about the significance level?
Answer:
This should be done *before* analyzing the data -- preferably before gathering the data. The choice of significance level should be based on the consequences of Type I and Type II errors.
If the consequences of a type I error are serious or expensive, then a very small significance level is appropriate.
Example 1: Two drugs are being compared for effectiveness in treating the same condition. Drug 1 is very affordable, but Drug 2 is extremely expensive. The null hypothesis is "both drugs are equally effective," and the alternate is "Drug 2 is more effective than Drug 1." In this situation, a Type I error would be deciding that Drug 2 is more effective, when in fact it is no better than Drug 1, but would cost the patient much more money. That would be undesirable from the patient's perspective, so a small significance level is warranted.
If the consequences of a Type I error are not very serious (and especially if a Type II error has serious consequences), then a larger significance level is appropriate.

13. Explain with an example when we need a large significance level.
Answer:
Two drugs are known to be equally effective for a certain condition. They are also each equally affordable. However, there is some suspicion that Drug 2 causes a serious side-effect in some patients, whereas Drug 1 has been used for decades with no reports of the side effect. The null hypothesis is "the incidence of the side effect in both drugs is the same", and the alternate is "the incidence of the side effect in Drug 2 is greater than that in Drug 1." Falsely rejecting the null hypothesis when it is in fact true (Type I error) would have no great consequences for the consumer, but a Type II error (i.e., failing to reject the null hypothesis when in fact the alternate is true, which would result in deciding that Drug 2 is no more harmful than Drug 1 when it is in fact more harmful) could have serious consequences from a public health standpoint. So setting a large significance level is appropriate in this case.

14. A coin is thrown 8 times. Null Hypothesis H_0 is p=1/2 and H_1 is p=2/3. Test procedure is a null hypothesis is rejected if 6 or more tosses give heads; p is the probability for getting head in each trial. Then determine the level of significance.
Answer:
Tossing of a coin, getting head or tail follows Binomial distribution. Hence suppose x is the number of heads, its probability mass function is given by

$$f(x) = n_{c_x} p^x q^{n-x}$$

Rejection region is 6 or more heads

$$f(x/H_0) = f(x/p=\frac{1}{2}) = 8_{c_x} (\frac{1}{2})^x (\frac{1}{2})^{8-x}$$

$$f(x/H_1) = f(x/p=\frac{2}{3}) = 8_{c_x}(\frac{2}{3})^x(\frac{1}{3})^{8-x}$$

Size of the test or Level of significance = P [Type I error] = P [Rejecting H_0 /H_0 is true]
=P [getting 6 or more heads / p=1/2]

= P [x≥6 / p=1/2] = $8_{c_6}(\frac{1}{2})^6(\frac{1}{2})^{8-6} + 8_{c_7}(\frac{1}{2})^7(\frac{1}{2})^{8-7} + 8_{c_8}(\frac{1}{2})^8(\frac{1}{2})^{8-8}$

$=(\frac{1}{2})^8[8_{c_6} + 8_{c_7} + 8_{c_8}] = \frac{37}{256} = 0.1445$

Hence level of significance is equal to 0.1445

15. An urn contains 6 balls of which 3 are white and others are black. H0:θ=3 and H1:θ=4 Test Procedure: 2 balls are chosen at random and it is decided to reject H0 if both are of the same colour. Determine the level of significance

Answer:

H0: θ=3; that is 3 white and 3 black balls
and H1:θ=4;that is 4 white and 2 black balls
Level of significance = P[Type I error] = P[Rejecting H_0 /H_0 is true]
=P [Choosing two balls of the same colour when 3 are white and 3 are black]
= P [Choosing two white and 2 black balls/ 3 are white and 3 are black]

$$= \frac{3_{c_2} + 3_{c_2}}{6_{c_2}} = \frac{2}{5} = 0.4$$

Hence the level of significance is 0.4