Frequently Asked Questions

1. What do you mean by a sample space?

Answer:

In testing of hypothesis suppose we take a sample of size n from a population X based on which a hypothesis must be tested. Then the set of all possible samples (x1, x2… xn) of size n from the population constitute a sample space. In symbols a sample space

$\Omega = \{ (x_{1,} x_2, \ldots\ldots, x_n) : x_i \in X \}$

2. How do you classify the sample space?

Answer:

Based on the samples drawn from the population a decision is taken about the acceptance and rejection of null hypothesis

The sample space Ω is divided into two groups ω and $\omega^c = \Omega - \omega$ depending on the null hypothesis and the test so that when and only when a sample point sample point falls in ω a null hypothesis shall be rejected. The set or region ω is known as a critical region. The set of all those sample observations which suggest the acceptance of a null hypothesis is called the acceptance region and is denoted by A.

3. What do you mean by a critical region or rejection region?

Answer:

The set of those sample observations which leads to the rejection of the null hypothesis is called a critical region or rejection region and is denoted by ω or C

Thus critical region or rejection region is that part of the sample space where if a sample point falls a null hypothesis is rejected.

If the observed data is in the critical region, then reject the maintained hypothesis. If the observed data is NOT in the critical region, then accept the maintained hypothesis.

4. How do you give a test procedure?

Answer:

Giving a test procedure is nothing but giving a critical region or an acceptance region.

We represent any test procedure as follows

$$\phi(x) = \begin{Bmatrix} > 0 \, if \, (x_1, x_{2,} \ldots\ldots, x_n) \in A \\ \leq 0 \, if \, (x_1, x_{2,} \ldots\ldots, x_n) \in C \end{Bmatrix}$$

A function Φ(x) is known as test function. A is the acceptance region and c is the critical region

5. Define a size of the critical region.

Answer:

Probability of a selected sample falls belongs to the critical region is called the size of the critical region. Size of the critical region is nothing but the size of the test or level of significance that is probability of rejecting a null hypothesis when it is true

6. What are the outcomes of a statistical test?

Answer:

There are only two outcomes of a statistical test. The data is either in the critical region or it is not in the critical region. If the data is NOT in the critical region, you accept the maintained (null). You reject the alternative. Reject the alternative must mean accept the maintained. Fail to reject the maintained means accept the maintained.

If 'fail to reject' had any real meaning other than accept, then 'fail to accept' would also have a different meaning. Now you would have four outcomes: accept the maintained, reject the maintained, fail to reject the maintained, fail to accept the maintained. But there are only two outcomes of a statistical test: the data is either in the critical region or it is not in the critical region. The only outcomes are to accept the maintained (reject the alternative) or accept the alternative (reject the maintained). Fail to reject must mean accept and fail to accept must mean reject.

7. Give one example for a critical region.

Answer:

A statistical hypothesis test specifies a critical region – a set of numbers. If the observed data is in the critical region, then reject the maintained hypothesis. If the observed data is NOT in the critical region, then accept the maintained hypothesis.

Example: Let the critical region be 0, 1, 2, 9, 10, 11 heads. If you flip the coin 11 times, **reject** the maintained hypothesis that the number of heads is a binomial distribution with probability .5 of heads and n=11 if you observer 0, 1, 2, 9, 10, 11 heads.

8. What do you mean by “fail to reject” in statistical hypothesis?

Answer:

For some statistical hypotheses, there are many tests. Homoscedasticity, non serially correlated errors and even normality of a random variable are all a hypothesis with many different tests. In some sense, saying ‘fail to reject’ means that the current test accepted the maintained but some other test might reject the maintained. Then ‘fail to reject’ is not about a hypothesis test, but about many hypothesis tests. But in such a case there are many critical regions (as many as there are tests). ACCEPT or REJECT is about one single critical region.

9. If accepting a hypothesis does not prove the hypothesis, then what does it do?

Answer:

Acceptance allows one to proceed as if the hypothesis were true. But there are two outcomes: Accepting a true hypothesis and accepting a false hypothesis. Accepting a true hypothesis would be a 'correct' decision while rejecting a true hypothesis would be an 'incorrect' decision. Rather than call it incorrect, we call it an **error**.

10. How do you know what tests are powerful against the alternatives?

Answer:

The **size** (or significance level) of a statistical test is the probability of a Type I error. The **power** of a test is 1 minus the probability of a Type II error. For a given size, we want a statistical test with highest power we can obtain. For most tests we encounter, we specify a size, we obtain a critical region and theoretical results indicate what alternatives have high power and what alternatives do not. For most tests we encounter, we never know the probability of a Type II error. We rely on prior research to tell us what tests are powerful against what alternatives.

11. How does the size and power of a test depend on the critical region?

Answer:

Both the size and the power are probabilities of the critical region. The difference is the assumption made to compute the probability. For the size, the probability is computed assuming the maintained or null hypothesis. For the power, the probability is computed assuming some alternative hypothesis. Both depends on the probabilities of rejection of null hypothesis

12. What do you mean by critical function?

Answer:

With each test for testing a null hypothesis against an alternative hypothesis a function is associated called the critical function of a test. The critical function of a test is that function which gives the probability of rejecting the null hypothesis. Suppose we have a population with an unknown parameter θ then the critical function is a function of θ which is given as P [Reject H_0 /θ]

13. Briefly explain critical value or values?

Answer:

The critical value(s) for a hypothesis test is a threshold to which the value of the test statistic in a sample is compared to determine whether or not the null hypothesis is rejected.

The critical value for any hypothesis test depends upon a test statistic, which is specific to the type of the test and the significance level α which defines the sensitivity of the test at which the test is carried out, and whether the test is one-sided or two-sided.

A value of α = 0.05 implies that the null hypothesis is rejected 5% of the time when it is in fact true. The choice of α is somewhat arbitrary, although in practice values of 0.1, 0.05, and 0.01 are common. Critical values are essentially cut-off values that define regions where the test statistic is unlikely to lie; for example, a region where the critical value is exceeded with probability α if the null hypothesis is true. The null hypothesis is rejected if the test statistic lies within this region which is often referred to as the rejection region(s).

14. What are one tailed tests?

Answer:

A test of a statistical hypothesis, where the region of rejection is on only one side of the sampling distribution, is called a **one-tailed test**. For example, suppose the null hypothesis states that the mean is less than or equal to 10. The alternative hypothesis would be that the mean is greater than 10. The region of rejection would consist of a range of numbers located on the right side of sampling distribution; that is, a set of numbers greater than 10.

A one-sided test is a statistical hypothesis test in which the values for which we can reject the null hypothesis, H0 are located entirely in one tail of the probability distribution.

In other words, the critical region for a one-sided test is the set of values less than the critical value of the test, or the set of values greater than the critical value of the test.

Accordingly we keep the level of significance completely either on the right tail or on the left tail depending upon the situation. If the alternative hypothesis is greater than type, then the critical region is towards the right side of the curve. On the other hand if our alternative hypothesis is less than type, then the critical region is towards the left side of the curve

A one-sided test is also referred to as a one-tailed test of significance.

The choice between a one-sided and a two-sided test is determined by the purpose of the investigation or prior reasons for using a one-sided test. Hence when the alternative hypothesis is less than or greater than type we come across with one tailed or one sided tests.

Example

Suppose we wanted to test a manufacturer's claim that there are, on average, 50 matches in a box. We could set up the following hypotheses

H0: $\mu = 50$,

against

H1: $\mu < 50$ or H1: $\mu > 50$

Either of these two alternative hypotheses would lead to a one-sided test. Presumably, we would want to test the null hypothesis against the first alternative hypothesis since it would be useful to know if there is likely to be less than 50 matches, on average, in a box (no one would complain if they get the correct number of matches in a box or more).

15. Briefly explain Two tailed tests.

Answer:

A test of a statistical hypothesis, where the region of rejection is on both sides of the sampling distribution, is called a **two-tailed test**. For example, suppose the null hypothesis states that the mean is equal to 10. The alternative hypothesis would be that the mean is less than 10 or greater than 10. The region of rejection would consist of a range of numbers located on both sides of sampling distribution; that is, the region of rejection would consist partly of numbers that were less than 10 and partly of numbers that were greater than 10.
A two-sided test is a statistical hypothesis test in which the values for which we can reject the null hypothesis, $H0$ are located in both tails of the probability distribution.
In this case we describe the level of significance on either side of the tail equally. For example if the level of significance $\alpha=0.05$, then we distribute this amount equally at the rate 0.025 on either side of the tail. The value of standard normal variate for $\alpha=0.025$ is 1.96. In other words, the critical region for a two-sided test is the set of values less than a first critical value of the test and the set of values greater than a second critical value of the test.
A two-sided test is also referred to as a two-tailed test of significance.
The choice between a one-sided test and a two-sided test is determined by the purpose of the investigation or prior reasons for using a one-sided test.
Example
Suppose we wanted to test a manufacturer's claim that there are, on average, 50 matches in a box. We could set up the following hypotheses
$H0$: $\mu = 50$,
An alternative hypothesis could be tested against the above null, leading this time to a two-sided test:
$H1$: μ not equal to 50
Here, nothing specific can be said about the average number of matches in a box; only that, if we could reject the null hypothesis in our test, we would know that the average number of matches in a box is likely to be less than or greater than 50.