Frequently Asked Questions

1. What do you mean by a Type II error?
Answer:
Type II error occurs when we accept the null hypothesis when it is actually false. That is a Type II error occurs when we don't reject a false null hypothesis [accept the null hypothesis]. That occurs when a guilty defendant is acquitted.

2. What are the Causes for Type II errors?
Answer:
- A Type II error occurs when the sample does not appear to have been affected by the treatment when, in fact, the treatment does have an effect
- In this case, the researcher will fail to reject the null hypothesis and falsely conclude that the treatment does not have an effect
- Type II errors are commonly the result of a very small treatment effect. Although the treatment does have an effect, it is not large enough to show up in the research study

3. What do you mean by the term "β" in test of hypothesis?
Answer:
The probability with which we may accept the null hypothesis when it is false is denoted by **β**. That is **β**=P [Accept H_0 /H_1 is true]

4. What is Power of a test?
Answer:
The **power** of a statistical test is the probability that the test will reject the null hypothesis when the null hypothesis is false (i.e. the probability of not committing a Type II error, or making a false negative decision). The power is in general a function of the possible distributions, often determined by a parameter, under the alternative hypothesis. As the power increases, the chance of occurrences of a Type II error decreases.
The probability of a Type II error occurring is referred to as the β. Therefore power is equal to 1 – β, which is also known as the sensitivity

5. Define a power function.
Answer:
With each test for testing a null hypothesis against an alternative hypothesis a function is associated called the power function of a test. The power function of a test is that function which gives the probability of rejecting the null hypothesis. The Power function of a test T is denoted by P_T and a power at point θ is denoted by $P_T(\theta)$
$P_T(\theta)$ = P [Reject H_0 /θ]
If H0 and H1 are the functions of the parameters θ (say) and if a power of the test $P_T(\theta)$ =1-**β** is regarded as a function of θ then it is known as a power function.

6. What is the objective when we make use of a test procedure?
Answer:
Our object is to determine a test procedure for which both the errors are minimum. But usually we find that as one error decreases the other increases. Therefore we fix one of the errors at a low permissible level and try to minimize the other. Usually the probability of type I error is fixed at a certain level that is α is fixed at 0.01, 0.02, 0.05 or 0.1 depending on the situation or requirements.

7. How can we decrease Probability of Type I and Type II errors?
Answer:

The probability of making a Type I (α) can be decreased by altering the level of significance. It will be more difficult to find a significant result if the power of the test will be decreased because the risk of a Type II error will be increased .The probability of making a Type II (β) can be decreased by increasing the level of significance which will increase the chance of a Type I error.

8. Suppose a researcher conducts an experiment to test a hypothesis. If she doubles her sample size, which of the following will increase?

- **The power of the hypothesis test**
- **The effect size of the hypothesis test**
- **The probability of making a Type II error**

Answer:

By increasing the sample size the power of the hypothesis test will increase. Increasing sample size makes the hypothesis test more sensitive - more likely to reject the null hypothesis when it is, in fact, false. Thus, it increases the power of the test. The effect size is not affected by sample size. And the probability of making a Type II error gets smaller, not bigger, as sample size increases.

9. How do you compute the power of the test?

Answer:

β = P (Type II error)

=P [H_0 accepted/ H_1 is true]

= $P[x \in A$ / H_1 is true] = $\int_A L_1(x_1, x_{2,.....,} x_n)dx_1dx_2.....dx_n = \beta$

Where $L_1(x_1, x_{2,.....,} x_n)$ is the likelihood function under the alternative hypothesis.

A is the Acceptance region of the null hypothesis.

If is the probability of Type II error then (1-ß) is called a Power of a test

That is we have = $P[x \in A$ / H_1 is true]= 1- $P[x \in C$ / H_1 is true]

which implies 1-ß= $P[x \in C$ / H_1 is true]

which implies 1-ß=P[Reject H_0 / H_1 is true]]

Where C is the Rejection region of the null hypothesis

Power of a test can be defined as a probability of rejecting H_0 when it is false.

10. What are the factors on which a Power of a test is dependent upon?

Answer:

Statistical power may depend on a number of factors. Some of these factors may be particular to a specific testing situation, but at a minimum, power nearly always depends on the following three factors:

- The statistical significance criterion used in the test
- The magnitude of the effect of interest in the population
- The sample size used to detect the effect

11. How do you interpret the power of a test?

Answer:

Power of a test gives us a chance of taking correct decisions. Although there are no formal standards for power (sometimes referred as P_T or π), most researchers assess the power of their tests using P_T =π=0.80 as a standard for adequacy. This convention implies a four-to-one trade off between β-risk and α-risk. (β is the probability of a Type II error; α is the probability of a Type I error, 0.2 and 0.05 are conventional values for β and α, being β=1-Pi or 1- P_T).

However, there will be times when this 4-to-1 weighting is inappropriate. In medicine, For example, tests are often designed in such a way that no false negatives (Type II errors) will

be produced. But this inevitably raises the risk of obtaining a false positive (a Type I error). The rationale is that it is better to tell a healthy patient "we may have found something - let's test further", than to tell a diseased patient "all is well".

12. Explain the factors that affect Power of a test?
Answer:
The power of a hypothesis test is affected by three factors.

- Sample size (*n*). Other things being equal, the greater the sample size, the greater the power of the test
- Significance level (α). The higher the significance level, the higher the power of the test. If you increase the significance level, you reduce the region of acceptance. As a result, you are more likely to reject the null hypothesis. This means you are less likely to accept the null hypothesis when it is false; i.e., less likely to make a Type II error. Hence, the power of the test is increased
- The "true" value of the parameter being tested. The greater the difference between the "true" value of a parameter and the value specified in the null hypothesis, the greater the power of the test. That is, the greater the effect size, the greater the power of the test
- Suppose the null hypothesis is false. One would want the hypothesis test to reject it all the time. Unfortunately, no test is foolproof, and there will be cases where the null hypothesis is in fact false but the test fails to reject it. In this case, a Type II error would be made. ß is the probability of making a Type II error and ß should be as small as possible. Consequently, 1 -ß is the probability of rejecting a null hypothesis correctly (because in fact it is false), and this number should be as large as possible

13. How do we use Power?
Answer:
Power analysis can be used to calculate the minimum sample size required so that one can be reasonably likely to detect an effect of a given size. Power analysis can also be used to calculate the minimum effect size that is likely to be detected in a study using a given sample size. In addition, the concept of power is used to make comparisons between different statistical testing procedures: for example, between a parametric and a nonparametric test of the same hypothesis.

14. A coin is thrown 8 times. Null Hypothesis H_0 is p=1/2 and H_1 is p=2/3. Test procedure is a null hypothesis is rejected if 6 or more tosses give heads; p is the probability for getting head in each trial. Then determine the power of the test.
Answer:
Tossing of a coin, getting head or tail follows Binomial distribution. Hence suppose x is the number of heads, its probability mass function is given by

$$f(x) = n_{c_x} p^x q^{n-x}$$

Rejection region is 6 or more heads

$$f(x/H_0) = f(x/p=\frac{1}{2}) = 8_{c_x}(\frac{1}{2})^x(\frac{1}{2})^{8-x}$$

$$f(x/H_1) = f(x/p=\frac{2}{3}) = 8_{c_x}(\frac{2}{3})^x(\frac{1}{3})^{8-x}$$

Power of the test = 1- P [Type II error]
=1- P [Accepting H_0 /H_1 is true]
= P [Rejecting H_0 /H_1 is true]
=P [getting 6 or more heads / p=2/3]

$$=P\ [x\geq 6 / p=2/3] = 8_{c_6}(\frac{2}{3})^6(\frac{1}{3})^{8-6} + 8_{c_7}(\frac{2}{3})^7(\frac{1}{3})^{8-7} + 8_{c_8}(\frac{2}{3})^8(\frac{1}{3})^{8-8}$$

$$=(\frac{2}{3})^6[8_{c6}(\frac{1}{3})^2+8_{c7}(\frac{2}{3})\ (\frac{1}{3})\ +8_{c8}(\frac{2}{3})^2(\frac{1}{3})^0]=\frac{3072}{6561}=0.4682$$

Hence the power of the test is equal to 0. 4682

15. An urn contains 6 balls of which 3 are white and others are black. H0: θ=3 and H1: θ=4

Test Procedure: 2 balls are chosen at random and it is decided to reject H0 if both are of the same colour. Determine the power of a test

Answer:

H0: θ=3; that is 3 white and 3 black balls; H1:θ=4;that is 4 white and 2 black balls

Power of the test =1- P [Type II error]

=1-P [Accepting H_0 /H_1 is true]

=1-P[Choosing two balls 1 white and 1 black when 4 are white and 2 are black]

=1-P [Choosing 1 white and 1 black ball/ 4 are white and 2 are black]

$$=1-\frac{(4_{c_1})2_{c_1}}{6_{c_2}}=1-\frac{8}{15}=\frac{7}{15}=0.4667$$

Hence the power of the test is 0.4667