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Topic no	16
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E-Learning Module on Use of CLT-Large Sample Test for Testing Single Mean and Equality of Two Means

Learning Objectives

At the end of this session, you will be able to:

- Explain large sample test procedure to test the mean of a population when the variance is known and unknown
- Explain large sample test procedure to test the difference between the two population means

Introduction

Consider a random sample of **n** measurements drawn from a population that has mean **μ** and standard deviation **σ**

We have seen that a sample mean is the best estimate of the actual value of **μ**

What value of the sample mean would ,lead us to believe that a null hypothesis is false and **μ**, is in fact greater than the hypothesized value?

The values of the sample mean that are extremely large would imply that **μ** is larger than the hypothesized value.

Hence we should reject the null hypothesis if the sample mean is too large.

The next problem is to define what is meant by too large.

The values of the sample mean lie too many standard deviations to the right of the mean are not very likely to occur.

Those values will have very little area to their right.

Hence we can define too large as too many standard deviations away from the hypothesized value. But what is too many?

This question may be answered using a significance level α

Since a sampling distribution of a sample mean is approximately Normal when **n** is large the number of standard deviations that a sample mean lies from the hypothesized value can be measured using the standardized test statistic

$$Z = \frac{\overline{x} - \mu_0}{s / \sqrt{n}}$$

which has an approximate standard Normal distribution when a null hypothesis is true

Hence when the sample drawn is large, that is,

n >30

most of the statistics are normally or at least approximately Normally distributed.

Hence if ‘t’ is the sample statistic (sample being large), then ‘t’ follows Normal distribution and

$$Z = \frac{t - E(t)}{\sqrt{V(t)}}$$

follows a standard Normal distribution

In this section will try to answer the following question: It has been known that some population mean is, say, 20, but we suspect that the population mean for a population that has "undergone some treatment" is different from 20, perhaps larger than 20.

We want to determine whether our suspicion is true or not.

We will follow the outline of a statistical test as described in the previous section, but adjust the four elements of the test to our situation of testing for a population mean.

From the previous chapter of Central Limit Theorem we do know the distribution of sample means.

According to Central Limit Theorem we know that the mean of the sample means is the same as the population mean, and the standard deviation is the original standard deviation divided by the square root of n (the sample size).

In other words, if the original mean is **μ** and the original standard deviation is **σ**, then the distribution of the sample means are

$$\mathbf{N(\mu, \sigma / \sqrt{n})}$$

Since we assumed the $\mathbf{H_0}$ was true, we actually know the population mean, and since nothing else is available, we use the S.D as computed from the sample to figure as the S.D we need.

Now we are ready to summarize our example into a procedure for testing for a sample mean as follows.

Statistical Test for the Mean (Large Sample Size n > 30):

Fix an error level you are comfortable with (something like 10%, 5%, or 1% is most common).

Denote that "comfortable error level" by "α" If no prescribed comfort level **α** is given, use 0.05 as a default value.

Then setup the test as follows:

To test the null hypothesis

$$\mathbf{H_0: \mu = \mu_0}$$

against the alternative hypothesis

$$\mathbf{H_1: \mu \neq \mu_0}$$

(mean is different from $\boldsymbol{\mu_0}$)

Test Statistic:

Select a random sample of size **n**, compute its sample mean $\overline{x}$ and the standard deviation **s**

Then compute the corresponding z-score as follows:

$$Z = \frac{\overline{x} - \mu_0}{s / \sqrt{n}}$$

Rejection Region (Conclusion)

If we are making use of Normal table manually then the test criterion is to reject the null hypothesis when

$$Z = \left| \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \right| \quad \text{exceeds} \quad Z_{\alpha/2}$$

Using software compute

$p = (2)P(z > |Z|)$
$= (2)(1 - \text{NORMSDIST}(\text{ABS}(Z)))$

If the probability p computed in the above step is less than **α** (the error level you were comfortable with initially, you **reject the null hypothesis** H_0 and accept the alternative hypothesis.

Otherwise you declare your test **inconclusive**.

Comments

- The **null hypothesis**, for this test, is that the population mean **is always equal to** a particular number. That number is usually thought of as the "default value", or the "status quo", or the "best guess" value. It is usually mentioned somewhere in the problem

- The **alternative hypothesis** could actually be split into 3 possibilities
  - mean less than $\boldsymbol{\mu_0}$
  - mean greater than $\boldsymbol{\mu_0}$
  - mean not equal to $\boldsymbol{\mu_0}$

- We make use of Standard Normal table to compute the probability in the last step or if we are making use of statistical software for testing, then we have to apply proper probability function

- Our conclusion is **always one of two options:** we either *reject the null hypothesis* or *declare our test invalid*

We **never** conclude anything else, such as *accepting* the null hypothesis. If the comfort level ɑ is not given in a particular problem, use 5%, or ɑ = 0.05

One Tailed Tests:

To test the null hypothesis

$$H_0: \mu = \mu_0$$

against the alternative hypothesis

$$H_1: \mu > \mu_0$$

(mean is greater than μ_0)

Compute the corresponding z score as follows:

$$Z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

Rejection Region (Conclusion)

If we are making use of Normal table manually then the test criterion is to reject the null hypothesis when

$$\frac{\bar{x} - \mu_0}{s / \sqrt{n}} \quad \text{exceeds} \quad Z_\alpha$$

To test the null hypothesis

$$\mathbf{H_0: \mu = \mu_0}$$

against the alternative hypothesis

$$\mathbf{H_1: \mu < \mu_0}$$

(mean is less than $\mathbf{\mu_0}$)

Compute the corresponding z score as follows:

$$Z = \frac{\overline{x} - \mu_0}{s / \sqrt{n}}$$

Rejection Region (Conclusion)

If we are making use of Normal table manually then the test criterion is to reject the null hypothesis when

$$\frac{\bar{x} - \mu_0}{s / \sqrt{n}} \quad \text{is less than } - z_{\alpha}$$

Technically speaking, this particular test works under the following assumptions:

- The standard deviations of the sample and the population are the same and are known

- The sample size is 30 or more

The probability computed in our test and used to determine the rejection region gives the **Level of Significance** of the test.

The smaller it is, the more likely you are to be correct in rejecting the null hypothesis.

To learn how to interpret the result of a test of hypotheses in the context of the original narrated situation, in this section we describe and demonstrate the procedure for conducting a test of hypotheses about the mean of a population in the case that the sample size ***n*** is at least 30

As we studied in the previous papers if we know **σ** then the

$$Z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$$

statistic is our test statistic.

If, as is typically the case, we do not know **σ** then we replace it by the sample standard deviation **s**

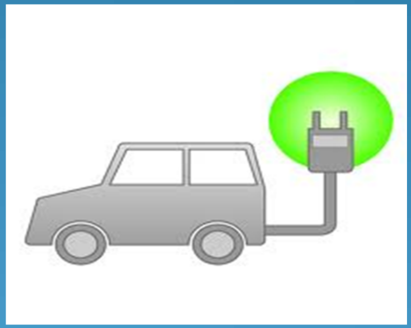
We observed in the previous topics that when the standard deviation is unknown we get t-statistic as a result of LRTP

By Central Limit Theorem, since the sample is large the resulting test statistic still has a distribution that is approximately standard normal.

Test for Difference Between Two Population Means (Large Sample Size)

In many situations a statistical question to be answered involves a comparison of two population means.

For example



US postal service is interested in reducing its massive 350 million gallons /year gasoline bill by replacing gasoline powered trucks with electric powered trucks

To determine whether significant savings in operating costs are achieved by changing to electric powered trucks, a pilot study should be undertaken using say 100 conventional gasoline powered mail trucks and 100 electric powered mail trucks operated under similar conditions.

The statistic that summarizes the sample information regarding the difference in the population means $\mu_1 - \mu_2$ is the difference in the $\overline{x_1} - \overline{x_2}$

Therefore in testing whether the difference in the sample mean indicates that the true difference in the population means differs from the specified value, $\mu_1 - \mu_2 = D_0$, one can use standard error of $\overline{x}_1 - \overline{x}_2$ as $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

estimated by the standard error $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

in the form of Z statistic.

If the sample size is large, then the normal approximation distribution and associated statistics can be used to determine a test for whether the difference in the sample means is equal to difference in the population means.

That is, when the sample sizes are greater than or equal to 30 we can use the **z** score statistics to compare the sample means against the population means using value of the sample standard deviations to estimate the sample standard deviations if they are not known.

There are three questions one may ask when comparing difference between two means

Question 1: : Is $\mu_1 \neq \mu_2$? H_1 (Two-tailed test)

Question 2: : Is $\mu_1 > \mu_2$? H_1 (Right-tailed test)

Question 3: : Is $\mu_1 < \mu_2$? H_1 (Left-tailed test)

Statistical Test for the Difference Between Two Populations Means (large sample size n > 30):

Fix an error level you are comfortable with (something like 10%, 5%, or 1% is most common).

Denote that "comfortable error level" by "**α**"

If no prescribed comfort level **α** is given, use 0.05 as a default value.

Now set up the test as follows:

To test the null hypothesis

$$H_0 : \mu_1 = \mu_2$$

against the alternative hypothesis

$$H_1 : \mu_1 \neq \mu_2$$

(means are different from each other) (*2-tailed test)*

Test Statistic:

Select a random sample of size $\mathbf{n_1}$ and $\mathbf{n_2}$ from the two populations, compute its sample mean $\overline{x}_1$ and $\overline{x}_2$ and the standard deviation $\mathbf{s_1}$ and $\mathbf{s_2}$

Then compute the corresponding z-score as follows:

$$z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Rejection Region (Conclusion)

If we are making use of Normal table manually then the test criterion is to reject the null hypothesis when

$$\left| \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \right| \quad \text{exceeds} \quad Z_{\alpha/2}$$

If the probability p computed as in the previous step is less than **α** (the error level we were comfortable with initially, we reject the null hypothesis H_0 and accept the alternative hypothesis.

Otherwise we declare our test inconclusive.

One Tailed Tests:

To test the null hypothesis

$$H_0: \mu_1 = \mu_2$$

against the alternative hypothesis

$$H_1: \mu_1 > \mu_2$$

Test Statistic:

Select a random sample of size $\mathbf{n_1}$ and $\mathbf{n_2}$ from the two populations, compute its sample mean $\overline{x}_1$ and $\overline{x}_2$ and the standard deviation $\mathbf{s_1}$ and $\mathbf{s_2}$

Then compute the corresponding z-score as follows:

$$z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Rejection Region (Conclusion)

If we are making use of Normal table manually then the test criterion is to reject the null hypothesis when

$$\frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \text{exceeds} \quad Z_{\alpha}$$

To test the null hypothesis

$$H_0: \mu_1 = \mu_2$$

against the alternative hypothesis

$$H_1: \mu_1 < \mu_2$$

The test criterion is to reject the null hypothesis if Z <- Z_{α}

Example :

A new antihypertensive drug is tested.

It is supposed to lower blood pressure more than other drugs.

Other drugs have been found to lower blood pressure by 10 mmHg on average, so we suspect (or hope) that our drug will lower blood pressure by more than 10 mm Hg.

At this stage we can setup the two competing hypothesis:

Null Hypothesis

$\mathbf{H_0}$**:** population mean = 10

Alternate Hypothesis

$\mathbf{H_1}$**:** population mean ≠ 10

We would like to know whether the new drug shows results different from the other drugs.

In particular whether it is better than the old drugs, i.e. does the new drug lower blood pressure more than other drugs?

To collect evidence, we select a random sample of:

Size n = 62 (say)

Sample mean = 11.3

Sample standard deviation = 5.1

Since the sample mean is 11.3, which is more than other drugs, it looks like this sample mean supports our suspicion (because the mean from our sample is indeed bigger than 10)

However knowing that we can never be 100% certain.

We must compute a probability and associate that with our conclusion.

Assuming that the null hypothesis is true we will try to compute the probability that a particular sample mean (such as the one we collected) could indeed occur.

Leaving the details of the computation aside for now, it turns out that the associated probability $p = 0.044$

Assuming that the null hypothesis is true, the probability of observing a random sample mean of 11.3 or more is quite small (less than 5%).

However we have observed a sample mean of 11.3, and there is no denying that fact.

So, something is not right:

Either we were extremely lucky to have hit the less than 5% case, or our assumption that the null hypothesis was true.

Since we don't believe in luck, we **choose to reject the null hypothesis** (even though there's a 4.4% chance based on our evidence that the null hypothesis could still be right).

The only practical consideration is: how do we compute the probability **p**?

We want to know the chance that a sample mean could be 11.3 (or more), given that we assume the population mean to be 10 (our null hypothesis).

In other words, we want to compute:

p (sample mean > 11.3)

We can obtain this if we only knew the distribution to use.

We know that, in our case:

Mean to use: 10

Standard deviation to use: $5.1/\sqrt{62}$

Therefore, using the Central Limit Theorem we get the required probability **p**