1. Introduction

Welcome to the series of E-learning modules on Harmonic Mean. In this module, we are going to cover the definition, calculation of Harmonic Mean for the various statistical series, applications, merits and limitations.

By the end of this session, you will be able to:

- Explain the concept of Harmonic mean
- Calculate the Harmonic Mean
- Discuss the applications of Harmonic Mean
- List the merits and demerits of Harmonic Mean

Means are mathematical formulations used to characterize the central tendency of a set of numbers. Most of the people are familiar with the "arithmetic mean" which is commonly called as an average.

Harmonic mean is another measure of central tendency and is based on mathematic footing like arithmetic mean and geometric mean. Like arithmetic mean and geometric mean, harmonic mean is also useful for quantitative data.

Harmonic mean is defined in following terms:
Harmonic mean is quotient of “number of the given values” and “sum of the reciprocals of the given values”.

The Harmonic Mean of any series is the reciprocal of the arithmetic mean of the reciprocal of the variate, that is, the harmonic mean by n.

It is given by
One by H is equal to arithmetic mean of one by x_1, one by x_2 and so on up to one by x_n
The harmonic mean is based on the reciprocals of numbers averaged. Thus by definition Harmonic mean equal to N by one by x1 plus one by x2 and so on up to plus 1 by xn

When the numbers of items is large, the computation of harmonic mean in the above manner becomes tedious. To simplify calculation, we obtain the reciprocals of the various items and apply the formulae.

Like any other measure of central tendency, harmonic mean can also be calculated in three different ways for three types of distribution. In the following section, we discuss how to compute the harmonic mean for an individual series, a discrete frequency distribution, and a continuous frequency distribution.

2. Computation of Harmonic Mean for Individual Series

Let us discuss the computation of harmonic mean for individual series.
An individual series is composed of raw or ungrouped data.
For a series like (x_1, x_2, x_3 ….., x_n) the harmonic mean is given by
Harmonic Mean is equal to number of items divided by sigma of the reciprocals of the variables

Let us take few examples to understand the calculation of the harmonic mean in individual series.
Calculate the harmonic mean of the following data:
2, 1.5, 3, 10, 250, 0.5, 0.905, 2000, 0.099

Figure 1

Size of the item (x)	Reciprocal (1/x)
2	0.5
1.5	0.6666
3	0.3333
10	0.1
250	0.004
0.5	2
0.905	1.1049
0.095	10.5263
2000	0.0005
0.099	10.1010

To compute the harmonic mean, we have to first calculate the reciprocals of each item that is for the item 2 the reciprocal is ½ which is equal to 0.5. For the item 1.5, the reciprocal is 1/1.5 is equal to 0.6666. Similarly, the reciprocal value of all the items is calculated and shown in the table.

Figure 2

Size of the item (x)	Reciprocal (1/x)
2	0.5
1.5	0.6666
3	0.3333
10	0.1
250	0.004
0.5	2
0.905	1.1049
0.095	10.5263
2000	0.0005
0.099	10.1010
N=10	Σ 1/x =25.3367

Then, the total of the reciprocal is taken i.e. the sigma of 1/x is equal to 25.3367 and the value for N is equal to 10 i.e. the number of data in the series.

Now, let us calculate the harmonic mean by substituting the value in the formula where N is equal to 10 the number of data and divided by 1/25.3367, which is equal to 0.39468.

Let us now discuss the example 2.

Find the harmonic mean for the following data:

2574, 475, 75, 5, 0.8, 0.08, 0.005, 0.0009

To compute the harmonic mean, we have to first calculate the reciprocals of each item i.e. for the item 2574 the reciprocal is1/2574, which is equal to 0.0004. For the item 475, the reciprocal is 1/475, which is equal to 0.0021. Similarly, the reciprocal value of all the items is calculated and shown in the table.

Figure 3

Size of the item (x)	Reciprocal (1/x)
2574	0.0004
475	0.0021
75	0.0133
5	0.2000
0.8	1.2500
0.08	12.5000
0.005	200.0000
0.0009	1111.1111
N=8	Σ 1/x = 1325.0769

Then, the total of the reciprocal is taken i.e. the sigma of 1/x is equal to 1325.0769 then, we calculate the harmonic mean is equal to 8 the number of data by 1325.0769 which is equal to 0.006.

3. Computation of Harmonic Mean of Discrete Frequency Distribution

Now, let us compute the Harmonic Mean of discrete frequency distribution.

A discrete frequency distribution is a distribution, which comprises of raw data grouped with frequencies. For a discrete frequency distribution, the formula should include the frequencies along with the individual values, which express the repetition of these values in the data.

A discrete frequency is composed of raw data grouped with the frequency.
For a series like (x_1, x_2, x_3 ….., x_n) having frequencies (f_1, f_2, f_3,….., f_n)the harmonic mean is given by
Harmonic Mean is equal to number of items divided by sigma of the reciprocals of the variables multiplied by the frequency

Let us list the steps to calculate the harmonic mean in discrete distribution frequency series.

1. Take the reciprocal of the various items of the variable x
2. Multiply the reciprocal by frequencies and obtain the total Σ (f x 1/x)
3. Substitute the values of N and Σ(f x 1/x)
4. Obtain the harmonic mean

Note: To make the calculation easier, instead of taking the reciprocals and multiplying it by the frequency, we will directly divide the frequency with the respective value of the variable.

Let us take a few examples to understand the calculation of the harmonic mean in discrete distribution frequency series.

From the following data compute the value of harmonic mean:

To compute the harmonic mean, we have to first calculate the reciprocal of each item i.e. the reciprocal of the items 20, 40, 50, 80 and 100 and then multiply it by the frequency. We will directly take the frequency and divide it with the respective value of the variable.

Figure 4

Marks (X)	No. of students (f)	f/x
20	40	2.000
40	60	1.500
50	100	2.000
80	30	0.375
100	10	0.100
	N = 240	Σ(f/x) = 5.975

Then, the total of the reciprocal is taken i.e. the sigma of frequency is multiplied by 1/x is equal to 5.975. Then, we calculate the harmonic mean is equal to 240 the total of the frequency divided by 5.975, which is equal to 40.16.

Now let us solve another example.
For the following data, compute the value of harmonic mean.

To compute the harmonic mean, we have to first calculate the reciprocals of each item i.e. the reciprocal of the items 10, 20, 25, 40 and 50 and then multiply it by the frequency. We will directly take the frequency and divide it with the respective value of the variable.

Figure 5

Marks (X)	No. of students (f)	f/x
10	20	2.000
20	30	1.500
25	50	2.000
40	15	0.375
50	5	0.100
	N = 240	Σ(f/x) = 5.975

Then, the total of the reciprocal is taken i.e. the sigma of frequency multiplied by 1/x is equal to 5.975. Then, we calculate the harmonic mean is equal to 120 the total of the frequency divided by 5.975, which is equal to 20.08.

Now, let us compute the Harmonic Mean of Continuous frequency distribution.
A continuous frequency distribution is a distribution where in the raw data is grouped into class intervals and the values of the data falling in each class interval is represented as the frequencies.

In case of a continuous frequency distribution, we use the formula as in discrete frequency distribution and use the midpoint of the class interval for representing the X variable. The mid value of the class interval is calculated as the total of the upper limit and the lower limit divided by 2.

A Continuous frequency distribution is composed of raw data grouped into class intervals with the frequency.

For a series like (x_1, x_2, x_3 ….., x_n) having frequencies (f_1, f_2, f_3,….., f_n)the harmonic mean is given by
Harmonic Mean is equal to number of items divided by sigma of the reciprocals of the variables multiplied by the frequency

4. Computation of Harmonic Mean of Continuous Frequency Distribution

Let us list the steps to calculate the harmonic mean in continuous frequency distribution.

1. Calculate the midpoints of the class interval m
2. Take the reciprocal of the various items of the variable m
3. Multiply the reciprocal by frequencies and obtain the total Σ (f x 1/m)
4. Substitute the values of N and Σ(f x 1/m)
5. Obtain the harmonic mean.

Note: To make the calculation easy, instead of taking the reciprocals and multiplying it by the frequency we will directly divide the frequency with the respective value of the variable m.

Let us take few examples to understand the calculation of the harmonic mean in continuous frequency distribution series.

For the following data, compute the value of harmonic mean.

To compute the harmonic mean, we have to first calculate the mid values of the class intervals 10-20, 20-30, 30-40, 40-50 and 50-60 like 10+20/2 = 15. Similarly, we get the mid values of the other class intervals as 25, 35, 45 and 55.

Figure 6

Class Interval	Midpoints (m)	Frequency(f)
10-20	15	4
20-30	25	6
30-40	35	10
40-50	45	7
50-60	55	3

Now, let us take the reciprocals of each item and multiply it with the frequency. However, here we will directly take the frequency and divide it with the respective value of the variable.

Figure 7

Class Interval	Midpoints (m)	Frequency(f)	f/m
10-20	15	4	0.267
20-30	25	6	0.240
30-40	35	10	0.286
40-50	45	7	0.156
50-60	55	3	0.055
		N=30	Σ (f/m) = 1.004

Then, the total of the reciprocal is taken i.e. the sigma of frequency multiplied by 1/m is equal to 1.004. Then, we calculate the harmonic mean is equal to 30 the total of the frequency divided by 1.004, which is equal to 29.88.

Let us discuss another example to calculate the harmonic mean in continuous frequency distribution series.

The following is the salary range and the number of employees for a manufacturing firm. Calculate the harmonic mean.

Figure 8

Salary (000)Rs.	50-60	60-70	70-80	80-90	90-100	100-110	110-120	120-130	Total
No of Employees	12	15	20	44	42	32	32	12	209

To compute the harmonic mean, we have to first compute the mid values of the class intervals 50-60, 60-70, 70-80, 80-90, and so on which is 50+60/2 = 55. Similarly, we get the mid values of the other class intervals as 65, 75, 85, 95, 105, 115 and125.

Figure 9

Salary	Midpoints (m)	Frequency(f)
50-60	55	12
60-70	65	15
70-80	75	20
80-90	85	44
90-100	95	42
100-110	105	32
110-120	115	32
120-130	125	12

Now, let us take the reciprocals of each item and multiply it with the frequency. However, here we will directly take the frequency and divide it with the respective value of the variable.

Figure 10

Salary	Midpoints (m)	Frequency(f)	f/m
50-60	55	12	0.2181
60-70	65	15	0.2307
70-80	75	20	0.2666
80-90	85	44	0.5176
90-100	95	42	0.4421
100-110	105	32	0.3047
110-120	115	32	0.2782
120-130	125	12	0.096
Total		N = 209	2.354

Then, the total of the reciprocal is taken that is the sigma of frequency multiplied by 1/m is equal to 2.354. Then we calculate the harmonic mean is equal to 209 the total of the frequency divided by 2.354, which is equal to 88.78.

5. Importance of Harmonic Mean

Let us discuss the importance of Harmonic mean.
Harmonic Mean is useful in problems in which the values of a variable are compared with a constant quantity of another variable.
The harmonic mean is a measure of central tendency restricted in its field of usefulness for data expressed as rates such as Kms, per hour, per litre, per semester, tones per month, etc.

It is useful for computing the average rate of increase in profits of a concern.
It is useful in computing the average speed at which a journey has been performed.
It is useful in computing the average price at which an article is sold.

The rate usually indicates the relation between two different types of measuring units that can be expressed reciprocally. It may be noted here that a rate represents a ratio.

For Example:
Price =money/quantity, Speed = distance/time, work done per hour = work done/time taken, etc.

The average of a rate is defined by the ratio p/q is given by the arithmetic mean of its value in different situations if the conditions are given in terms of q and by the harmonic mean if the conditions are given in terms of p.

Let us take an example to understand the importance of harmonic mean.
If a bus travels 100km in 4 hours, then its speed can be expressed as
100km/4hrs = 25 kmph

Here, the unit of distance travelled is kilometers and the unit of time is hours this can be reciprocally expressed as
4hrs/100km = 0.4hpkm
Therefore, when the average of different speeds is to be compounded the harmonic mean is an appropriate averaging tool.

Let us discuss another example for understanding the importance of harmonic mean.

An individual purchases three qualities of pencils. The relevant data are given here:

Figure 11

Quality	Price per pencil (Rs.)	Money spent (Rs.)
A	1.00	50
B	1.50	30
C	2.00	20

Here we are given:
Total expenditure = Rs(50+30+20) = Rs 100
Total number of pencils purchased = 50/1+30/1.50+20/2 =80
Average price per pencil = total expenditure/total no. of pencils = 100/80 = Rs 1.25

Remarks: Average price of Rs.1.25 can also be obtained by finding the weighted harmonic mean (HM) of 1, 1.5, and 2 with corresponding weights 50, 30 and 20 respectively.

Let us list the merits and demerits of Harmonic Mean.
Harmonic mean is based upon all the observations and given weightage to smaller values. Its application is limited because of its shortcomings.
It is useful in cases where small items need to be given very high weightage.

Following are the merits of harmonic mean:

- It is based on all observations
- It is a suitable algebraic treatment
- It gives more importance to smaller values

Let us list the demerits of Harmonic Mean.

- It is difficult to compute
- When there is a positive and negative value in a series, it is difficult to compute
- When there are one or more zeroes in a series, it is difficult to compute because it gives more importance for smaller value, which becomes a hindrance in its wide use to economical data

Here's a summary of our learning in this session:

- We have understood the concept of Harmonic Mean
- The calculation of Harmonic Mean for various statistical series
- Application of Harmonic Mean