1. Introduction

Welcome to the series of e-learning modules on Standard Error of Estimators and Estimation of Standard Errors. In this module we are going cover the basic need of standard error, Variance of sample mean and estimated population total under Stratified Random Sampling with and without Replacement, Standard errors of these estimators and the estimation of standard errors of estimated population mean and total.

By the end of this session, you will be able to explain

- The need for Standard Error
- Variance of sample mean and total under stratified random sampling with SRSWR and its standard error
- Variance of sample mean and total under stratified random sampling with SRSWOR and its standard error
- Estimation of standard errors of estimated mean and total

The term ‘Standard Error’ is used in relation to a statistic, which is a measure based on the sample observations. As such, we may speak of the standard error only in connection with sampling.
A group of observations constituting one sample is likely to be different from that of another sample; the value of a statistic varies from sample to sample. A measure of this variability of statistic is called Standard Error.

A group of observations constituting one sample is likely to be different from that of another sample, and the value of a statistic could vary from sample to sample. A measure of this variability of statistic is called Standard Error.
Of course, the variability is measured by Standard Deviation. Thus, Standard Error is a standard deviation of all possible values of a statistic in repeated samples of a fixed size from a given population.

Standard error depends on:

i) Size of the sample
ii) Nature of the statistic, Eg: Mean, Variance
iii) The mathematical form of the sampling distribution
iv) The values of some of the parameters used in the sampling distribution.

The reciprocal of the standard error is sometimes used to measure the precision of the statistic as an estimate of a parameter.

Standard error can be defined as the standard deviation of a distribution.
The reciprocal of the standard error is sometimes used to measure the precision of the statistic as an estimate of a parameter.
For example: Suppose, ‘t’ is an estimator, then Standard Error of t is equal to the square root of variance of t.

The formulae for the standard errors of the estimated population mean and total are used primarily for three purposes:
One, to compare the precision obtained by stratified simple random sampling with that given by other methods of sampling

Two, to estimate the size of the sample needed in a survey that is being planned, and
Three, to estimate the precision actually attained in a survey that has been completed.

2. Variance of Unbiased Estimator of Population Mean

<u>Variance of Unbiased Estimator of Population Mean</u>
<u>Theorem 1:</u>
The variance of the unbiased estimator of the population mean under Stratified Sampling using Simple Random Sample With Replacement is given by:
Variance of y bar st is equal to summation Wh square into sigma h square by nh.

Let us draw a Simple Random Sampling With Replacement sample of size n from k stratum of the population of size N.
Consider , Variance of y bar st is equal to Variance of summation, h runs from 1 to k, Wh into y bar h.
Which is equal to summation, h runs from 1 to k, Wh square into variance of y bar h plus summation, h runs from 1 to k, summation h dash runs from 1 to k, h is not equal to h dash , Wh into Wh dash into Covariance of y bar h coma ybar h dash
Here, Covariance of y bar h coma ybar h dash is equal to zero.

We know that, a sample mean for the observations drawn from the h-th stratum and the sample mean for the observations drawn from the h-dash th stratum are independent of each other.
Therefore,
Variance of y bar st is equal to summation, h runs from 1 to k, Wh square into variance of y bar h
Since the sample in each stratum is a Simple Random Sample With Replacement we have, Variance of y bar h under Simple Random Sampling With Replacement is equal to sigma h square divided by nh.

Where sigma h square is equal to summation i runs from 1 to Nh, Y-h-i minus Y bar h whole square divided by Nh
We know that Wh is equal to Nh by N
Therefore Variance of y bar st under Simple Random Sample With Replacement is equal to summation, h runs from 1 to k, Wh square into sigma h square by nh
Simplifying further, this implies Variance of y bar st under Simple Random Sample With Replacement is equal to summation, h runs from 1 to k, Nh divided by N whole square into sigma h square by nh.

Therefore, variance of y bar st under Simple Random Sample With Replacement is equal to summation, h runs from 1 to k, Nh square into sigma h square divided by N square into nh
Thus, under Stratified Random Sampling With Simple Random Sample With Replacement variance of y bar st depends on sigma h square.

Theorem 2:
Variance of an unbiased estimator of population total is given by:
N square into summation, h runs from 1 to k, Wh square into sigma h square divided by nh

The population total under stratified random sampling is given by:
Y is equal to summation h runs from 1 to k, summation i runs from 1 to Nh, Y-h-i which is equal to N into Y bar
Since, Y bar is equal to summation h runs from 1 to k, summation i runs from 1 to Nh, Y-h-i by N
Unbiased estimator of population total Y is equal to N into Y bar cap which is equal to N into y bar st.

Hence Variance of Y cap is equal to Variance of N into ybar st
Which is equal to N square into variance of y bar st
Hence, Variance of Y cap is equal to N square into summation h runs from 1 to k, Wh square into sigma h square by nh
Which can be further simplified as Variance of Y cap is equal to N square into summation h runs from 1 to k, Nh square into sigma h square by N square into nh
Which is equal to summation h runs from 1 to k, Nh square into sigma h square by nh

3. Standard Errors of estimated mean and total under Stratified SRSWR

Standard Errors of estimated mean and total under Stratified SRSWR

a) The Standard Error of estimated population mean:

Standard Error of y bar st is equal to square root of Variance of y bar st which is equal to square root of summation, h runs from 1 to k, Wh square into sigma h square divided by nh.

b) An estimate of Standard Error of estimated population mean

Estimate of Standard Error y bar st is equal to Estimate of the square root of variance of y bar st

which is equal to square root of Estimate of summation, h runs from 1 to k, Wh square into sigma h square divided by nh which is equal to square root of summation, h runs from 1 to k, Wh square into sh square divided by nh.

c) Standard Error of the estimated population total

Standard Error of Y cap is equal to square root of variance of estimated population total

Which is equal to square root of N square into summation, h runs from 1 to k, Wh square into sigma h square divided by nh

Which is equal to square root of summation, h runs from 1 to k, Nh square into sigma h square divided by nh.

d) An estimate of S.E. of estimated population total

An estimate of Standard Error of Y cap is equal to estimate of square root of variance of estimated population total

Which is equal to Estimate of square root of N square into summation, h runs from 1 to k, Wh square into sigma h square divided by nh

Which is equal to square root of N square into summation, h runs from 1 to k, Wh square into estimate of sigma h square divided by nh

Which is equal to square root of N square into summation, h runs from 1 to k, Wh square into sh square divided by nh

Which is equal to square root of summation, h runs from 1 to k, Nh square into sh square divided by nh.

Theorem 3:

The variance of the unbiased estimator of the population mean under Stratified sampling using Simple Random Sample Without Replacement is given by:

Variance of y bar st is equal to summation , h runs from 1 to k, Wh square into Nh minus nh into S h square divided by Nh into nh. Call this as (1)

Variance of y bar st is equal to summation , h runs from 1 to k, Wh square into S h square divided by nh minus summation, h runs from 1 to k, Wh square into S h square divided by Nh. Call this as (2)

Variance of y bar st is equal to 1 by N square into summation, h runs from 1 to k, Nh into Nh minus nh into S h square divided by nh. Call this as (3)

Proof:
Consider Variance of y bar st is equal to Variance of summation, h runs from 1 to k, Wh into y bar h
Which is equal to summation, h runs from 1 to k, Wh square into variance of y bar h plus summation , h runs from 1 to k, summation h dash runs from 1 to k, h is not equal to h dash , Wh into Wh dash into Covariance of y bar h coma y bar h dash
Here, Covariance of y bar h coma y bar h dash is equal to zero.

We know that, a sample mean for the observations drawn from the h^{th} stratum and the sample mean for the observations drawn from the h dash th stratum are independent of each other
Therefore, Variance of y bar st is equal to summation , h runs from 1 to k, Wh square into variance of y bar h

Since the sample in each stratum is Simple Random Sample Without Replacement , we have Variance of y bar h under Simple Random Sample Without Replacement is equal to Nh minus nh into S h square divided by Nh into nh where S h square summation i runs from 1 to Nh, yhi minus Y bar h whole square divided by Nh minus 1

We know that Wh is equal to Nh by N
Therefore Variance of y bar st under Simple Random Sample Without Replacement is equal to summation, h runs from 1 to k, Wh square into Nh minus nh nto S h square divided by Nh into nh.
This is equation (1)
Variance of y bar st under Simple Random Sample Without Replacement is equal to summation, h runs from 1 to k, Wh square into S h square divided by nh minus summation, h runs from 1 to k, Wh square into S h square divided by Nh. This is equation (2)
Variance of y bar st under Simple Random Sample Without Replacement is equal to summation, h runs from 1 to k, Nh into Nh minus nh into Sh square whole divided by N square into Nh into nh.
Which is equal to 1 by N square into summation, h runs from 1 to k, Nh into Nh minus nh into Sh square divided by nh.
This is equation (3)

Remarks: In general Sh square are not known . Since a simple Random sample is drawn from each stratum ,
Expected value of sh square is equal to Sh square; h equals to 1,2, etc,k. Accordingly an unbiased estimate of Variance of y bar st is given by replacing Sh square by sh square in the formulae.
Thus we see that Variance of y bar st depends on Sh square, the heterogeneity within the strata. Thus if Sh square are small , that is strata are homogeneous then stratified sampling provides estimates with greater precision.

4. Standard Errors of estimated mean and total under Stratified SRSWR- Theorem 4

Theorem 4:
The variance of the unbiased estimator of the population Total under Stratified sampling using Simple Random Sample Without Replacement is given by:
Variance of Y cap is equal to N square into summation, h runs from 1 to k, Wh square into Nh minus nh nto S h square divided by Nh into nh.
OR
Variance of Y cap is equal to N square into summation, h runs from 1 to k, Wh square into S h square divided by nh minus summation, h runs from 1 to k, Wh square into S h square divided by Nh.
OR
Variance of Y cap is equal to summation , h runs from 1 to k, Nh into Nh minus nh into S h square divided by nh.

<u>Proof</u>:

The population total under stratified random sampling is given by

Y is equal to summation h runs from 1 to k, summation i runs from 1 to Nh Y-h-i which is equal to N into Y bar

Since, Y bar is equal to summation h runs from 1 to k, summation i runs from 1 to Nh, Y-h-i by N

Unbiased estimator of population total Y is Y cap is equal to N into Y bar cap which is equal to N into y bar st
Hence Variance of Y cap is equal to Variance of N into ybar st Which is equal to N square into variance of y bar st

Which is equal to N square into summation h runs from 1 to k, Wh square into Nh minus nh into S h square divided by Nh into nh.
Which is equal to summation h runs from 1 to k, Nh square into Nh minus nh nto S h square whole divided by N square into Nh into nh.
Which is equal to summation h runs from 1 to k, Nh into Nh minus nh into S h square divided by nh.
(31)
Variance of Y cap is equal to N square into summation h runs from 1 to k, Wh square into Sh square by nh minus summation h runs from 1 to k, Wh square into Sh square by Nh
Which is equal to summation h runs from 1 to k, Nh square into Sh square by nh minus summation h runs from 1 to k, Nh into Sh square.

Variance of Y cap is equal to N square into 1 by N square into summation h runs from 1 to k,

Nh into Nh minus nh into Sh square by nh
Which is equal to summation h runs from 1 to k, Nh into Nh minus nh into Sh square by nh

5. Standard Errors Of Estimate population mean and total under Simple Random Sampling Without Replacement

Standard Errors Of Estimate population mean and total under Simple Random Sampling Without Replacement

a. The Standard Error of Estimated population mean

Standard Error of y bar st is equal square root of Variance of y bar st is equal to square root of summation , h runs from 1 to k, Wh square into Nh minus nh into Sh square divided by Nh into nh

An estimate of S.E. of estimated population mean

Estimate of Standard error of y bar st

Is equal to Estimate of square root of Variance of y bar st is equal to square root of summation, h runs from 1 to k, wh square into Nh minus nh into sh square divided by Nh into nh

Which is equal to square root of summation, h runs from 1 to k, Nh into Nh minus nh into sh square divided by nh

c) A Standard Error of the estimated population total

Standard Error of Y cap is equal to square root of variance of estimated population total

Which is equal to square root of summation, h runs from 1 to k, Nh square into S h square divided by nh minus summation, h runs from 1 to k, Nh into S h square OR

Standard Error of Y cap is equal to square root of variance of estimated population total

Which is equal to square root of summation, h runs from 1 to k, Nh into Nh minus nh into Sh square divided by nh

d) An estimate of S.E. of estimated population total

Estimate of Standard error of estimated population total

Is equal to Estimate of square root variance of estimated population total

Which is equal to estimate of square root of summation, h runs from 1 to k, Nh square into Sh square divided by nh minus summation, h runs from 1 to k, Nh into Sh square

OR

Estimate of Standard error of estimated population total is equal to

Estimate of square root variance of estimated population total

Which is equal to Estimate of square root of summation, h runs from 1 to k, Nh into Nh minus nh into Sh square divided by nh

Which is equal to square root of summation, h runs from 1 to k, Nh into Nh minus nh into sh square divided by nh.

Here’s a summary of our learning in this session:

- Basic need of standard error
- Derivation of variance of sample mean and estimated population total under Stratified Random Sampling with SRSWR
- Derivation of variance of sample mean and estimated population total under Stratified Random Sampling with SRSWOR
- Standard Error of estimated population mean and total
- Estimation of Standard error of mean and total