Frequently Asked Questions

1. Write a note on Standard Error?

Answer:

The term Standard error is however used in relation to a statistic, which is a measure based on the sample observations. As such we may speak of the standard error only in connection with sampling.

As the group of observations constituting a sample is likely to be different from that of another sample, the value of a statistic varies from sample to sample. A measure of this variability of statistic is called Standard error.

Of course the variability measured by Standard deviation. Thus standard error is a Standard deviation of all possible values of statistic in repeated samples of a fixed size from a given population. In other words Standard error is a standard deviation calculated from the sampling distribution.

For example, suppose t is an estimator then Standard Error of t equals to square root of variance of t

2. Name the factors on which the Standard error is dependent on?

Answer:

Standard error depends on i) Sample size ii) Nature of the statistic eg: Mean, Variance,…iii) The mathematical form of the sampling distribution iv) The values of some of the parameters used in the sampling distribution.

3. Prove that the variance of the unbiased estimator of the population mean under Stratified sampling using SRSWR is

$$V(\bar{y}_{st}) = \sum W_h^{\,2} \frac{\sigma_h^{\,2}}{n_h}$$

Answer:

Consider $V(\bar{y}_{st}) = V(\sum_{h=1}^{k} W_h \bar{y}_h)$

$$= \sum_{h=1}^{k} W_h^{\,2} V(\bar{y}_h) + \sum_{h} \sum_{h''} W_h W_{h''} \operatorname{cov}(\bar{y}_h, \bar{y}_{h''})$$

Here $\operatorname{cov}(\bar{y}_h, \bar{y}_{h'}) = 0$ since a sample mean in the hth stratum and the sample mean of the h′th stratum are independent of each other.

Therefore $V(\bar{y}_{st}) = \sum_{h=1}^{k} W_h^{\,2} V(\bar{y}_h)$

Since the sample in each stratum is Simple Random Sample With Replacement we have

$$V(\bar{y}_h)_{wr} = \frac{\sigma_h^{\,2}}{n_h}$$

Where

$$\sigma_h^2 = \frac{\sum_{i=1}^{Nh}(Y_{hi} - \overline{Y_h})^2}{N_h}$$

We know that $w_h = \frac{N_h}{N}$

$$V(\bar{y}_{st})_{wr} = \sum_{h=1}^{k} W_h^2 \frac{\sigma_h^{\,2}}{n_h}$$

$$V(\bar{y}_{st})_{wr} = \sum_{h=1}^{k} \left[\frac{Nh}{N}\right]^2 \frac{\sigma_h^{\,2}}{n_h}$$

$$V(\bar{y}_h)_{wr} = \sum \frac{N_h^2 \sigma_h^{\,2}}{N^2 n_h}$$

4. Derive an expression for the Variance of an_unbiased estimator of population total under Stratified SRSWR

Answer:

The population total under stratified random sampling is given by

$$Y = \sum_k \sum_i Y_{hi} = N\bar{Y} \qquad [\because \bar{Y} = \sum\sum Y_{hi}/N]$$

Unbiased estimator of population total $\hat{Y} = N\hat{\bar{Y}} = N\bar{y}_{st}$

Hence $V(\hat{Y}) = V(N\bar{y}_{st}) = N^2 V(\bar{y}_{st})$

$$= N^2 \sum_{h=1}^{k} W_h^2 \frac{\sigma_h^{\,2}}{n_h} =$$

$$N^2 \sum \frac{N_h^2 \sigma_h^{\,2}}{N^2 n_h} = \sum \frac{N_h^2 \sigma_h^{\,2}}{n_h}$$

5. Obtain expressions for the Standard Error of the estimated population mean and its estimate.

Answer:

Standard error of estimated population mean

$$S.E\,(\bar{y}_{st}) = \sqrt{V(\bar{y}_{st})} = \sqrt{\frac{\sum_{h=1}^{k} W_h^2 \sigma_h^2}{n_h}}$$

6. Derive expressions for the Standard Error of the estimated population total under Stratified SRSWR

Answer:

A Standard Error of the estimated population total

$$S.E.(\hat{Y}) = \sqrt{V(\hat{Y})} = \sqrt{N^2 \sum W_h^2 \sigma_h^2 / n_h}$$

$$S.E.(\hat{Y}) = \sqrt{\sum_{h=1}^{k} N_h^2 \sigma_h^2 / n_h}$$

7. Prove that the variance of the unbiased estimator of the population mean under Stratified sampling using SRSWOR is any one of the following

$$V(\bar{y}_{st}) = \sum W_h^2 \left(\frac{N_h - n_h}{N_h}\right) \frac{S_h^2}{n_h} \longrightarrow 1$$

$$= \sum \frac{W_h^2 S_h^2}{n_h} - \sum \frac{W_h^2 S_h^2}{N_h} \longrightarrow 2$$

$$= \left(\frac{1}{N^2}\right) \sum N_h (N_h - n_h) \frac{s_h^2}{n_h} \longrightarrow 3$$

Answer:

Consider $V(\bar{y}_{st}) = V(\sum_{h=1}^{k} W_h \bar{y}_h)$

$$= \sum_{h=1}^{k} W_h^2 V(\bar{y}_h) + \sum_{h} \sum_{h''} W_h W_{h''} \operatorname{cov}(\bar{y}_h, \bar{y}_{h''})$$

Here $\operatorname{cov}(\bar{y}_h, \bar{y}_{h'}) = 0$ since a sample mean in the hth stratum and the sample mean of the h′th stratum are independent

Therefore $V(\bar{y}_{st}) = \sum_{h=1}^{k} W_h^2 V(\bar{y}_h)$

Since the sample in each stratum is Simple Random Sample Without Replacement we have

$$V(\bar{y}_h)_{wor} = \left(\frac{N_h - n_h}{N_h}\right) \frac{S_h^2}{n_h}$$

Where $S_h^2 = \frac{\sum_{i=1}^{Nh} (Y_{hi} - \overline{Y_h})^2}{N_h - 1}$

We know that $w_h = \frac{N_h}{N}$

$$V(\bar{y}_{st})_{wor} = \sum_{h} W_h^2 \left(\frac{N_h - n_h}{N_h}\right) \frac{S_h^2}{n_h} \longrightarrow 1$$

$$= \sum \frac{W_h^2 S_h^2}{n_h} - \sum \frac{W_h^2 S_h^2}{N_h} \longrightarrow 2$$

$$V(\bar{y}_{st})_{wor} = \Sigma \frac{N_h}{N^2} \frac{(N_h - n_h)}{N_h} \frac{s_h^2}{n_h}$$

$$= \left(\frac{1}{N^2}\right) \sum N_h (N_h - n_h) \frac{s_h^2}{n_h} \longrightarrow 3$$

8. Derive expression for the Variance of an unbiased estimator of population total under Stratified SRSWOR

Answer:

The population total under stratified random sampling is given by

$$Y = \sum_{k} \sum_{i} Y_{hi} = N\bar{Y} \qquad [\because \bar{Y} = \sum \sum Y_{hi}/N]$$

Unbiased estimator of population total Y = $\hat{Y} = N\hat{\bar{Y}} = N\bar{y}_{st}$

Hence $V(\hat{Y}) = V(N\bar{y}_{st}) = N^2 V(\bar{y}_{st})$

$$= N^2 \sum W_h^2 \left(\frac{N_h - n_h}{N_h}\right)\frac{S_h^2}{n_h}$$

$$= N^2 \sum \frac{N_h^2}{N^2}\left(\frac{N_h - n_h}{N_h}\right)\frac{S_h^2}{n_h}$$

$$= \sum N_h (N_h - n_h)\frac{S_h^2}{n_h} \quad \text{--------(4)}$$

$$V(\hat{Y}) = N^2 \sum \frac{W_h^2 S_h^2}{n_h} - \sum \frac{W_h^2 S_h^2}{N_h}$$

$$= \sum \frac{N_h^2 S_h^2}{n_h} - \sum N_h S_h^2$$

$$V(\hat{Y}) = N^2 \left(\frac{1}{N^2}\right)\sum N_h (N_h - n_h)\frac{S_h^2}{n_h}$$

$$= \sum N_h (N_h - n_h)\frac{S_h^2}{n_h}$$

9. Deduce Standard error of the estimated population mean under Stratified SRSWOR

Answer:

Standard Error of estimated population mean is its Standard deviation

$$S.E(\bar{y}_{st}) = \sqrt{V(\bar{y}_{st})} = \sqrt{\sum W_h^2 \left(\frac{N_h - n_h}{N_h}\right)\frac{S_h^2}{n_h}} = \sqrt{\sum_{h=1}^{k} \frac{N_h}{N^2} (N_h - n_h)\frac{S_h^2}{n_h}}$$

10. Deduce an expression for the estimate of Standard error of the estimated population n under Stratified SRSWOR

Answer:

An estimate of S.E. of estimated population mean

$$S.E.(\hat{\bar{y}}_{st}) = \sqrt{\hat{V}(\bar{y}_{st})} = \sqrt{\sum W_h^{\,2} \left(\frac{N_h - n_h}{N_h}\right) \frac{s_h^2}{n_h}} = \sqrt{\sum N_h (N_h - n_h) \frac{s_h^{\,2}}{n_h}}$$

11. Obtain an expression for the Standard Error of the estimated population total under Stratified SRSWOR

Answer:

A Standard Error of the estimated population total

$$S.E.(\hat{Y}) = \sqrt{V(\hat{Y})} = \sqrt{\sum_{h=1}^{k} \frac{N_h^2 S_h^2}{n_h} - \sum_{h=1}^{k} N_h S_h^2} \quad \text{OR}$$

$$S.E.(\hat{Y}) = \sqrt{V(\hat{Y})} = \sqrt{\sum_{h=1}^{k} N_h (N_h - n_h) \frac{S_h^2}{n_h}}$$

12. Obtain an expression for the estimate of Standard Error of the estimated population total under Stratified SRSWOR

Answer:

An estimate of S.E. of estimated population total

$$\widehat{S.E.}(\hat{Y}) = \sqrt{\hat{V}(\hat{Y})} = \widehat{\sqrt{\sum_{h=1}^{k} \frac{N_h^2 S_h^2}{n_h} - \sum_{h=1}^{k} N_h S_h^2}} = \sqrt{\sum_{h=1}^{k} \frac{N_h^2 s_h^2}{n_h} - \sum_{h=1}^{k} N_h s_h^2}$$

OR

$$\widehat{S.E.}(\hat{Y}) = \sqrt{\hat{V}(\hat{Y})} = \widehat{\sqrt{\sum_{h=1}^{k} N_h (N_h - n_h) \frac{S_h^{\,2}}{n_h}}} = \sqrt{\sum_{h=1}^{k} N_h (N_h - n_h) \frac{s_h^{\,2}}{n_h}}$$

Because under SRSWOR $E(s_h^2) = S_h^2$

13. Obtain an expression for the estimate of Standard Error of the estimated population total under Stratified SRSWR

Answer:

An estimate of S.E. of estimated population total

$$S.E.(\hat{Y}) = \sqrt{\hat{V}(\hat{Y})} = \sqrt{N^2 \sum_{h=1}^{k} W_h^2 \hat{\sigma}_h^2 \Big/ n_h} = \sqrt{\frac{N^2 \sum_{h=1}^{k} W_h^2 \hat{\sigma}_h^{\,2}}{n_h}} = \sqrt{\frac{N^2 \sum_{h=1} W_h^2 s_h^{\,2}}{n_h}} = \sqrt{\frac{\sum N_h^2 s_h^2}{n_h}}$$

Because under SRSWR $E(s_h^2) = \sigma_h^2$

14. Deduce an expression for the estimate of Standard Error of the estimated population total under Stratified SRSWR

Answer:

An estimate of S.E. of estimated population mean

$$S.E.(\hat{\bar{y}}_{st}) = \sqrt{\hat{V}(\bar{y}_{st})} = \sqrt{\sum_{h=1}^{k} Wh^2 \hat{\sigma}_h^{\,2} \Big/ n_h} = \sqrt{\frac{\sum_{h=1}^{k} W_h^2 s_h^{\,2}}{n_h}}$$

Because under SRSWR $E(s_h^2) = \sigma_h^2$

15. Stratified Sampling with SRSWOR gives estimates of greater precision. Give reasons

Answer:

One can observe that $V(\bar{y}_{st})$ under SRSWOR depends on S_h^2, the heterogeneity within the strata. Thus if $S_h^2$are small, that is strata are homogeneous then stratified sampling provides estimates with greater precision. In Stratified Random Sampling since a population is stratified such that units within the strata are as homogeneous as possible,we can expect small S_h^2 for all strata. Hence Stratified sampling with SRSWOR gives estimates of greater precision.