1. Introduction

Welcome to the series of E-learning modules on Practical - Systematic Sample. In this module, we are going to cover the basic problems of Systematic Random Sampling, comparison of the technique with that of Stratified Random Sampling and SRSWOR, the estimates of the Standard errors.

By the end of this session, you will be able to:

- Obtain an estimate of population mean, total, variance of the estimators and its Standard error under Systematic Random Sampling
- Compare Systematic Sampling with Stratified Random Sampling and SRSWOR

Problem 1:
The data for a small artificial population exhibiting a fairly steady trend of around forty observations are:

Figure 1

2	14	2	13	8	8	18	11
41	11	3	16	9	17	8	14
3	12	15	16	17	9	26	10
0	15	2	9	7	19	9	1
10	5	11	7	16	6	16	2

i. Write down all possible systematic samples of size 4
ii. Show that expected value of yr bar is equal to the population mean
iii. Obtain the variance of sample mean for the sample of size 4 under:
   a) Systematic sampling
   b) SRSWOR
   c) Stratified Random Sampling with SRSWOR and compare the efficiency
iv. Obtain Variance of the estimated population total

Solution:
i. Given a sample size n is equal to four

Population size N is equal to forty
Sampling Interval k is equal to (N by n) which is equal to (forty by four) which is equal to ten
Suppose, r is any random number from 1 to k then the systematic samples consists of the observations corresponding to,
r, (r plus k), (r plus 2k), up to [r plus (n minus 1)] into k.

Figure 2

	Systematic Samples									
Strata	1	2	3	4	5	6	7	8	9	10
i	2	8	11	16	3	6	15	9	1	7
ii	14	18	4	9	12	17	0	7	10	16
iii	2	8	11	17	4	9	15	19	5	6
iv	13	18	3	8	15	20	2	9	11	16
Total	31	52	39	50	34	52	32	44	27	45

The table presented here gives all possible systematic sample of size 4.
There are 10 possible samples each of size 4.
For example:
Systematic Sample number 1 consists of the values 2, fourteen, 2 and thirteen, which are obtained by taking r as 1. The corresponding positions of the observations in the population are obtained first by assigning r equal to 1 in the expressions r, (r plus k), (r plus 2k), etc., [r plus (n minus 1)] k.
When r is equal to 1, k is equal to ten and n is equal to 4, the corresponding positions of the observations in the population, which forms a first systematic sample are:
First, eleventh, twenty first and thirty first observations of the population which are respectively 2, fourteen, 2 and thirteen.
Similarly, ten systematic samples are generated by taking respectively r equal to 2, 3, up to ten. Hence, all the forty observations of the population form ten systematic samples of size 4 each, which are presented in the table.

Figure 3

Stratum no.	Stratum means $\bar{y}_h$	$\frac{\sum(y_{hi}-\bar{y}_h)^2}{N_h-1}$
I	7.8	27.10444
II	10.7	34.45556
III	9.6	33.37778
IV	11.5	36.72222
Total	40.6	14.62889

Systematic Samples		
	$\bar{y}_r$	$\sum(y_{ri}-\bar{y}_r)^2$
1	7.75	132.75
2	13	100
3	7.25	56.25
4	12.5	65
5	8.5	105
6	13	130
7	8	198
8	11	88
9	6.75	64.75
10	11.25	90.75

In this table for each strata sample mean (yh bar), the sum of the squared deviations of the observations from the respective sample means are obtained.

Similarly, means of each systematic samples are obtained as (yr bar) which is equal to summation (y ri by n).

Next sum of the squared deviations of the observations from the respective systematic sample means are obtained.

2. Illustration 1 (Contd.)

ii. To show that expected value of systematic sample mean is equal to the population mean , that is:

E of (yr bar) is equal to (Y bar)
Consider a population mean,
(Y bar) is equal to summation (r runs from 1 to k), summation (i runs from 1 to n) (yri) divided by (N) which is equal to (three hundred and ninety six divided by forty which is equal to (nine point nine).
Expected value of (y r bar) is equal to summation, (r runs from 1 to 10) (yr bar) divided by (k) which is equal to (ninety nine divided by 10) which is equal to (nine point nine) which is nothing but the value of the population mean (Y bar).
Hence, Expected value of y r bar is equal to the population mean (Y bar).

iii. To compare the efficiency of Systematic sampling with Stratified Random Sampling and Simple Random Sampling without Replacement

Variance of the systematic sample mean y r bar is equal to (N minus 1) into S square divided by N minus (n minus 1) into S square systematic divided by n
Where S square systematic is equal to 1 by k into (n minus 1) into summation r runs from 1 to k summation i runs from 1 to n (yri minus yr bar) whole square
Which is equal to one thousand and thirty one divided by ten into three which is equal to thirty four point three six six seven

S square is equal to summation (r runs from 1 to 10) summation (i runs from 1 to 4) (yri minus Y bar) whole square divided by (N minus 1) which is equal to summation (r runs from 1 to 10) summation (i runs from 1 to 4), (yri square) minus (N) into (Y bar square) divided by (N minus 1)
Which is equal to (one thousand two hundred and fifty five point six divided by thirty nine) which is equal to (thirty two point one nine four eight seven)
Hence, Variance of (yr bar) is equal to (N minus 1) into (S square) divided by (N) minus (n minus 1) into (S square) systematic divided by (n) which is equal to (forty minus 1) into (thirty two point one nine four eight seven) divided by forty minus (4 minus 1) into (thirty four point three six six six seven) by 4 which is equal to (five point six one four nine nine)

Variance of y bar under SRSWOR is equal to (N minus n) into S square divided by (N into n) which is equal to (forty minus four) into (thirty two point one nine four eight seven) divided by (forty into four) which is equal to (seven point two four three nine)
(Sh square) is equal to summation (i runs from 1 to k) (Yhi minus Ybarh whole square) divided by (Nh minus 1)
Summation, (h runs from 1 to n), (Sh square) is equal to (fourteen point six two eight)
Variance of (y bar st) is equal to (k minus 1) into summation h runs from 1 to n, Sh square divided by n square into k
Which is equal to (10 minus 1) into (fourteen point six two eight) by (4 square into 10) which is equal to zero point eight two two eight three

Hence, one may observe that

Variance of (y r bar) systematic is equal to (five point six one four nine nine)
Variance of (y bar st) is equal to (zero point eight two two eight)

Variance of (y bar under SRSWOR) is equal to (seven point two four three nine)
Hence, we can observe that variance of (y bar st) is less than the variance of (y r bar) systematic which is less than Variance of (y bar under SRSWOR)
Therefore, Stratified Random Sampling is more efficient than Systematic Sampling. But Systematic Random Sampling is more efficient than Simple Random Sampling without Replacement.

iv. To find Variance of estimated Population total under Systematic sampling
Variance of the estimated population total is equal to Variance of (Y cap) which is equal to (N square) into variance of (y r bar) which is equal to (forty square into five point six one four nine nine) which is equal to (eight nine eight three point nine nine two eight)

3. Illustration 2

Problem 2:
The following is a small artificial population consisting of thirty five units. List all one in 7 systematic samples. Compute sample mean under systematic y bar sy for each sample

- Show that expected value of y bar sys is equal to the population mean Y bar.
- Compute Variance of y bar sys and the Standard error of the systematic sample mean
- Estimate the standard error of the estimated population total
- Compare Systematic sampling with SRSWOR and Stratified Random Sampling

A population which consists of thirty five observations are given as follows:

Figure 4

63, 69, 65, 71, 67, 59, 61, 35,
41,37,43,39,31,33,07,13,09,15,11,03,
05,21, 27, 23, 29, 25, 17, 19, 49, 55,
51, 57, 53, 45, 47

Solution:

i. Given population size N is equal to thirty five, the number of systematic samples is equal to 7, and then k is equal to N by n which is equal to thirty five by n which is equal to 7. Then n must be equal to 5

Suppose, r is any random number from 1 to k then the systematic samples consists of the observations corresponding to
r, r plus k,r plus 2k, up to r plus (n minus 1) into k.

The following table gives all possible systematic sample of size 5.

Figure 5

	Systematic Samples						
Strata	**1**	**2**	**3**	**4**	**5**	**6**	**7**
I	63	69	65	71	67	59	61
II	35	41	37	43	39	31	33
III	07	13	09	15	11	03	05
IV	21	27	23	29	25	17	19
V	49	55	51	57	53	45	47
Total	175	205	185	215	195	155	165

The required values in this problem are computed as done in the previous one.

Figure 6

Stratum no.	Stratum means $\overline{y}_h$	$\frac{\sum (y_{hi} - \overline{y}_h)^2}{N_h - 1}$
I	65	18.66667
II	37	18.66667
III	9	18.66667
IV	23	18.66667
V	51	18.66667
Total		93.33333

Systematic Samples		
	$\overline{y}_r$	$\sum (y_{ri} - \overline{y}_r)^2$
1	35	1960
2	41	1960
3	37	1960
4	43	1960
5	39	1960
6	31	1960
7	33	1960

ii. To show that expected value of systematic sample mean is equal to the population mean, that is E of (yr bar) is equal to (Y bar)

Consider a population mean

(Y bar) is equal to summation (r runs from 1 to k), summation (i runs from 1 to n) (yri) divided by (N) which is equal to (one thousand two hundred and ninety five divided by thirty five) which is equal to (thirty seven).

Expected value of (y r bar) is equal to summation, (r runs from 1 to 7 yr bar) divided by (k) which is equal to (two hundred fifty nine nine divided by 7) which is equal to (thirty seven) which is nothing but the value of the population mean (Y bar)

Hence, Expected value of (y r bar) is equal to the population mean (Y bar).

4. Illustration 2 (Contd.)

iii. To compare the efficiency of Systematic sampling with Stratified Random Sampling and Simple Random Sampling without Replacement

Variance of the systematic sample mean y r bar is equal to (N minus 1) into S square divided by N minus (n minus 1) into S square systematic divided by n
Where S square systematic is equal to 1 by k into (n minus 1) into summation r runs from 1 to k summation i runs from 1 to n (yri minus yr bar) whole square
Which is equal to thirteen thousand seven hundred and twenty divided by seven into four which is equal to four hundred and ninety

S square is equal to summation r runs from 1 to 7 summation i runs from 1 to 5 (yri minus Y bar) whole square divided by (N minus 1) which is equal to summation r runs from 1 to 7 summation i runs from 1 to 5 , yri square minus N into Y bar square divided by N minus 1
Which is equal to sixty two thousand one hundred and ninety five divided by thirty four which is equal to four hundred and twenty.
Variance of the systematic sample mean y r bar is equal to (N minus 1) into S square divided by N minus (n minus 1) into S square systematic divided by n
Which is equal to (thirty five minus 1) into four hundred and twenty divided by thirty five minus (5 minus 1) into four hundred and ninety divided by 5 which is equal to sixteen
Standard Error of yr bar is square root of variance of y r bar which is equal to square root of sixteen equals to 4
Standard Error of Y cap is square root of N square into variance of y r bar which is equal to square root of nineteen thousand six hundred equals to hundred and forty.

Variance of y bar under SRSWOR is equal to (thirty five minus five) into four hundred and twenty divided by thirty five into five which is equal to seventy two.
Sh square is equal to summation i runs from 1 to k (Yhi minus Ybarh) whole square divided by Nh minus 1
Summation , h runs from 1 to n, Sh square is equal to ninety three point three three three three
Variance of y bar st is equal to (k minus 1) into summation h runs from 1 to n, Sh square divided by n square into k
Which is equal to (7 minus 1) into (ninety three point three three three three) by (5 square into 7) which is equal to three point one nine nine nine

Hence one may observe that
Variance of y r bar systematic is equal to sixteen
Variance of y bar st is equal to three point one nine nine nine
Variance of y bar under SRSWOR is equal to seventy two
Hence we can observe that variance of y bar st is less than the variance of y r bar systematic which is less than Variance of y bar under SRSWOR
Therefore, Stratified Random Sampling is more efficient than Systematic Sampling. As far as the efficiency of the technique is concerned the Systematic Random Sampling is the next choice.

5. Illustration 3

Problem 3:
The data for a small artificial population exhibiting a fairly steady trend are. There are forty observations which forms a population.

Figure 7

**0,1,1,2,5,4,7,7,8,6,6,8,9,10,13,
12,15,16,16,17,18,19,20,20,24,
23,25,28,29,27,26,30,
31,31,33,32,35,37,38,38**

i. Write down all possible systematic samples of size 4
ii. Obtain the variance of sample mean for the sample of size 4 under
    a) Systematic sampling
    b) SRSWOR
    c) Stratified Sampling

Solution:
Given a sample size n is equal to four
Population size N is equal to forty
Sampling Interval k is equal to N by n which is equal to forty by four which is equal to ten
Starting from any random number selected in between 1 to 10, we get 10 possible systematic random samples of size 4 each.
Suppose r is any random number from 1 to k then the systematic samples consists of the observations corresponding to
r, r plus k,r plus 2k, etc, r plus (n minus 1) into k.

Figure 8

	Systematic Samples									
Strata	1	2	3	4	5	6	7	8	9	10
I	0	1	1	2	5	4	7	7	8	6
II	6	8	9	10	13	12	15	16	16	17
III	18	19	20	20	24	23	25	28	29	27
IV	26	30	31	31	33	32	35	37	38	38
Total	50	58	61	63	75	71	82	88	91	88

For example : The first sample staring from numbers 0,6, eighteen and twenty six by taking a random start r is equal to 1

The following table gives all possible systematic sample of size 4

Similarly, ten systematic samples are generated by taking respectively r equal to 2,3 up to ten. Hence all the forty observations of the population forms ten systematic samples of size 4 each which are presented in the table.

Figure 9

Stratum no.	Stratum means $\overline{y}_h$	$\frac{\sum (y_{hi} - \overline{y}_h)^2}{N_h - 1}$
I	4.1	8.544444
II	12.2	14.62222
III	23.3	15.56667
IV	33.1	15.21111
Total	72.7	53.94444

Systematic Samples		
	$\overline{y}_r$	$\sum (y_{ri} - \overline{y}_r)^2$
1	12.5	32.20563
2	14.5	13.50563
3	15.25	8.555625
4	15.75	5.880625
5	18.75	0.330625
6	17.75	0.180625
7	20.5	5.405625
8	22	14.63063
9	22.75	20.93063
10	22	14.63063

In this table for each strata sample mean (yh bar), the sum of the squared deviations of the observations from the respective sample means are obtained

Similarly, means of each systematic samples are obtained as (yr bar) which is equal to summation (yri by n)

Next sum of the squared deviations of the observations from the respective systematic sample means are obtained.

Consider a population mean
Y bar is equal to summation r runs from 1 to k, summation i runs from 1 to n yri divided by N which is equal to seven hundred and twenty seven divided by forty which is equal to eighteen point one seven five
Variance of yr bar systematic is equal to Expected value of y r bar minus Y bar whole square
Which is equal to summation r runs from 1 to k, (yr bar minus Y bar) whole square multiplied by 1 by k
Which is equal to one hundred and sixteen point two five six three divided by 10
Which is equal to eleven point six two five three

Sh square is equal to summation i runs from 1 to k (Yhi minus Ybarh) whole square divided by Nh minus 1
Summation , h runs from 1 to n, Sh square is equal to fifty three point nine four four four four
Variance of y bar st is equal to (k minus 1) into summation h runs from 1 to n, Sh square divided by n square into k
Which is equal to (10 minus 1) into (fifty three point nine four four four four) by (4 square into 10) which is equal to three point zero three four four

S square is equal to summation r runs from 1 to 10 summation i runs from 1 to 4 (yri minus Y bar) whole square divided by (N minus 1)
Which is equal to one hundred and thirty six point two five zero six four one
Variance of y bar under SRSWOR is equal to (N minus n) into S square divided by N into n which is equal to (forty minus four) into one hundred and thirty six point two five zero six four one divided by forty into four which is equal to thirty point six five six four.

Hence,
Variance of y r bar systematic is equal to eleven point six two five three Variance of y bar st is equal to three point zero three four four
Variance of y bar under SRSWOR is equal to to thirty point six five six four
Hence we may observe that variance of y bar st is less than the variance of y r bar systematic which is less than Variance of y bar under SRSWOR
Hence, stratified sampling gives précised estimates than Systematic and SRSWOR for the given population.

Here’s a summary of our learning in this session:

- Demonstrated a Systematic Random sampling technique estimate the parameters of the population with the standard error
- Compared Systematic Sampling with Stratified Sampling and SRSWOR