Frequently Asked Questions

1. Name the estimators used to estimate population mean, population total and variance of the estimated population mean using SRSWR.

Answer:

Population mean is estimated using sample mean $\bar{y} = \sum y_i/n$

Population mean is estimated as $\hat{Y}$ = N$\bar{y}$

Variance of the estimated population mean using SRSWR can be estimated using the formula V ($\bar{y}$) = $\frac{s^2}{n}$

2. How do you estimate population mean, population total and variance of the estimated population mean under SRSWOR?

Answer:

Population mean is estimated using sample mean $\bar{y} = \sum y_i/n$

Population mean is estimated as $\hat{Y}$ = N$\bar{y}$

Variance of the estimated population mean using SRSWOR can be estimated using the formula V($\bar{y}$) = $\frac{N-n}{N}\frac{s^2}{n}$

3. How the standard error of an estimator can be estimated under Simple Random sampling?

Answer:

Standard error of any estimator is given by the Standard deviation of that estimator. That is if $\bar{y}$ is an estimator of the population mean then its Standard error is given by S.E ($\bar{y}$) = $\sqrt{v(\bar{y})}$

4. State how many possible random samples of size n can be drawn under SRSWR and SRSWOR technique from a population of size N?

Answer:

Under SRSWR suppose we have to draw a random sample of size n from a population of size N then we can draw N^n possible samples

Whereas under SRSWOR we can draw N_{C_n} possible samples

5. Draw all the possible SRSWR samples of size 2 from a population containing 4 units as Y1, Y2, Y3 and Y4.

Answer:

Under SRSWR we can draw a random samples of size n from a population of size N in N^n possible ways and hence we get we get 4^2= 16 possible samples.

They are

Sl. No	Samples	Sl. No.	Samples	Sl. No.	Samples	Sl. No.	Samples
1	(Y1,Y1)	5	(Y2,Y1)	9	(Y3,Y1)	13	(Y4,Y1)
2	(Y1,Y2)	6	(Y2,Y2)	10	(Y3,Y2)	14	(Y4,Y2)
3	(Y1,Y3)	7	(Y2,Y3)	11	(Y3,Y3)	15	(Y4,Y3)
4	(Y1,Y4)	8	(Y2,Y4)	12	(Y3,Y4)	16	(Y4,Y4)

6. Draw all the possible SRSWOR samples of size 2 from a population containing 4 units as Y1, Y2, Y3 and Y4.

Answer:

Under SRSWOR we can draw a random samples of size n from a population of size N in N_{C_n} possible ways and hence we get we get 4_{C_2} = 6 possible samples.

They are

Sl. No	Samples	Sl. No.	Samples
1	(Y1,Y2)	4	(Y2,Y3)
2	(Y1,Y3)	5	(Y2,Y4)
3	(Y1,Y4)	6	(Y3,Y4)

7. A population consists of the five numbers 2,3,6,8 and 11 Consider all possible samples of size 2 that can be drawn using SRSWOR from this population. Find

I) the mean of the population
II) Standard deviation of the population
III) The mean of the sampling distribution of the means
IV) The Standard error of the means

Answer:

There are ${}^5C_2$ = 10 samples of size 2 can be drawn without replacement (this means that we draw one number and then another number different from the first)

Sl. No	Samples	Means	Sl. No.	Samples	Means
1	(2,3)	2.5	6	(3,8)	5.5
2	(2,6)	4	7	(3,11)	7
3	(2,8)	5	8	(6,8)	7
4	(2,11)	6.5	9	(6,11)	8.5
5	(3,6)	4.5	10	(8,11)	9.5

The mean of the sampling distribution of the mean is 2.5+4+5+…+9.5/10 = 6

The variance of the sampling distribution of mean is

$$\sigma^2_{\bar{y}} = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n} = (2.5-6)^2 + (4-6)^2 + \ldots + (9.5-6)^2/10 = 4.05$$

The standard deviation $\sigma_{\bar{y}} = \sqrt{\sigma^2_{\bar{y}}} = 2.01$

8. A simple random sample of size 100 is drawn from a population of size 300 .A fraction of students that have held part time jobs during the college are given as follows

Xi equals to zero if she/ he has not held a job

Xi equals to one if she/ he has held a job

Student: 1, 2 , 3, 4, ………………………….., 99 ,100

Xi : 0, 1, 1, 1 …1,1,1

$$\sum_{i=1}^{n} x_i = 65$$

Summation xi equals to sixty five

Estimate the proportion of students who have held a part time job during the college careers.

Answer:

$$\hat{P} = \bar{x} = \frac{\sum_{i=1}^{100} x_i}{100} = \frac{65}{100} = 0.65$$

65% of the students had a part time job

9. From the 500 students of an institution a simple random sample of 100 students is selected without replacement and 25 of them were found to be with two wheelers. Estimate the total number of students with two wheelers in an institution and estimate the standard error of the estimate.

Answer:

Total number of students with two wheelers in an institution can be estimated as follows

Let X denote the total number of students with two wheelers in an institution

N is the population size = 500

Sample size n =100

Let x denote the number of students with two wheelers in the sample selected =25

Then, the sample proportion p = x/n = 25/100=0.25

q = 1 – p = 0.75

Estimate of number of students with two wheelers in an institution

$\hat{X} = Np$ = 500 * 0.25 = 125

Standard error of the estimate can be obtained as follows

$$\text{S.E}(\hat{X}) = \sqrt{N^2 V(p)}$$

$$= \sqrt{N^2 \frac{(N-n)pq}{N(n-1)}} = \sqrt{(500)^2 \frac{(500-100)(0.25)(0.75)}{500(100-1)}} \textbf{=19.4625}$$

10. A random sample of size 2 households was drawn from a small population of 5 households having weekly income in rupees as follows:

Household	1	2	3	4	5
Income	159	149	166	164	155

Draw all the possible SRSWOR samples of size 2 and verify that

i)E($\bar{y}$)=$\bar{Y}$ ii) E(s^2)=S^2 iii)V($\bar{y}$)=(N-n)S^2/Nn

Answer:

Here N=5 and n=2

Under SRSWOR we can draw N_{C_n}possible samples i,e; 5_{C_2}=10

Samples	Corresponding Income(y_i)	y_i	S^2	y_i^2
(1,2)	(159,149)	154	50	23716
(1,3)	(159,166)	162.5	24.5	26406.25
(1,4)	(159,164)	161.5	12.5	26082.25
(1,5)	(159,155)	157	8	24649
(2,3)	(149,166)	157.5	144.5	24806.25
(2,4)	(149,164)	156.5	112.5	24492.25
(2,5)	(149,155)	152	18	23104
(3,4)	(166,164)	165	2	27225
(3,5)	(166,155)	160.5	60.5	25760.25
(4,5)	(164,155)	159.5	40.5	25440.25
	Total	1586	473	251681.5

i)E(($\bar{y}$)=$\bar{Y}$

$E(\bar{y})= \sum\bar{y} * \frac{1}{N_{C_n}}$

$= 1586.\frac{1}{10}$

$= 158.6$

$\bar{Y} = \frac{\sum yi}{N}$

$= \frac{793}{5}$

$= 158.6$

ii) $E(s^2)=S^2$

$E(s^2) \;=\; \sum s^2 * \frac{1}{N_{C_n}}$

$\frac{473}{10} = 47.3$

$S^2 = \frac{1}{N-1}[\sum y_i^2 - N\bar{Y}^2]$

$= \frac{1}{5-1}[125959-5*25153.96]$

$= 47.3$

iii) $V(\bar{y}) = E(\bar{y}^2) - [E(\bar{y})]^2$

$= \sum y_i^{2} * \frac{1}{N_{C_n}} - \bar{Y}^2$

$= 251681.5 * \frac{1}{10} - 25155.96$

$= 14.19$

$(N-n)S^2/N\,n = \frac{(5-2)*47.3}{5*2}$

$= \; 14.19$

Therefore, $V(\bar{y})=(N-n)S^2/Nn$

RESULT:-

i) $E((\bar{y})=\bar{Y} = 158.6$

ii) $E(s^2)=S^2 = \; 47.3$

iii) $V(\bar{y})=(N-n)S^2/Nn = \; 14.9$

11. A population consists of 4 units 10,11,12,13 a random sample of size 2 is taken under SRSWR.

i) Write down all possible samples.

ii) Verify that $E(\bar{y}) = \bar{Y}$

Answer:

Here N=4 and n=2.

Under SRSWR the total number of possible samples of size that can be drawn from a population of size N is N^n hence in this case we will get 4^2=16.Under SRSWR

sample	$\bar{y}$	sample	$\bar{y}$	sample	$\bar{y}$	sample	$\bar{y}$
(10,10)	10	(11,10)	10.5	(12,10)	11	(13,10)	11.5
(10,11)	10.5	(11,11)	11	(12,11)	11.5	(13,11)	12
(10,12)	11	(11,12)	11.5	(12,12)	12	(13,12)	12.5
(10,13)	11.5	(11,13)	12	(12,13)	12.5	(13,13)	13

Yi :10,11,12,13

$\bar{Y} = \frac{\sum Y}{N} \quad = 46/4$

$=11.5$

$E(\bar{y}) = \sum \bar{y}.\frac{1}{N^n}$

$= \frac{184}{16}$

$=11.5=\bar{Y}$

Hence, $E(\bar{y})= \bar{Y}$

12. For the samples drawn in the previous problem (question No.11), calculate sample mean square for all samples and verify that $E(s^2)= \sigma^2$, variance of the population.

Answer:

sample	$\bar{y}$	s^2	sample	$\bar{y}$	s^2	sample	$\bar{y}$	s^2	sample	$\bar{y}$	s^2
(10,10)	10	0	(11,10)	10.5	0.5	(12,10)	11	2	(13,10)	11.5	4.5
(10,11)	10.5	0.5	(11,11)	11	0	(12,11)	11.5	0.5	(13,11)	12	2
(10,12)	11	2	(11,12)	11.5	0.5	(12,12)	12	0	(13,12)	12.5	0.5
(10,13)	11.5	4.5	(11,13)	12	2	(12,13)	12.5	0.5	(13,13)	13	0

$\sum s^2$ =20

Yi :10,11,12,13

$\bar{Y} = \frac{\sum Y}{N}$ = 46/4

=11.5

$\sigma^2 = \frac{[\sum Y^2 - N\bar{Y}^2]}{N} = \frac{[534-4\ (11.5)(11.5)]}{4}$=1.25

E(s^2) =$\sum s^2 . \frac{1}{N^n}$

=20/16= 1.25= σ^2

Hence, E(s^2)= σ^2.

13. The units of a population are 205, 208, 212, 206, and 207. Draw a sample of size 3 by SRSWOR and show that

a) A sample mean is an unbiased estimate of a population mean

b) Sample mean square is unbiased for population mean square

Answer:

Under SRSWOR we can draw N_{C_n}possible samples i.e. 5_{C_3} =10 possible samples

Samples	Sample Mean $\bar{y}$	Sample Mean square s^2
(205,208,212)	208.3333	12.3333
(205,212,206)	207.6667	14.3334
(205,206,207)	206	1
(205,208,206)	206.3333	2.3334

(205,208,207)	206.6667	2.3334
(205,212,207)	208	13
(208,212,206)	208.6667	9.3334
(208,206,207)	207	1
(208,212,207)	209	7
(212,206,207)	208.3333	10.3334

Yi: 205,208,212,206,207

$\bar{Y} = \frac{\sum Y}{N}$ = 1038/5

=207.6

E($\bar{y}$)= $\sum \bar{y} * \frac{1}{^{N}C_{n}}$

= 2076X$\frac{1}{10}$

= 207.6

Hence, sample mean is an unbiased estimate of a population mean

$S^2 = \frac{1}{N-1}[\sum y_i^2 - N\bar{Y}^2]$=7.3

$E(s^2) = \sum s^2 * \frac{1}{^{N}C_{n}}$

$\frac{73.00025}{10}$ = 7.300025 =S^2

$E(s^2)=S^2$

Hence, Sample mean square is unbiased for population mean square

14. Obtain variance of the sample mean for the problem given in the previous question (question No.13).

Answer

$S^2 = \frac{1}{N-1}[\sum y_i^2 - N\bar{Y}^2]$= 7.3

$$V(\bar{y}) = \frac{(N-n)S^2}{Nn} = \frac{(5-3)*7.3}{5*3}$$

= 0.97333

15. A SRSWOR sample of size 50 houses was drawn from a village of 500 households. Only eight households possess a transistor radio out of. Estimate the total number of households possessing a transistor and standard error of your estimate.

Answer:

Let X denote the total number of households possessing a transistor in a village
N is the population size = 500
Sample size n =50

Let x denote the number of households possessing a transistor in the sample selected =8

Then the sample proportion p = x/n = 8/50=0.16

q = 1 – p = 0.84

$\hat{X} = Np$ = 500 * 0.16 = 80

Hence, an estimate of the total number of households possessing a transistor in a village is 80

Standard error of the estimate can be obtained as follows:

S.E ($\hat{X}$) = $\sqrt{N^2 V(p)}$

$$= \sqrt{N^2 \frac{(N-n)pq}{N(n-1)}} = \sqrt{(500)^2 \frac{(500-50)(0.16)(0.84)}{500(50-1)}}$$

=24.8424