Summary

- In probability theory, there exist several different notions of convergence of random variables

- A sequence $\{X_1, X_2, \dots\}$ of random variables is said to converge in distribution, or converge weakly, or converge in law to a random variable X if

$$\lim_{n\to\infty} F_n(x) = F(x)$$

  For every number x in R at which F is continuous. Here Fn and F are the cumulative distribution functions of random variables Xn and X correspondingly.

- Convergence in distribution may be denoted as

$$Xn \xrightarrow{d} X, Xn \xrightarrow{D} X, Xn \xrightarrow{L} X$$

- Convergence in distribution is the weakest of all of these modes of convergence
- The central limit theorem, one of the two fundamental theorems of probability, is a theorem about convergence in distribution
- The basic idea behind convergence in probability is that the probability of an “unusual” outcome becomes smaller and smaller as the sequence progresses
- A sequence $\{X_n\}$ of random variables converges in probability towards X if for all $\varepsilon > 0$

$$\lim_{n\to\infty} \Pr\left(|X_n - X| \geq \varepsilon\right) = 0$$

- Convergence in probability implies convergence in distribution
- Convergence in probability does not imply almost sure convergence
- In the opposite direction, convergence in distribution implies convergence in probability only when the limiting random variable X is a constant
- The continuous mapping theorem states that for every continuous function $g(\cdot)$, if $X_n \xrightarrow{P} X$, then $g(X_n) \xrightarrow{P} g(X)$