1. Introduction

Welcome to the series of E-learning modules on illustrations on construction of Confidence Intervals for variance and ratio of variances. In this module we are going cover the Interval Estimation procedure, to estimate the population variance when the population mean is known and unknown, and also ratio of two variances when the respective means are known and unknown through some practical examples.

By the end of this session, you will be able to:

- Understand interval estimation of variance and ratio of variances
- Describe the procedure to construct confidence interval for the population variance when mean is known and unknown through examples
- Demonstrate interval estimation technique for the estimation of ratio of two population variances to some practical problems
- Variability is a very important characteristic of data
- Measures of variability can help one to create a mental picture of the spread of the data
- One of the important measures of variability is variance
- An estimate of population variance is usually needed before you can make inferences about population means

Practical problems very often lead to estimation of sigma square, the variance of the population. A confidence interval for a variance specifies a range of values within which the unknown population parameter, in this case the variance, or ratio of variances may lie.

One hundred into (one minus alpha) percent confidence interval for the variance when the mean is known is given by,
[summation (xi minus mu) whole square by B , summation (xi minus mu) whole square by A] where B is equal to chi square (alpha by two) with n degrees of freedom and A is equal to chi square (one minus alpha by two) with n degrees of freedom

One hundred into (one minus alpha) percent Confidence Interval for the population variance sigma square when the mean is unknown is given by,[summation (yi minus y bar) whole square by B , summation (yi minus y bar) whole square by A]

But since s square is equal to summation yi minus y bar whole square by n minus one which implies summation (yi minus y bar) whole square is equal to (n minus one) into s square.

Hence hundred into (one minus alpha) percent Confidence Interval for the population variance sigma square when the mean is unknown is given by
[(n minus one) into s square by B , (n minus one) into s square by A]

Hence hundred into (one minus alpha) percent Confidence Interval for the ratio of variances when the means are known is computed as , [n into summation (xi minus mu one) whole square by B into m into summation (yi minus mu two) whole square, N into summation (xi

minus mu one) whole square by A into m into summation (yi minus mu two) whole square], where B is equal to F (alpha by two) with (m, n)degrees of freedom and A is equal to F (one minus alpha by two) with(m,n)degrees of freedom.

One hundred into (one minus alpha) percent Confidence Interval for the ratio of variances of two populations with unknown means is given by [s one square by B into ‘s’ two square, ‘s’ one square by A into ‘s’ two square], where B is equal to F (alpha by two) with (m minus one, n minus one)degrees of freedom and A is equal to F (one minus alpha by two) with(m minus one, n minus one)degrees of freedom.

2. Illustrative Examples

Illustrative examples:
Problem 1:
The following capacities were recorded from ten batteries of a new type.
One hundred forty six, one hundred forty one, one hundred thirty five, one hundred forty two, one hundred forty, one hundred forty three, one hundred thirty eight, one hundred thirty seven, one hundred forty two and one hundred thirty six.
Establish ninety percent confidence interval for the Standard deviation of the capacities of a new type of batteries.

Solution :
One hundred into (one minus alpha) percent Confidence Interval for the variance when the mean is unknown is,
[(n) into s n square by B , (n) into s n square by A] where s n square is equal to summation (x i minus x bar) whole square by n.
Given One hundred into (one minus alpha) percent is equal to ninety percent. This implies one minus alpha is equal to zero point nine zero implies alpha is equal to zero point one zero and alpha by two is equal to zero point zero five.

From the table of chi square distribution we get the values for A and B as follows, B is equal to chi square (alpha by two) with (n minus one) degrees of freedom . This is equal to Chi square zero point zero five with nine degrees of freedom which is equal to sixteen point nine one nine.
A is equal to chi square (one minus alpha by two) with (n minus one) degrees of freedom which is equal to chi square (zero point nine five) with nine degrees of freedom which is equal to two point zero eight eight. s n square is equal to summation (x i minus x bar) whole square by n which is equal to summation xi square by n minus x bar square which is equal to one lakh ninety six thousand one hundred and eight by ten minus one hundred and forty square which is equal to ten point eight.

Therefore ninety percent confidence interval for the variance when the mean is unknown is,
[ten into ten point eight by sixteen point nine one nine, ten into ten point eight by two point zero eight eight]his is equal to [six point three eight three three five six, fifty one point seven two for one four].
Hence ninety percent confidence interval for the variance is [six point three eight three three five six, fifty one point seven two four one four]
Ninety percent confidence interval for the standard deviation is [two point five two six five, seven point one nine two zero]

3. Examples 2 and 3

Problem 2:
A random sample of size ten is drawn from a Normal population with mean twenty two as given below. Obtain ninety eight percent Confidence interval for the variance.
The sample is as follows, twenty, sixteen, twenty six, twenty seven, twenty three, eighteen, twenty four, twenty five, nineteen and seventeen.

Solution:
Given n is equal to ten and mu is equal to twenty two.
One hundred into (one minus alpha) percent confidence interval for the variance when the mean is known is given by,
[summation (xi minus mu) whole square by B , summation (xi minus mu) whole square by A] where B is equal to chi square (alpha by two) with n degrees of freedom and A is equal to chi square (one minus alpha by two) with n degrees of freedom
Given One hundred into (one minus alpha) percent is equal to ninety eight percent implies one minus alpha is equal to zero point nine eight implies alpha is equal to zero point zero two and alpha by two is equal to zero point zero one.

From the table of chi square distribution we get the values for A and B as follows, B is equal to chi square (alpha by two) with (n) degrees of freedom which is equal to C hi square zero point zero one with ten degrees of freedom which is equal to twenty three point two zero nine.
A is equal to chi square (one minus alpha by two) with (n minus one) degrees of freedom which is equal to chi square (zero point nine nine) with ten degrees of freedom which is equal to two point five five eight.

Therefore ninety eight percent confidence interval for the variance when the mean is known is
[One forty five by twenty three point two zero nine, one forty five by two point five five eight] which is equal to [six point two four seven six, fifty six point six eight four nine.
Hence ninety eight percent confidence interval for the variance when the mean is known for the given data is [six point two four seven six, fifty six point six eight four nine].

Problem 3:
Two random samples from two Normal populations with means twenty two and thirty five have been selected. Assuming Normality, obtain ninety eight percent confidence interval for the ratio of variances. The observations of sample one is, twenty, sixteen, twenty six, twenty seven, twenty three, twenty two, eighteen, twenty four, twenty five and nineteen.
The observations of sample two are twenty seven, thirty three, forty two, thirty five, thirty two, thirty four, thirty eight, twenty eight, forty one, forty three and thirty.

Solution:
One hundred into (one minus alpha) percent confidence interval for the ratio of variances when the means are known is computed as
[n into summation , i runs from one to m, (xi minus mu one) whole square by B into m into summation, i runs from one to n, (yi minus mu two) whole square, n into summation, i runs from one to m, (xi minus mu one) whole square by A into m into summation, i runs from one

to n, (yi minus mu two) whole square]
Where B is equal to F (alpha by two) with (m, n) degrees of freedom and A is equal to F (one minus alpha by two) with (m, n) degrees of freedom.

Given mu one is equal to twenty two , mu two is equal to thirty five , m is equal to ten , n is equal to twelve.
Given One hundred into (one minus alpha) percent is equal to ninety eight percent implies one minus alpha is equal to zero point nine eight implies alpha is equal to point zero two and alpha by two is equal to zero point zero one.
From the table of F distribution we get the values for A and B as follows, B is equal to F (alpha by two) with (m, n)degrees of freedom which is equal to F zero point zero one (ten , twelve) degrees of freedom which is equal to four point three three.
A is equal to F (one minus alpha by two) with(m,n)degrees of freedom which is equal to one by F (alpha by two) with (m, n)degrees of freedom which is equal to one by B which is equal to zero point two three.

For the given data,
Summation , i runs from one to m, (xi minus mu one) whole square is equal to summation , i runs from one to m, (xi minus twenty two) whole square which is equal to one hundred and twenty.
Summation , i runs from one to n, (yi minus mu two) whole square is equal to summation , i runs from one to n, (yi minus thirty five) whole square which is equal to three hundred and fourteen.

Therefore ninety eight percent Confidence interval for the ratio of variances of two populations with known means is given by
[twelve into one hundred and twenty by (four point three three) into ten into three hundred and fourteen, twelve into one twenty by (zero point two three) into ten into three hundred and fourteen] which is equal to [zero point one zero five nine, one point nine eight five]

4. Example 4

Problem 4:

Life expectancy in nine regions of Brazil in nineteen hundred and in eleven regions of Brazil in nineteen seventy is given in the table below. Assuming Normality find ninety eight percent confidence interval for the ratio of the variances sigma one square by sigma two square where sigma one square and sigma two square are the variances of the population in nineteen hundred and nineteen seventy.
Life expectancy in nineteen hundred is as follows, forty two point seven, forty three point seven, thirty four, thirty nine point two, forty six point one, forty eight point seven, forty nine point four, forty five and fifty five point three.
Life expectancy in nineteen seventy is as follows fifty four point two, fifty point four, forty four point two, forty nine point seven, fifty five point four, fifty seven, fifty eight point two, fifty six point six, sixty one point nine, fifty seven point five and fifty three point four.

Solution:
One hundred into (one minus alpha) percent Confidence interval for the ratio of variances of two populations with unknown means is given by
[s one square by B into s two square, s one square by A into s two square]
Where, s one square is equal to summation (x i minus x bar) whole square by (m minus one) which is equal to one by (m minus one) into summation xi square minus m into x bar square].
S two square is equal to summation (y i minus y bar) whole square by (n minus one) which is equal to one by (n minus one) into summation yi square minus n into y bar square].

For the given data, summation xi square is equal to eighteen thousand five hundred and twenty seven point seven eight, summation yi square is equal to thirty two thousand seven hundred and ninety nine point nine one, x bar is equal to forty five , y bar is equal to fifty four point four zero nine.
Therefore s one square is equal to one by (nine minus one) into eighteen thousand five hundred and twenty seven point seven eight minus nine into (forty five) square] equal to one by eight into eighteen thousand five hundred and twenty seven point seven eight minus nine into two thousand and twenty five is equal to thirty seven point eight four five.
s two square is equal to one by (eleven minus one) into thirty two thousand seven hundred and ninety nine point nine one minus eleven into (fifty four point four zero nine square] equal to one by ten into thirty two thousand seven hundred and ninety nine point nine one minus eleven into (two thousand nine hundred and sixty point three three nine which is equal to twenty three point six zero five .

Given One hundred into (one minus alpha) percent is equal to ninety eight percent implies alpha by two is equal to point zero one.
From the table of F distribution we get the values for A and B as follows
B is equal to F (alpha by two) with (m minus one, n minus one)degrees of freedom equal to F point zero one (eight, ten) degrees of freedom equal to five point zero six.
A is equal to F (one minus alpha by two) with(m minus one, n minus one)degrees of freedom, which is equal to one by B equal to zero point one nine seven six.

Therefore ninety eight percent confidence interval for the ratio of variances of two populations with unknown means is given by,
[thirty seven point eight four five by (twenty three point ix zero five) into

(five point zero six), thirty seven point eight four five by (twenty three point six zero five) into (zero point one nine seven six) which is equal to zero point three one six eight , [eight point one one three six]
Hence ninety eight percent confidence interval for the ratio of variances of two populations is [zero point three one six eight, eight point one one three six].

5. Example 5

Problem 5:
You randomly select and weigh thirty samples of an allergy medication. The sample standard

deviation is one point two milligrams. Assuming the weights are normally distributed, construct ninety nine percent confidence intervals for the population variance and standard deviation.

Solution:
Given n is equal to thirty , s is equal to one point two and (one minus alpha) is equal to ninety nine percent which is equal to zero point nine nine.
Alpha is equal to one minus zero point nine nine which is equal to zero point zero one.
Then alpha by two is equal zero point zero zero five and one minus alpha by two is equal to zero point nine nine five.
From the table of Chi square distribution B is equal to chi square point zero zero five (with twenty nine degrees of freedom) is equal to fifty two point three three six.
A is equal to chi square point nine nine five (with twenty nine degrees of freedom) is equal to thirteen point one two one.

Hence hundred into one minus alpha percent confidence interval for the population variance sigma square when the mean is unknown is given by
[(n minus one) into s square by B, (n minus one) into s square by A]. This is equal to[zero point seven nine eight, three point one eight three].
So, with ninety nine percent confidence, you can say that the population variance is between zero point seven nine eight and three point one eight three. The population standard deviation is between zero point eight nine and one point seven eight milligrams.

Here's a summary of our learning in this session where we have :

- Obtained confidence limits for the unknown population variance when mean mu is known and unknown
- Derived confidence limits for the ratio of two population variances when the respective means are known and unknown