Frequently Asked Questions

1. List the various interval estimators of the population variance and ratio of two population variances?
   Answer:

   a) 100 (1- α) % C.I for the population variance σ^2 when the mean is known as μ is given by

   [∑ (yi-μ)^2 / B , ∑(yi-μ)^2 / A] where B= $\chi^2_{\alpha/2}(n)$ and

   A= $\chi^2_{(1-\alpha/2)}(n)$

   b) 100 (1- α) % C.I for the population variance σ^2 when the mean is unknown is given by

   [∑(yi- $\bar{y}$)^2 / B , ∑(yi- $\bar{y}$)^2 / A] where values B= $\chi^2_{\alpha/2}(n-1)$ and A= $\chi^2_{(1-\alpha/2)}(n-1)$

   C) 100 (1- α) % C.I for the ratio of variances of two populations with unknown means is given by

   [$\frac{s_1^2}{Bs_2^2}$, $\frac{s_1^2}{As_2^2}$] Where B= $F_{\alpha/2}(m,n)$ and A= $F_{(1-\alpha/2)}(m,n)$

   d) Then the confidence interval for the ratio of variances when the means are known is computed as

   $$\left[\frac{n\sum_{i=1}^{m}(x_i-\mu_1)^2}{Bm\sum_{i=1}^{n}(y_i-\mu_2)^2} , \frac{n\sum_{i=1}^{m}(x_i-\mu_1)^2}{Am\sum_{i=1}^{n}(y_i-\mu_2)^2} \right]$$

   Where B= $F_{\alpha/2}(m-1,n-1)$ and A= $F_{(1-\alpha/2)}(m-1,n-1)$

2. Establish 95% confidence interval for the variance of a normal population with unknown mean, if the random sample of 16 values from it has standard deviation 3 and mean 12.8
   Answer:

   100(1-α) % CI for the variance when the mean is unknown is $[\frac{ns_n^2}{B}, \frac{ns_n^2}{A}]$

Given 100(1-α) %=90% implies 1- α=0.90 implies α=.10 and α/2=0.05

From the table of χ^2 distribution we get the values for A and B as follows

$$B = \chi^2_{\alpha/2}(n-1) = \chi^2_{0.05}(9) = 16.919$$

$$A = \chi^2_{1-\alpha/2}(n-1) = \chi^2_{0.99}(9) = 2.088$$

$$s_n^2 = \frac{\sum_{i=1}^{n}(xi-\bar{x})^2}{n} = \frac{\sum xi^2}{n} - \bar{x}^2 = \frac{196108}{10} - (140)^2 = 10.8$$

Therefore 90% CI for the variance when the mean is unknown is

$$[\frac{ns_n^2}{B}, \frac{ns_n^2}{A}] = [\frac{10*10.8}{16.919}, \frac{10*10.8}{2.088}] = [6.383356, 51.72414]$$

Hence 90% CI for the variance is $[6.383356, 51.72414]$

3. An experimenter is convinced that her measuring instrument had variability. During the experiment she recorded the measurements 4.1, 5.2 1nd 10.2.Construct 90% confidence interval to estimate the true value of the population variance.
Answer:

Given n=3, and (1-α) = 90%=0.90 then α= 1-0.90 =0.10

Then α/2=0.05 and 1- α/2=0.95

From the table of Chi square distribution B= $\chi^2_{0.05}(2) =$ 5.99147

and A= $\chi^2_{0.95}(2) =$ 0.102587

s^2 =10.57

Hence 100 (1- α) % C.I for the population variance σ^2 when the mean is unknown as μ is given by

[$(n-1)s^2$ / B, $(n-1)s^2$ / A] = [(3-1) 10.57/5-99147, (3-1)10.57/0.102587]

[3.53, 206.07]

So, with 90% confidence, we can say that the population variance is between 3.53and 206.07. This very wide CI indicates how little information on the population variance is obtained from a sample of only three measurements.

4. A random sample of size 25 from the population gives the sample Standard deviation to be 8.5. Obtain 95% Interval estimate for the population Standard deviation.
Answer:

Given, n=25, s =8.5 AND s^2 =72.25

Hence 100 (1- α) % C.I for the population variance σ^2 when the mean is unknown is given by

$[(n-1)s^2 /$ B , $(n-1)s^2 /$ A]

100 (1- α) %=95%

1 -α = 0.95 which implies α=0.05 ,α/2=0.025 and 1-α/2=.975

B= $\chi^2_{\alpha/2}(n-1) = \chi^2_{0.025}(24) =$ 39.3641

$A = \chi^2_{(1-\alpha/2)}(n-1) = \chi^2_{(0.975)}(24) =$ 12.4011

By substituting the given values in the above we get

[(25-1) 72.25/39.3641, (25-1) 72.25/12.4011] = [44.05,139.83]

The 95% confidence interval for the population standard deviation is between 0.54 and 11.83

5. An experimenter is concerned that the variability of responses using 2 different experimental procedures may not be the same. Before conducting his research he conducts a pre study with a random sample of 10 and 8 responses and gets s_1^2 =7.14 and s_2^2 =3.21 respectively. Construct 90% CI for σ_1^2 / σ_2^2
Answer:

Therefore 100 (1- α) % C.I for the ratio of variances of two populations with unknown means is given by:

[$\frac{s_1^2}{Bs_2^2}$, $\frac{s_1^2}{As_2^2}$] where s_1^2 =7.14 and s_2^2 =3.21

Given 100(1- α) %=90%. Then α=0.1 and α/2= 0.05

From the table of F – distribution

B= $F_{\alpha/2}(m-1,n-1) = F_{0.05}(9,7)=3.68$ and $A = 1/B = 0.2717$

Substituting these values into the formula for CI we get

$$\left[\frac{7.14}{3.68(3.21)}, \frac{7.14}{0.2717(3.21)}\right] = [0.6044, 8.18]$$

6. Among 11 patients in a certain study, the standard deviation of the property of interest was 5.8. In another group of 4 patients, the standard deviation was 3.4. Construct a 95 percent confidence interval for the ratio of the standard deviations of these two.

   Answer:

   Given: $s_1 = 5.8$, $s_2 = 3.4$, $m=11$, $n=4$

   Therefore 100 (1- α) % C.I for the ratio of variances of two populations with unknown means is given by:

$$\left[\frac{s_1^2}{Bs_2^2}, \frac{s_1^2}{As_2^2}\right] \text{ where } s_1^2 = 33.64 \text{ and } s_2^2 = 11.56$$

Given 100(1- α) %=95%. Then α=0.05 and α/2= 0.025

From the table of F – distribution,

B= $F_{\alpha/2}(m-1, n-1) = F_{0.025}(10, 3) = 14.42$ and $A = F_{1-\alpha/2}(m-1, n-1) = F_{0.975}(10, 3) = 1/B = 0.0694$

Substituting these values into the formula for CI we get

$$\left[\frac{33.64}{14.42(11.56)}, \frac{33.64}{0.0694(11.56)}\right] = [0.2018, 41.93]$$

Therefore 95 % C.I for the ratio of standard deviations of two populations with unknown means is given by: [0.4492, 6.4754]

7. The following are the length of 18 items manufactured by a company. Assuming Normality obtain 98% confidence interval for the variance of the population. From the CI what can you say about the hypothesis $H_0: \sigma^2 = 0.5$? the observation is :
   18.51, 18.1, 18.61, 18.32, 18.33, 18.46, 18.12, 18.34, 18.57, 18.22, 18.63, 18.43, 18.37, 18.64, 18.58, 18.34, 18.43, 18.63

   Answer:

100(1-α) % CI for the variance when the mean is unknown is $[\frac{ns_n^2}{B}, \frac{ns_n^2}{A}]$ where

$$s_n^2 = \frac{\sum_{i=1}^{n}(xi - \bar{x})^2}{n}$$

Given 100(1-α) %=98% implies 1- α=0.98 implies α=.02 and α/2=0.01

From the table of χ^2 distribution we get the values for A and B as follows

$$B = \chi^2_{\alpha/2}(n-1) = \chi^2_{0.01}(17) = 33.409$$

$$A = \chi^2_{1-\alpha/2}(n-1) = \chi^2_{0.99}(17) = 6.408$$

$$s_n^2 = \frac{\sum_{i=1}^{n}(xi - \bar{x})^2}{n} = \frac{\sum xi^2}{n} - \overline{xn}^2 = 0.02915$$

Therefore 98% CI for the variance when the mean is unknown is

$$[\frac{ns_n^2}{B}, \frac{ns_n^2}{A}] = [\frac{18 * 0.02915}{33.409}, \frac{18 * 0.02915}{6.408}] = [0.015, 0.08]$$

We can observe that in the estimated interval 0.5 is not an interior point. Therefore we reject the hypothesis H_0:σ^2 =0.5 at 2% level of significance.

Hence 98% CI for the variance is $[0.015, 0.08]$

8. A random sample of size 10 is drawn from a Normal population with mean 22 as given below. Obtain 98% Confidence interval for the variance. The observation is : 20, 16, 26, 27, 23, 18, 24, 25, 19, 17

Answer:

Given n=10 and μ=22

100(1-α) % CI for the variance when the mean is known is

$[\frac{\sum_{i=1}^{n}(xi - \mu)^2}{B}, \frac{\sum_{i=1}^{n}(xi - \mu)^2}{A}]$ Where $B = \chi^2_{\alpha/2}(n)$ and $A = \chi^2_{1-\alpha/2}(n)$

Given 100(1-α) %=98% implies 1- α=0.98 implies α=.02 and α/2=0.01

From the table of χ^2 distribution we get the values for A and B as follows

$$B = \chi^2_{\alpha/2}(n) = \chi^2_{0.01}(10) = 23.209$$

$$A = \chi^2_{1-\alpha/2}(n) = \chi^2_{0.99}(10) = 2.558$$

Therefore 98% CI for the variance when the mean is known is

$$[\frac{\sum_{i=1}^{n}(xi-\mu)^2}{B}, \frac{\sum_{i=1}^{n}(xi-\mu)^2}{A}] = [\frac{145}{23.209}, \frac{145}{2.558}] = [6.2476, 56.6849]$$

Hence 98% CI for the variance when the mean is known for the given data is:

[6.2476, 56.6849]

9. One night New York City was victimized by a power failure most of the homes were without power for 12 hours or more. 25 homes were approached and asked how long they were without power. It was observed that the variance of the sample was 4. Find 98% CI for its population variance in New York City. What is your conclusion regarding $H_0:\sigma^2 > 2.3$?

Answer:

Given n=25 and $s_n^2=4$

100(1-α) % CI for the variance when the mean is unknown is $[\frac{(n-1)s_n^2}{B}, \frac{(n-1)s_n^2}{A}]$

where $s_n^2 = \frac{\sum_{i=1}^{n}(xi-\bar{x})^2}{n-1}$

Given 100(1-α) %=98% implies 1- α=0.98 implies α=.02 and α/2=0.01

From the table of χ^2 distribution we get the values for A and B as follows

$$B = \chi^2_{\alpha/2}(n-1) = \chi^2_{0.01}(24) = 42.98$$

$$A = \chi^2_{1-\alpha/2}(n-1) = \chi^2_{0.99}(24) = 10.856$$

Therefore 98% CI for the variance when the mean is unknown is

$$[\frac{(n-1)s_n^2}{B}, \frac{(n-1)s_n^2}{A}] = [\frac{(25-1)4}{42.98}, \frac{(25-1)4}{10.856}] = [2.233, 8.843]$$

Since 2.3 is an interior point of an estimated interval we accept the hypothesis $H_0:\sigma^2 > 2.3$ at 2% level of significance

Hence 98% CI for the population variance is [2.233, 8.843]

10. The mean and variance of heights of 13 men are 172.7 cms and 12 cms respectively. The mean and variance of 9 women are 163.9 cm and 10.5 cm respectively. Construct 98% confidence interval for the ratio of the variances of heights of men to variance of heights of women.

Answer:

Given: $s_1^2 = 12$, $s_2^2 = 10.5$, m=13, n=9

Therefore 100 (1- α) % C.I for the ratio of variances of two populations with unknown means is given by

$$[\frac{s_1^2}{Bs_2^2}, \frac{s_1^2}{As_2^2}]$$ where $s_1^2 = 12$, $s_2^2 = 10.5$

Given 100(1- α) %=98%. Then α=0.02 and α/2= 0.01

From the table of F – distribution

$B = F_{\alpha/2}(m-1, n-1) = F_{0.01}(12, 8) = 5.67$ and $A = F_{1-\alpha/2}(m-1, n-1) = F_{0.99}(12, 8) = 1/B = 0.1764$

Substituting these values into the formula for CI we get

$$[\frac{12}{5.67(10.5)}, \frac{12}{0.1764(10.5)}] = [0.2016, 6.4788]$$

Therefore 98 % C.I for the ratio of the variances of heights of men to variance of heights of women is [0.2016, 6.4788]

11. The feeding habits of two-species of net-casting spiders are studied. The species, the *deinopis* and *menneus*, coexist in eastern Australia. The following summary statistics were obtained on the size, in millimeters, of the prey of the two species. Estimate, with 95% confidence, the ratio of the two population variances.

Adult Deinopis	Adult Menneus
m=10	n = 10
$\overline{x} = 10.26$	$\overline{y} = 9.02$
$S_x^2 = (2.51)^2$	$S_y^2 = (1.90)^2$

Answer:

In order to estimate the ratio of the two population variances, we need to obtain two *F*-values from the *F*-table, namely:

B= F $_{\alpha/2}$(m-1, n-1) = $F_{0.025}$ (9, 9) =4.03 and A = $F_{0.975}$ (9, 9) =1/B = 0.2481

Therefore 100 (1- α) % C.I for the ratio of variances of two populations with unknown means is given by

$$\left[\ \frac{s_x^2}{Bs_y^2}\ ,\ \frac{s_x^2}{As_y^2}\ \right]$$ where $S_x^2 = (2.51)^2$ and $s_y^2 = (1.90)^2$

Then, the 95% confidence interval for the ratio of the two population variances is:

$$\left[\ \frac{(2.51)^2}{4.03(1.90)^2}\ ,\ \frac{(2.51)^2}{0.2481(1.90)^2}\ \right]$$

Simplifying, we get: [0.433,7.033]

That is, we can be 95% confident that the ratio of the two population variances is between 0.433 and 7.033. (Because the interval contains the value 1, we cannot conclude that the population variances differ.)

12. The following are the weights of 10 college students with the population mean 65. Assuming Normality obtain 90% confidence intervals for the S.D. the observation is :
65.5, 49.6, 81.3, 54.5, 39.9, 62.9, 59.6, 73.4, 78.6, 64.5
Answer:

Given n=10 and μ=22

100(1-α)% CI for the variance when the mean is known is

$$\left[\frac{\sum_{i=1}^{n}(xi-\mu)^2}{B},\frac{\sum_{i=1}^{n}(xi-\mu)^2}{A}\right]$$ Where $B = \chi^2_{\alpha/2}(n)$ and $A = \chi^2_{1-\alpha/2}(n)$

Given 100(1-α)%=90% implies 1- α=0.90 implies α=.1 and α/2=0.05

From the table of χ^2 distribution we get the values for A and B as follows

$$B = \chi^2_{\alpha/2}(n) = \chi^2_{0.05}(10) = 18.307$$

$$A = \chi^2_{1-\alpha/2}(n) = \chi^2_{0.95}(10) = 3.940$$

Therefore 90% CI for the variance when the mean is known is

$$[\frac{\sum_{i=1}^{n}(xi-\mu)^2}{B}, \frac{\sum_{i=1}^{n}(xi-\mu)^2}{A}] = [\frac{153.27}{18.307}, \frac{153.27}{3.940}] = [8.372, 38.9010]$$

Hence 90% CI for the variance when the mean is known for the given data is $[8.372, 38.9010]$

Therefore 90% CI for the S.D when the mean is known for the given data is $[2.893, 6.2371]$

13. A random sample of 10 students is selected from a college and their weights are noted down. Set up 99% Confidence Interval estimate for the standard deviation of the weights of the students. The observation is :38,40,45,53,47,43,55,48,52,49
Answer:

Given n=10 and μ=22

100(1-α) % CI for the variance when the mean is known is

$$[\frac{\sum_{i=1}^{n}(xi-\mu)^2}{B}, \frac{\sum_{i=1}^{n}(xi-\mu)^2}{A}]$$ Where $B = \chi^2_{\alpha/2}(n)$ and $A = \chi^2_{1-\alpha/2}(n)$

Given 100(1-α) %=90% implies 1- α=0.90 implies α=.1 and α/2=0.05

From the table of χ^2 distribution we get the values for A and B as follows

$$B = \chi^2_{\alpha/2}(n) = \chi^2_{0.05}(10) = 18.307$$

$$A = \chi^2_{1-\alpha/2}(n) = \chi^2_{0.95}(10) = 3.940$$

Therefore 90% CI for the variance when the mean is known is

$$[\frac{\sum_{i=1}^{n}(xi-\mu)^2}{B}, \frac{\sum_{i=1}^{n}(xi-\mu)^2}{A}] = [\frac{153.27}{18.307}, \frac{153.27}{3.940}] = [8.372, 38.9010]$$

Hence 90% CI for the variance when the mean is known for the given data is $[8.372, 38.9010]$

Therefore 90% CI for the S.D when the mean is known for the given data is $[2.893, 6.2371]$

14. A random sample of size 15 from a Normal population is found to have variance 16.4. Find a 95% C.I for the population variance.

Answer:

Given n=15 and s_n^2 =16.4

100(1-α) % CI for the variance when the mean is unknown is $[\frac{ns_n^2}{B}, \frac{ns_n^2}{A}]$ where

$$s_n^2 = \frac{\sum_{i=1}^{n}(xi-\bar{x})^2}{n}$$

Given 100(1-α) %=95% implies 1- α=0.95 implies α=.05 and α/2=0.025

From the table of χ^2 distribution we get the values for A and B as follows

$$B = \chi^2_{\alpha/2}(n-1) = \chi^2_{0.025}(14) = 26.119$$

$$A = \chi^2_{1-\alpha/2}(n-1) = \chi^2_{0.975}(14) = 5.629$$

Therefore 95% CI for the variance when the mean is unknown is

$$[\frac{ns_n^2}{B}, \frac{ns_n^2}{A}] = [\frac{15*16.4}{26.119}, \frac{15*16.4}{5.629}] = [9.42, 43.7]$$

Hence Population variance lies between 9.42 and 43.7

15. Two random samples from two Normal populations with means 22 and 35 have been selected. Assuming Normality obtain 98% confidence interval for the ratio of variances.

Sample I	20	16	26	27	23	22	18	24	25	19		
Sample II	27	33	42	35	32	34	38	28	41	43	30	37

Answer:

100 (1- α) % confidence interval for the ratio of variances when the means are known is computed as :

$$\left[\frac{n\sum_{i=1}^{m}(x_i-\mu_1)^2}{Bm\sum_{i=1}^{n}(y_i-\mu_2)^2} \quad , \quad \frac{n\sum_{i=1}^{m}(x_i-\mu_1)^2}{Am\sum_{i=1}^{n}(y_i-\mu_2)^2} \right]$$

Given μ1= 22, μ2=35, m=10, n=12

Given 100(1-α) %=98% implies 1- α=0.98 implies α=.02 and α/2=0.01

From the table of F-distribution we get the values for A and B as follows

$$B = F_{\alpha/2}(m,n) = F_{0.01}(10,12) = 4.33$$

$$A = F_{1-\alpha/2}(m,n) = 1/F_{\alpha/2}(m,n) = 1/B = 0.23$$

For the given data $\sum_{i=1}^{m}(x_i-\mu_1)^2 = \sum_{i=1}^{m}(x_i-22)^2 = 120$

$$\sum_{i=1}^{n}(y_i-\mu_2)^2 = \sum_{i=1}^{n}(y_i-35)^2 = 314$$

Therefore 98 % C.I for the ratio of variances of two populations with known means is given by

$$\left[\frac{n\sum_{i=1}^{m}(x_i-\mu_1)^2}{Bm\sum_{i=1}^{n}(y_i-\mu_2)^2} \quad , \quad \frac{n\sum_{i=1}^{m}(x_i-\mu_1)^2}{Am\sum_{i=1}^{n}(y_i-\mu_2)^2} \right]$$

$$=\left[\frac{12(120)}{(4.33)(10)(314)} , \frac{12(120)}{(0.23)(10)(314)} \right]$$

= [0.1059, 1.985]