1. Introduction

Welcome to the series of E-learning modules on Point Estimation. In this module, we are going to cover the basic concept of point estimation, essentials of best point estimates, methods of point estimation and limitations and applications of point estimation.

By the end of this session, you will be able to:

- Explain the types of estimation
- Explain the role of point estimation
- Explain the criteria for the best point estimates
- Explain the methods of obtaining point estimates
- List the limitations and applications of point estimation

Point Estimation is to explore the possibility of making inferential statements about a population based on the information contained in a random sample.

This is a diagrammatic presentation of the estimation process. That is from a population a random sample is drawn (i.e. each and every unit of the population has an equal chance of being selected in the sample). Function of the population observations is known as a parameter, whereas that of a sample is known as a statistic. Estimation process is used to estimate the parameters using statistic. We have to make an appropriate choice of the statistic.

One of the prime techniques of statistics is sampling, whose object is to study the features of the population based on the sample observations. A careful selection of the sample is expected to reveal these features and hence we shall infer about the population from a statistical analysis of the sample. This process is known as Statistical Inference.
Hence, by statistical inference we mean that certain conclusions drawn about certain parameters of the population under consideration based on the study of sample observations.

There are two types of problems in statistical inference.
Firstly, we may have no information about some characteristics of the population, especially the values of the parameters in the distribution and it is required to obtain the estimates of these parameters. The true parameter will be unknown and one objective of sampling could be to estimate its value.This is a problem of estimation.
Secondly, some information or hypothetical values of the parameters may be available and it is required to test how far the hypothesis is tenable in the light of information provided by the sample. This is the problem of test of hypothesis or tests of significance given point estimator (s) from samples. We may wish to infer about the reproducibility of results, or if any statistical differences exist.

Example:
Suppose you measure two samples.
Common Question: Is it reasonable to conclude that no statistically significant difference exists?
Mu1 cap minus mu2 cap or sigma1 cap square minus sigma 2 cap square

2. Theory of Estimation and Two Types of Estimation

Theory of Estimation:
An estimator of a population parameter is a random variable that depends on the sample information whose realizations provide approximations to this unknown parameter. A specific realization of that random variable is called an estimator and estimate.

Let us consider the estimation of mean income of all the families in a neighbourhood based on the random sample of 20 families. It seems reasonable to base our conclusion on the sample mean income, so we say that the estimator of the population mean is the sample mean. Let us consider that the sample mean is x bar. Naturally, x bar is the estimator and the value of this sample mean is the estimate of the population mean (mu).

In a population (Random variable), the distribution has a known mathematical form involving an unknown parameter theta. Suppose a random sample $x_{1,}$ x_2, up to x_n of size n is drawn from the population whose distribution involves an unknown parameter theta. It is required to find an estimate of theta based on sample values.

The estimation is done in two different ways:

- Point Estimation and
- Interval Estimation

In point estimation, the estimated value is given by a single quantity, which is the function of sample observations (that is a statistic). This function is called the estimator of the parameter and the value of the estimator is called an estimate.

In interval estimation, an interval within which the parameter is expected to lie is given by using two quantities based on the sample values. This is known as a confidence Interval.
A number of functions or statistic (such as mean, median, minimum of observations, etc.) can be defined based on the random samples drawn from a population, which involves an unknown parameter theta and one of the parameters can be taken as an estimator of the unknown parameter theta.

Each value of each function is used in estimating the parameter, which is the point estimate of theta. In general, an estimate of an unknown parameter is called as a point estimate.

Estimator of a population parameter is a function of the sample information that yields a single number. The corresponding realization is called the point estimate of the parameter. That means when we provide a single numerical estimate of the population parameter (theta) based on the sample information, it is known as point estimation.

Hence, statisticians use sample statistics to estimate population parameters. For example, sample means are used to estimate population means and sample proportions to estimate population proportions.

As an example of a point estimate, assume you wanted to estimate the mean time it takes for the12-year-olds to run 100 yards. The mean running time of a random sample of 12-year-old would be an estimate of the mean running time for all 12-year-olds. Thus, the sample mean, x bar, would be a point estimate of the population mean, mu.

3. Criteria for a Good Estimator

Point Estimation- Criteria for Good Estimators:
Many functions of sample observations may be proposed as estimators of the same parameter. For example, either the mean or median or mode of the sample values may be used to estimate the parameter mu of the normal population. Apparently, we would like to use the one, which has the smallest error.

Unfortunately, we have no way to calculate what the error is for a particular estimator (if we could calculate the error exactly, we could compensate for it exactly and there would be no further need for statistical analysis), and so there is no direct way to decide which is the best estimator to use in a specific situation.

Goal of point estimation is to obtain the "best estimator". However, statisticians have developed some criteria, which they find useful in deciding which estimator might be the more advantageous in specific circumstances.

According to R. A. Fisher the criteria for a good estimator are:
i) Unbiasedness
If an estimator *tn* estimates *theta,* then difference between them (*tn minus theta)* is called the estimation error. Bias of the estimator is defined as the expectation value of this difference.
B of theta is equal to Expected value of (tn minus theta), which is equal to Expected value of tn minus theta.
If the bias is equal to zero, then the estimation is called unbiased. For example, sample mean is an unbiased estimator.
ii) Consistency
Note that estimator is a sample statistic. That is, it is a function of the sample elements. For many estimators variance of the sampling distribution of an estimator decreases as sample size increases. We would like that estimator stays as close as possible to the parameter it estimates as sample size increases.
The property of consistency is a limiting property.
iii) Efficiency
Estimators whose sampling distributions have smaller variances are considered to be superior. The idea here is that different random samples will give different values of the estimator, consistent with the sampling distribution.
iv) Sufficiency
An estimator is sufficient if it makes so much use of information in the sample that no other estimator could extract additional information from the sample about the population parameter being estimated.

Statistical theory concerned with the properties of estimators. That is, with defining properties that can be used to compare different estimators for the same quantity, based on the same data. Such properties can be used to determine the best rules to use under given circumstances.

However, in Robust Statistics, the statistical theory considers the balance between having good properties, if rightly defined assumptions hold, and having less good properties that hold under wider conditions.

How well a specific estimator satisfies each of these criteria depends very much on the details of the population distribution. Evaluating specific estimators in relation to these criteria often involves mathematical techniques. Some estimators are better estimators than others are. Fortunately, we can evaluate the quality of a statistic as an estimator by using the above four criteria.

Problem of statistics is not to find estimates but to find estimators. Estimator is not rejected because it gives one bad result for one sample. It is rejected when it gives bad results in a long run. That is, if it gives bad result for many, many samples. Estimator is accepted or rejected depending on its sampling properties. Estimator is judged by the properties of the distribution of estimates it gives rise.

In general, it is not possible for an estimator to have all these properties.

Since estimator gives rise to an estimate that depends on sample points (x1, x2, up to xn), estimate is a function of sample points. Sample points are random variable. Therefore, estimate is random variable and has probability distribution.

Let (x1, x2, up to xn) be a sample of size n drawn from a given population. Then, a function t is equal to t of (x1, x2, up to xn) of the random sample can be taken as an estimator of the unknown parameter theta.

T is equal to t of (x1, x2, up to xn), the value of the function is an estimate of theta, where (x1, x2, up to xn) are the sample observations.

In general, theta may be a vector of real numbers and it can be written as theta curl is equal to theta 1, theta2, up to theta k. (Often k is equal to 1). Sometimes, statistics are defined to estimate a function phi of theta of theta.

4. Methods of Point Estimation and Types of Variables

Methods of Point Estimation
Point estimation refers to the process of estimating a parameter from a probability distribution based on observed data from the distribution. There are many methods available for the estimation of the population parameters using point estimation method. Some of them are:

- Method of maximum Likelihood
- Method of Moments
- Method of minimum variance
- Method of Chi-square
- Bayesian's estimators etc.

Hence, a "point estimate" is a one-number summary of data.
For practical examples:

- Dose finding trials: Maximum Tolerable Dose (MTD)
- Safety and Efficacy Trials: response rate, median survival

Types of variables
Already we are familiar with the types of variables that we use in statistical analysis. The point estimate we choose depends on the "nature" of the outcome of interest. Some of the point estimates suggested for different types of variables are:

- Continuous Variables
  - Examples: Change in tumour volume or tumour diameter
  - Commonly used point estimates: mean, median

- Binary Variables
  - Examples: response, progression, greater than fifty percent reduction in tumour size
  - Commonly used point estimate: proportion, relative risk, odds ratio

- Time-to-Event (Survival) Variables
  - Examples: time to progression, time to death, time to relapse
  - Commonly used point estimates: median survival, *k*-year survival, hazard ratio

- Other types of variables: nominal categorical, ordinal categorical
  - Mode for nominal and median for ordinal categorical variables

Some of the potential point estimators:
Mean - population parameter is mu and the point estimate is x bar
Variance - population parameter is sigma square and the point estimate is s square
Proportion - population parameter is phi or P and the point estimate is p
correlation coefficient- population parameter is rho and the point estimate is r
Similarly,

For the difference between the two proportions - population parameter is phi 2 minus phi 1 or (P2-P1) and the point estimate is (p2 minus p1)

Last two entries of this table would be used when we want to compare two populations, a very important type of problem in statistics. By now, other entries should be quite familiar to you. Notice that all of the point estimators, theta cap, listed in this table are necessarily random variables, because they are statistics for random samples. Since such random variables have distributions with some width (that is, different random samples will generally give different values of theta cap), we realize that the determination of a value of theta cap does not guarantee us that we have the exact value of the corresponding population parameter, theta.

In fact, in general, we can write
Theta is equal to theta cap plus error
Where, 'error' stands for the difference between the observed value of theta cap and the actual "true" value of theta. This emphasizes one very serious danger in reporting point estimates of population parameters -- readers may forget that there is an unstated and perhaps large error, and so may mistakenly attribute greater accuracy to the estimate than is really permitted.

The consequences of unwittingly using erroneous results can often be worse than not having any result to use at all. One reason that interval estimates are favoured much more than point estimates in statistical inference is that they make the presence of potential estimation error much more explicit.

5. Limitations and Applications of Point Estimation

Limitations of Point Estimation:
A point estimate is often insufficient because it is either right or wrong. If we are told that, our point estimator of any parameter is wrong and we do not know how wrong it is, then we cannot be certain of the estimate's reliability. Therefore, a point estimate is much more useful if it is accompanied by an estimate of the error that might be involved.

It is obvious that a point estimate is normally different from the actual value of the parameter. The reason is that the point estimate is derived from the random sample whose values vary from a sample to sample, where x bar is the estimator of the mean mu. The values of x bar will vary in a manner that most of them are spread about closely on both sides of mu.

As the parameter to be estimated is unknown, neither the error in the point estimate is noted nor it is accurately measured. This greatly reduces the practical utility of point estimation. Accuracy can be expressed in probabilistic terms by stating how likely or probable a particular value of an estimator theta cap is equal to the parameter theta to be estimated. This necessitates that an estimator should be expressed in the form of an interval rather than a single numerical value.

In practice, confidence interval estimates are used more commonly by far than point estimates. Nevertheless, point estimates are used in certain important ways in statistics, and carry with them some important concepts and terms, we need to look at them briefly.
Some people refer to point estimates as a "best guess" value. The term "guess" is a bit pessimistic, but it does give you the sense that there is a degree of uncertainty in relying on point estimates.

Application of Point Estimates

Often point estimates are used as parts of other statistical calculations. For example, a point estimate of the **standard deviation** is used in the calculation of a confidence **interval** for mu. Point estimates of parameters are often used in the formulas for **significance testing.**
Point estimates are not usually as informative as confidence intervals. Their importance lies in the fact that many statistical formulas are based on them.

Nevertheless, the convenience of point estimates outweighs their deficiencies in some instances. Two examples are:

i. Interval estimates of certain fundamental physical constants would be very difficult or inconvenient to work with in calculations. Thus, although quantities such as the gravitational acceleration, g; Avogadro's number, N; and so forth are numbers, which are experimentally determined and thus subject to sampling errors of one sort or another, we normally use point estimates of them rather than interval estimates. Of course, many of these fundamental constants have been estimated with high precision so that errors in their estimates are not significant for many applications.

ii. As you will see shortly, when we carry out various procedures of statistical inference focusing on one population parameter of greatest interest, the formulas that result may involve the values of other population parameters. In such situations, we can usually obtain adequately accurate results by using point estimates for the parameters of secondary interest in order to derive formulas for an interval estimate of the parameter of greatest interest.

For example, in deriving formulae for interval estimates of the population mean, mu, we require the value of the population standard deviation, sigma. Since mu is unknown, it is very unlikely that we will know the value of sigma (though in some instances we might). Rather than backing up one more step and determining an interval estimate for sigma, it is more usual to use the available value of s as a point estimate of sigma in the formula for the interval estimate of mu.

Here’s a summary of our learning in this session, where we understood:

- The statistical definition of Point Estimation
- The criteria for the best point estimators
- The methods of point estimation
- The types of variables and the respective point estimates
- The limitations and applications of point estimation