1. Introduction

Welcome to the series of E-learning modules on Practical-Construction of Confidence Intervals for mean and difference of means. In this module, we are going to cover the Interval Estimation- procedure to estimate the population mean and difference in the population means, when the population standard deviations are known and unknown through some practical problems for dependent and independent samples.

By the end of this session, you will be able to:

- Explain the interval estimation
- Illustrate the construction of confidence interval for the population mean when variance is known and unknown
- Illustrate the construction of confidence interval for the difference of population means when the respective variances are known and unknown
- Apply interval estimation technique for the estimation of unknown population mean in case of correlated samples

Interval estimation is a rule for calculating two numbers say A and B. This is to create an interval that we are fairly certain, which contains the parameter of interest that is population mean mu. Hence, a confidence interval gives an estimated range of values for the population parameter, which is likely to include an unknown population parameter and the estimated range is calculated from a given set of sample data.

Consider a distribution with probability
density function f of (x, theta), where theta is an unknown parameter. Our problem is to find a confidence interval for the parameter theta. In an interval estimation problem of finding confidence interval for the parameter theta with certain amount of confidence (1 minus alpha), we need to find two quantities A and B based on the sample observations such that Probability of [A less than theta less than B] is equal to 1 minus alpha

Practical problems very often lead to the estimation of mu, the mean of the population.
One hundred into (one minus alpha) percent Confidence Interval for the population mean mu when the variance is known as sigma square is given by
[y bar minus Z alpha by two into Standard Error of y bar, y bar plus Z alpha by two into Standard error of y bar]
That is
[y bar minus Z alpha by two into sigma by root n, y bar plus Z alpha by two into sigma by root n]

Ninety five percent confidence Interval
For Ninety five percent confidence,
Alpha is equal to point zero five and alpha by two is equal to point zero two five. The value of Z point zero two five is found by looking in the standard normal table. This area in the table is associated with a Z value of one point nine six.
In other words, of all the possible y-bar values along the horizontal axis of the normal distribution curve, Ninety five percent of them should be within a Z score of one point nine six

from the mean.
Ninety five percent Confidence Interval for the population mean mu is
[y bar minus one point nine six into root of sigma square by n, y bar plus one point nine six into root of sigma square by n]

In most practical research, the standard deviation for the population of interest is not known. In this case, the standard deviation sigma is replaced by the estimated standard deviation s. One hundred into (1 minus alpha) percent confidence interval for the population mean μ when the variance is unknown is given by
[y bar minus t alpha (n minus 1) into s by root n, y bar plus t alpha (n minus 1) into s by root n]

2. Problem on C.I for Mean

Problem 1:
A sample of size 10 from a Normal distribution with Standard deviation 3 is as follows. Establish ninety percent confidence interval for the mean of the distribution
6 point 5, 5 point 5, 4 point 8, 5 point 6, 4, 4 point 7, 10 point 5, 8, 7 point 3, 6 point 8

Solution:

We have to establish ninety percent confidence interval for the population mean when the standard deviation is known.
Let Xi follow Normal distribution with mean mu and variance sigma square

Given sigma is equal to 3 and n is equal to 10

One hundred into (1minus alpha) percent confidence interval for the population mean mu when the variance is known as sigma square is given by
[y bar minus Z alpha by two into square root of sigma square by n , y bar plus Z alpha by two into square root of sigma square by n]

Given One hundred into (1minus alpha) percent is equal to ninety percent implies 1 minus alpha is equal to zero point nine zero implies alpha is equal to zero point one then alpha by 2 is equal to zero point zero five.

From the table of Standard Normal Probabilities, we get Z alpha by 2 is equal to Z zero point zero five is equal to one point six four
y bar is equal to six point three seven and root n is equal to three point one six

Therefore, ninety percent Confidence Interval for the mean of the distribution
[y bar minus Z alpha by 2 into sigma by root n, y bar plus Z alpha by 2 into sigma by root n]
Which is equal to [six point three seven minus 1 point six four into three by three point one six, six point three seven plus 1 point six four into three by three point one six] which is equal to [four point eight one four, seven point nine two five eight]

Therefore, ninety percent Confidence Interval for the mean of the distribution is
[four point eight one four, seven point nine two five eight]

3. Problem on C.I for the Mean Life Expectancy

Problem 2:
The life expectancy of the people in Brazil in the year Nineteen seventy three is given below, which is after a survey conducted in 11 regions of Brazil. Obtain ninety eight percent confidence interval for the mean life expectancy.
Life expectancy in the year is given as follows:
54 point 2, 50 point 2, 44 point 2, 49 point 7, 55 point 4, 57, 58 point 2, 56 point 6, 61 point 9, 57 point 5, 53 point 4

Solution:
Xi follows Normal with mean mu and variance sigma square.
Given n is equal to 11
Given one hundred into (1 minus alpha) percent is equal to ninety eight percent implies 1 minus alpha is equal to zero point nine eight implies alpha is equal to zero point zero two
From the table of Students t distribution, the value of t for (n minus1) which is equal to (eleven minus 1) which is equal to 10 degrees of freedom at zero point zero two level of significance is t alpha (n minus 1) which is equal to t zero point zero two with 10 degrees of freedom is equal to 2 point seven six four
y bar is equal to fifty four point three nine and s square is equal to summation i runs from 1 to n, (yi minus y bar) whole square by (n minus 1) which is equal to twenty three point eight seven nine eight and s is equal to four point eight eight six seven and root n is equal to three point three one six six

One hundred into (1 minus alpha) percent Confidence Interval for the population mean mu when the variance is unknown is given by
[y bar minus t alpha (n minus 1) into s by root n, y bar plus t alpha (n minus 1) into s by root n]
Where, y bar is equal to summation i runs from 1 to n, (yi by n) and s square is equal to summation i runs from 1 to n, (yi minus y bar) whole square by n minus 1)
Which is equal to [fifty four point three nine minus 2 point seven six four into four point eight eight six seven by three point three one six six , fifty four point three nine plus 2 point seven six four into four point eight eight six seven by three point three one six six
Which is equal to [fifty point three two, fifty eight point four six]
Therefore, ninety eight percent Confidence Interval for the mean of the distribution is
[fifty point three two, fifty eight point four six]

4. Problem on C.I for Difference (Part-1)

Problem 3:

Two salesmen are working in a shop. The number of items sold by them in a week is as given in the table below.

Figure 1

First Salesman	25	32	30	32	24	14	32
Second Salesman	24	34	22	30	42	31	40

Establish a ninety nine percent confidence interval for the difference between the average sales of the two salesmen.

Solution:
We have to establish ninety nine percent confidence interval for the difference between the average sales of the two salesmen, where the population variances are unknown.
One hundred into (1 minus alpha) percent confidence interval for the difference of means of two populations with unknown but common Standard Deviations is given by
[(x bar minus y bar) minus t alpha (m plus n minus 2) into Sp into square root of (1 by m plus 1 by n), (x bar minus y bar) plus t alpha (m plus n minus 2) into Sp into square root of (1 by m plus 1 by n)]

Where m is equal to n is equal to 7; x bar is equal to summation xi by m which is equal to one eighty nine by seven, which is equal to twenty seven and y bar is equal to summation yi by n equal to two twenty three by 7 which is equal to thirty one point eight five seven one four
Sp is equal to square root of (m minus 1) into s1 square plus (n minus 1) s2 square by (m plus n minus 2) which is equal to square root of summation (xi minus x bar) square plus summation (yi minus y bar) square by (m plus n minus 2)
Sp is equal to square root of two hundred and sixty six plus three hundred and thirty six point eight five seven one by 12, which is equal to 7 point zero eight seven eight eight four
Given one hundred into (1 minus alpha) percent is equal to ninety nine percent implies 1 minus alpha is equal to zero point nine nine implies alpha is equal to zero point zero one

From the table of Students t distribution for (m plus n minus 2) is equal to 12 degrees freedom, we get t zero point zero one (12) is equal to three point zero six
By substituting in the above expression, we get
[twenty seven minus thirty one point eight five seven one four minus (three point zero six) into (7 point zero eight seven eight eight four) into square root of 1 by seven plus 1 by seven,
[twenty seven minus thirty one point eight five seven one four plus (three point zero six) into

(7 point zero eight seven eight eight four) into square root of 1 by seven plus 1 by seven
Is equal to [minus sixteen point four five zero four, six point seven three six zero seven five]
Therefore, ninety nine percent confidence interval for the difference of means of two populations with unknown but common Standard Deviations is given by [minus sixteen point four five zero four, six point seven three six zero seven five]

4. Problem on C.I for Difference (Part-1)

Problem 4:

The sales data of an item in six shops before and after a special promotional campaign are as given in the table.

Figure 2

Shops	A	B	C	D	E	F
Before campaign	53	28	31	48	50	42
After campaign	58	29	30	55	56	45

Obtain a ninety five percent confidence interval for the difference between the average sales.

Solution:

Figure 3

xi	53	28	31	48	50	42	Total
y_i	58	29	30	55	56	45	
$d_i=x_i-y_i$	-5	-1	1	-7	-6	-3	-21
di^2	25	1	1	49	36	9	121

The given set of observations is correlated variables. Hence, we find di for the given values. Let xi denote the sales before campaign and yi denote the sales after campaign.

Therefore, hundred into (1 minus alpha) percent confidence interval for the population mean theta in case of correlated variables is given by
[d bar minus t alpha (n minus 1) into sd by root n,[d bar plus t alpha (n minus 1) into sd by root n]
Where, sd square is equal to summation i runs from 1 to n, (di minus d bar) whole square by (n minus 1) and d bar is equal to summation di by n
d bar is equal to summation d i by n is equal to minus twenty one by 6, which is equal to minus three point five

sd square is equal to summation i runs from 1 to n, (di minus d bar) whole square by (n minus 1) which is equal to summation di square minus n into d bar square by (n minus 1) which is equal to nine point five and sd is equal to three point zero eight
One hundred into (1 minus alpha) percent is equal to ninety five percent, which implies alpha is equal to zero point zero five

From the table of t-distribution, we get t alpha (n minus 1) is equal to t zero point zero five (with 5 degrees of freedom) is equal to 2 point five seven

Therefore, ninety five percent confidence interval for the difference in average sales is given by
[d bar minus t alpha (n minus 1) into sd by root n, [d bar plus t alpha (n minus 1) into sd by root n]
[minus three point five minus 2 point five seven into three point zero eight by root six, minus three point five plus 2 point five seven into three point zero eight by root six], which is equal to [minus six point seven three one five three, minus zero point two six eight four seven]
Hence, ninety five percent Confidence Interval for the difference in the average sales is [minus six point seven three one five three, minus zero point two six eight four seven]

Problem 5:

Construct ninety five percent confidence interval for the difference of means given that m is equal to 10, n is equal to 15, x bar is equal to 16, y bar is equal to thirteen point 5. Assume that the population variances are 8 and 10 respectively.

Solution:

One hundred into (1 minus alpha) percent confidence interval for the difference of means of two populations with Known Standard Deviations is given by [(x bar minus y bar) minus Z alpha by 2 into square root of (sigma 1 square by m plus sigma 2 square by n), (x bar minus y bar) plus Z alpha by 2 into square root of (sigma 1 square by m plus sigma 2 square by n)]

We have to get ninety five percent Confidence Interval ,
One hundred into (1 minus alpha) percent is equal to ninety five percent, which implies alpha is equal to zero point zero five and alpha by 2 is equal to zero point zero two five

From the table of standard Normal probabilities, Z alpha by 2 is equal to Z zero point zero two five, which is equal to one point nine six. Therefore,

[(16 minus 13 point 5) minus (1 point 9 6) square root of (8 by 10 plus 10 by 15), (16 minus 13 point 5) plus (1 point 9 6) square root of (8 by 10 plus 10 by 15)], which is equal to [zero point one two six three, four point eight seven three seven]
Hence, ninety five percent confidence interval for the difference of means is [zero point one two six three , four point eight seven three seven]

Here's a summary of our learning in this session, where we have understood:

- The method of obtaining the confidence limits for the unknown population average when sigma is known and unknown
- The method of obtaining the confidence limits for the difference of means of two populations when the variances are known and unknown
- The illustration of the interval estimation procedure for the estimation of the population mean in case of dependent samples