Frequently Asked Questions

1. What do you mean by method of moments?

Answer:
The method of moments is the oldest method of deriving point estimators. It almost always produces some asymptotically unbiased estimators, although they may not be the best estimators.

The method was moments were discovered by Karl Pearson. As the name itself suggests in this technique moments are utilized for the estimation of the unknown parameters. This method is based on the principle that the unknown population parameters can be estimated by making use of sample moments. By equating the sample moments to the population raw moments the unknown parameters of the population may be estimated.

2. Write a note on the principle behind the method of moments.

Answer:

The method of moments is based on the moments of population as well as that of sample.

Let $f(x,\theta 1,\theta_2,\ldots, \theta_k)$ is the density function of the population under consideration. To estimate the unknown parameters $\theta_1,\theta_2,\ldots, \theta_k$ if $\mu_r^{'}$ denotes the r the moment (about zero) then by definition

$\mu_r^{'} = \int x^r f(x,\theta_1,\theta_2,\ldots, \theta_k)dx$, r=12,.....

That is

$\mu_1^{'} = \int x f(x,\theta 1,\theta_2,\ldots, \theta_k)dx$

$\mu_2^{'} = \int x^2 f(x,\theta 1,\theta_2,\ldots, \theta_k)dx$

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$\mu_{k'} = \int x^k f(x,\theta 1,\theta_2,\ldots, \theta_k)dx$

$\mu_1^{'}, \mu_2^{'}, \mu_3^{'},\ldots \mu_k^{'}$ are in general functions of the parameters $\theta_1,\theta_2,\ldots, \theta_k$. Thus the above is a set of k equations involving k unknown parameters $\theta_1,\theta_2,\ldots, \theta_{k.}$

Now solving the equations $\theta_1,\theta_2,\ldots, \theta_k$ can be written as the functions of $\mu_1^{'}, \mu_2^{'}, \mu_3^{'},\ldots \mu_k^{'}$. But in general $\mu_1^{'}, \mu_2^{'}, \mu_3^{'},\ldots \mu_k^{'}$ are unknown and hence their estimators by sample moments $m_1^{'}, m_2^{'}, m_3^{'},\ldots m_k^{'}$ respectively where $m_{r'} = \sum x_i^r /n$, x1,x2,...xn being sample observations are determined.

In case of frequency distribution of the sample observations r^{th} sample moment m_r1 is given by $m_{r'} = \sum_{fixi}^{r} /N$. Thus the method of moments consistes in equating rth raw moments about the origin in the population to the r th raw moments about the origin in the sample by giving values r=1,2,…. And obtaining various equations containing parameters and solving these equations to obtain the estimate of the parameters.

3. Find the moment estimator of θ based on the following sample taken from the population.

   $f(x,\theta) = \theta x^{\theta-1}$, $0<x<1$, $\theta > 0$

   0.35, 0.65, 0.27, 0.04, 0.48, 0.72, 0.49, 0.31, 0.56, 0.82

Answer:

We can find the estimate of θ using the moment μ_1'

$$\mu_1' = \int_0^1 x f(x,\theta) dx = \int_0^1 x\theta x^{\theta=1} dx = \int_0 \theta x^{\theta} dx = \frac{\theta}{\vartheta + 1}$$

Therefore $\theta = \dfrac{\mu_1'}{1-\mu_1'}$

By the method of moments a moment estimate of θ is obtained such that $\mu_1' = m_1' = \bar{x} = 0.469$

Therefore, Moment estimator of θ is given by $\hat{\theta} = \dfrac{m_1'}{1-m_1'} = \dfrac{\bar{x}}{\overline{\overline{1-x}}} = 0.8832$

4. Obtain the moment estimate of λ in $F(x,\lambda) = (\lambda^4 x^3 e^{-\lambda x})/6$; $x \geq 0$ based on the sample observations: 1.4, 2.6,3.1. 0.9, 1.5, 2.4, 2.5

Answer:

Given

$f(x,\lambda) = (\lambda^4 x^3 e^{-\lambda x})/6$; $x \geq 0$

We can find the estimate of λ using the moment μ_1'

$$\mu_1' = \int_0^\infty x f(x,\lambda) dx = \int_0^\infty x \frac{\lambda^4 x^3 e^{-\lambda x}}{6} dx = \frac{4}{\lambda} \Rightarrow \lambda = \frac{4}{\mu_1'}$$

By the method of moments a moment estimate of λ is obtained such that $\mu_1' = m_1' = \bar{x}$

Therefore $\hat{\lambda} = \dfrac{4}{m_1'} = \dfrac{4}{\bar{x}} = 1.9444$

5. The lives of 5 tyres are 2100,4300,4380,3860 and 3150 kilo meters. If life of tyres has a p.d.f, f(x) = = $\theta e^{-\theta x}$, $x > 0$. Find the moment estimator of θ.

Answer:

We can find the estimate of θ using the moment μ_1'

$$\mu_1^{'} = \int xf(x,\theta)dx = \int_0 x\theta e^{-\theta x} dx = \frac{1}{\vartheta}$$

Therefore $\hat{\theta} = \frac{1}{\mu_1^{'}}$

By the method of moments a moment estimate of θ is obtained such that $\mu_1^{'} = m_1^{'} = \bar{x}$

Therefore Moment estimator of θ is given by $\hat{\theta} = \frac{1}{m_1^{'}} = \frac{1}{\bar{x}} = 0.00028$

6. Given the observations from U (θ1 - θ2, θ1 + θ2). Obtain the moment estimates of θ1 and θ2. 1.5, -1.2, 2.6, 0.6, 0, -2.5, 3.8, -1.7, 0.8, 0.5

Answer:

The density function f(x) = 1/ ($\theta_1 + \theta_2 -(\theta_1 - \theta_2)$) = 1/2 θ_2

We can find the estimates of $\theta_1$and θ_2 based on the given sample observations. Hence we need to find μ_1' and μ_2'

$$\mu_1^{'} = \int_{\theta_1-\theta_2}^{\theta_1+\theta_2} x\, f(x,\theta_1,\theta_2)dx = \int_{\theta_1-\theta_2}^{\theta_1+\theta_2} x\, \frac{1}{2\theta_2} dx = \theta_1$$

$$\mu_2^{'} = \int_{\theta_1-\theta_2}^{\theta_1+\theta_2} x^2 f(x,\theta_1,\theta_2)dx = \int_{\theta_1-\theta_2}^{\theta_1+\theta_2} x^2 \frac{1}{2\theta_2} dx = \frac{\theta_2^2 + 3\theta_1^{\,2}}{3}$$

By substituting $\mu_1' = \theta_1$

We get $\theta_2 = \sqrt{3(\mu_2^{'} - (\mu_1^{'})^2)}$

By the method of moments $\mu_1^{'} = m_1^{'} = \bar{x}$ and $\mu_2^{'} = m_2^{'}$

Therefore $\hat{\theta}_1 = \bar{x} = \frac{\sum xi}{n} = 0.44$

$$\hat{\theta}_2 = \sqrt{3(m_2^{'} - (m_1^{'})^2)} = \sqrt{3(\frac{\sum xi^2}{n} - (\bar{x})^2)} = 3.1628$$

Therefore the moment estimate of θ_1 is 0.44 and

The moment estimate of θ_2 is 3.1628

7. Find the moment estimate of θ in the distribution $f(x) = 2x / \theta^2$, $0<xi< \theta$ using the sample 1.51, 1.76, 2.31, 2.76, 1.57, 1.98, 1.23

Answer:

We can find the estimate of θ using the moment μ_1'

$$\mu_1^{'} = \int xf(x,\theta)dx = \int x\frac{2x}{\theta^2}dx = \frac{2}{3}\theta$$

Therefore $\hat{\theta} = \dfrac{3\mu_1^{'}}{2}$

Hence a moment estimate of θ is obtained such that $\mu_1^{'} = m_1^{'} = \bar{x}$

Therefore Moment estimator of θ is given by $\hat{\theta} = \dfrac{3m_1^{'}}{2} = 3\dfrac{\overline{(x)}}{2} = 2.81$

8. Find the moment estimate of θ of the geometric distribution with parameter θ using the data 1,7, 6, 2, 3,1, 2,7,6, 5,7, 1, 9, 8, 1, 2, 3

Answer:

The probability mass function of a geometric variate x is

$F(x, \theta) = \theta(1- \theta)^x$

The first population moment $\mu_1' = E(x) = (1- \theta)/ \theta$

The first sample moment $m_1^{'} = \bar{x}$

Then $\mu_1^{'} = \bar{x} \Rightarrow \dfrac{1-\theta}{\theta} = \bar{x} \Rightarrow \dfrac{1}{\theta} - 1 = \bar{x} \Rightarrow \theta = \dfrac{1}{1+\bar{x}}$

Hence the moment estimator of the parameter θ is $1/(1+\bar{x})$

Hence the moment estimate of the parameter θ is 0.193

9. Find the moment estimates of θ for the distribution $f(x, \theta) = (x / \theta^2)\, e^{-x/\theta}$, $x >0$
   Given the following observations: 5.35, 4.27, 6.02, 9.63, 4.67, 3.49,2.95,4.02,8.12,7.58

Answer:

We can find the estimate of θ using the moment μ_1'

$$\mu_1' = \int xf(x,\theta)dx = \int x\frac{x}{\theta^2}e^{-x/\theta}dx = 2\theta$$

Therefore $\hat{\theta} = \dfrac{\mu_1'}{2}$

Hence a moment estimate of θ is obtained such that $\mu_1' = m_1' = \bar{x}$

Therefore Moment estimator of θ is given by $\hat{\theta} = \dfrac{m_1'}{2} = \dfrac{\bar{x}}{2} = 2.805$

10. Find the moment estimator of θ in the Beta population of second kind with parameters 1 and θ and hence find the estimate of θ based on the observations 1.4, 2.6, 3.1, 0.9, 1.5, 2.4, 2.5

Answer:

$x_i \sim \beta_2(1,\theta)$

$$f(xi,\theta) = \frac{1}{\beta(1,\theta)}\frac{x^{1-1}}{(1+x)^{\theta+1}} = \frac{1}{\beta(1,\theta)}\frac{1}{(1+x)^{\theta+1}}$$

When $x_i \sim \beta_2(m,n)$

f(xi,m,n) = $\dfrac{1}{\beta(m,n)}\dfrac{x^{m-1}}{(1+x)^{m+n}}$,0<x< ∞

$$\mu_r' = E(x^r) = \int x^r \frac{1}{\beta(m,n)}\frac{x^{m-1}}{(1+x)^{m+n}}dx$$

=

$$\frac{1}{\beta(m,n)}\int\frac{x^{m+r-1}}{(1+x)^{m+n}}dx = \frac{1}{\beta(m,n)}\int\frac{x^{m+r-1}}{(1+x)^{m+r+n-r}}dx = \frac{1}{\beta(m,n)}\beta(m+r,n-r)$$

$$=\frac{\Gamma(m+n)}{\Gamma(n)\Gamma(m)}\frac{\Gamma(m+r)\Gamma(n-r)}{\Gamma(m+n)}=\frac{1}{\Gamma(n)\Gamma(m)}\Gamma(m+r)\Gamma(n-r)$$

Putting r=1,m=1,n=θ

We get

$$\mu_1^{'}=\frac{\Gamma(1)\Gamma(\theta-1)}{\Gamma(1)\Gamma(\theta)}=\frac{1}{\theta-1}$$

$$\mu_1^{'}=m_1^{'}=\bar{x}\Rightarrow\frac{1}{\theta-1}=\bar{x}\Rightarrow\theta=\frac{1+\bar{x}}{\bar{x}}$$

Hence the moment estimator of θ is $\widehat{\theta}=\dfrac{1+\bar{x}}{\bar{x}}$

$$\bar{x}=\frac{\sum xi}{n}=\frac{14.4}{7}=2.057$$

$$\widehat{\theta}=\frac{1+\bar{x}}{\bar{x}}=\frac{1+2.057}{2.057}=1.486$$

11. For the following distribution estimate the parameters α and Θ by the method of moments $f(x,\theta)=\dfrac{1}{\alpha!\theta^{\alpha+1}}x^{\alpha}e^{-x/\theta},\ 0<x<\infty$

Answer:

We have to estimate α and Θ based on the sample observations $x_1,x_2,\ldots,xn$. Hence we need to find μ_1' and μ_2'

$$\mu_1^{'}=\int xf(x,\theta)dx=\int x\frac{1}{\alpha!\theta^{\alpha+1}}x^{\alpha}e^{-x/\theta}dx=\frac{1}{\alpha!\theta^{\alpha+1}}\int x^{\alpha+1}e^{-x/\theta}dx$$

$$=\frac{1}{\alpha!\theta^{\alpha+1}}\frac{\Gamma(\alpha+2)}{(1/\theta)^{\alpha+2}}=(\alpha+1)\theta---(1)$$

$$\mu_2' = \int x^2 f(x,\theta)dx = \int x^2 \frac{1}{\alpha!\theta^{\alpha+1}} x^{\alpha} e^{-x/\theta} dx = \frac{1}{\alpha!\theta^{\alpha+1}} \int x^{\alpha+2} e^{-x/\theta} dx$$

$$= \frac{1}{\alpha!\theta^{\alpha+1}} \frac{\overline{)(\alpha+3)}}{(1/\theta)^{\alpha+3}} = (\alpha+1)(\alpha+2)\theta^2 ---(2)$$

Equation(2) by equation (1)

$$\Rightarrow \frac{\mu_2'}{\mu_1'} = (\alpha+2)\theta ---(3)$$

Equation (3) – equation (1) $\Rightarrow \frac{\mu_2'}{\mu_1'} - \mu_1' = (\alpha+2)\theta - (\alpha+1)\theta = \theta$

$$\Rightarrow \theta = \frac{\mu_2' - (\mu_1')^2}{\mu_1'}$$

Substituting this Θ in equation (1)

μ_1' = (α+1) [$\frac{\mu_2' - (\mu_1')^2}{\mu_1'}$] $\Rightarrow$ α= $\frac{(\mu_1')^2}{\mu_2' - (\mu_1')^2} - 1$

By the method of moments $\mu_1' = m_1'$ and $\mu_2' = m_2'$

$$\hat{\theta} = \frac{\hat{\mu}_2' - (\hat{\mu}_1')^2}{\hat{\mu}_1'} = \frac{m_2' - (m_1')^2}{m_1'} = \frac{(\sum xi^2/n) - \bar{x}^2}{\bar{x}}$$

$$\hat{\alpha} = \frac{(\hat{\mu}_1')^2}{\hat{\mu}_2' - (\hat{\mu}_1')^2} - 1 = \frac{(m_1')^2}{(m_2') \ -((m_1')^2} - 1 = \frac{\bar{x}^2}{(\sum xi^2/n) - (\bar{x}^2)} - 1$$

12. Find the moment estimate of Θ in the following function whose p.d.f is

f(x,Θ) = $\frac{\theta(\theta+1)x^{\theta-1}}{(1+x)^{\theta+2}}, 0 < x < \infty$

Using the sample 75,68,55,62,50,82,77,60,72,50. Compute the moment estimate of θ.

Answer:

We can find the estimate of Θ using the moment

$$\mu_1^{'} = \int xf(x,\theta)dx = \int x\frac{\theta(\theta+1)x^{\theta-1}}{(1+x)^{\theta+2}}dx = \theta(\theta+1)\int\frac{x^{\theta}}{(1+x)^{\theta+2}}dx = \theta(\theta+1)\beta(\theta+1,1)$$

$$= \theta(\theta+1)\frac{\Gamma(\theta+1)\Gamma 1}{\Gamma(\theta+2)} = \frac{\theta(\theta+1)\theta\Gamma\theta}{(\theta+1)\theta\Gamma\theta} = \theta$$

Hence a moment estimate of Θ is obtained as $\mu_1^{'} = m_1^{'} = \bar{x}$

Therefore Moment estimator of Θ is given by $\hat{\theta} = \bar{x}$

$$\bar{x} = \frac{\sum xi}{n} = \frac{651}{10} = 65.1$$

Hence a moment estimate of Θ is 65.1

13. Obtain an moment estimator of Θ of a Uniform population with parameters 0 and Θ given the following observations 2.5, 1.63, 5.21, .82, 6.31, 4.34, 1.52, 3.62, 4.73, 3.95, 5.63, 6.5

Answer:

We know that when $x_i \sim U(0,\theta)$

The p.d.f f(x) = 1/ Θ; 0<xi< Θ

The first population moment $\mu_1^{'} = \theta/2$

Now from the method of moments unknown parameter Θ is estimated such that $\mu_1' = m_1'$

But $m_1^{'} = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}$

Hence $\mu_1^{'} = \theta/2 = m_1^{'} = \bar{x}$

Therefore Moment estimator of Θ is given by $\hat{\theta} = 2\bar{x} = 2\frac{\sum xi}{n} = 2\frac{46.76}{12} = 7.7933$

Hence moment estimate of θ is 7.7933

14. For the following distribution estimate the parameter α and θ from the data given below

$$f(x,\alpha,\theta)=\frac{1}{\alpha!\theta^{\alpha+1}}x^{\alpha}e^{-x/\theta}$$

Observations: 3.1,10.27,8.60,7.1,5.82,0.04

Answer:

We have to estimate α and θ based on the given sample observations. Hence we need to find μ_1' and μ_2'

$$\mu_1'=\int_0^\infty x f(x,\alpha,\theta)dx=\int_0^\infty x\frac{1}{\alpha!\theta^{\alpha+1}}x^{\alpha}e^{-x/\theta}dx=(\alpha+1)\theta---(1)$$

$$\mu_2'=\int_0^\infty x^2 f(x,\alpha,\theta)dx=\int_0^\infty x^2\frac{1}{\alpha!\theta^{\alpha+1}}x^{\alpha}e^{-x/\theta}dx=(\alpha+1)(\alpha+2)\theta^2---(2)$$

Equation (2) by equation (1) implies

$$\frac{\mu_2'}{\mu_1'}=(\alpha+2)\theta----(3)$$

Equation (3) - equation (1) implies

$$\frac{\mu_2'}{\mu_1'}-\mu_1'=(\alpha+2)\theta-(\alpha+1)\theta=\theta\Rightarrow\hat{\theta}=\frac{\mu_2'-(\mu_1')^2}{\mu_1'}$$

Substituting this value of θ in equation (1)

$$\mu_1'=(\alpha+1)(\frac{\mu_2'-(\mu_1')^2}{\mu_1'})\Rightarrow\alpha=\frac{(\mu_1')^2}{\mu_2'-(\mu_1')^2}-1$$

Moment estimators of θ and α are

$$\hat{\theta}=\frac{\mu_2'-(\mu_1')^2}{\mu_1'}=\frac{m_2'-(m_1')^2}{m_1'}$$

$$= \frac{\dfrac{\sum xi^2}{n} - \left(\dfrac{\sum xi}{n}\right)^2}{\dfrac{\sum xi}{n}} = \frac{35.38782 - (5.8217)^2}{5.8217} = 0.256905$$

$$\hat{\alpha} = \frac{(\hat{\mu}_1^{'})}{\hat{\theta}} - 1 = \frac{(m_1^{'})}{\hat{\theta}} - 1 = \frac{(\dfrac{\sum xi}{n})}{\hat{\theta}} - 1 = \frac{5.8217}{0.256905} - 1 = 21.6608$$

Therefore moment estimate of θ= 0.256905 and

Moment estimate of α = 21.6608

15. Assuming the distribution to be Normal estimate the parameters of the distribution for the following data by the method of moments

Central value :	30	35	40	45	50	55	60
Frequency :	5	9	15	32	28	10	8

Answer:

Let μ and σ^2 be the parameters of the Normal population then we know that $\mu_1' = \mu$ and $\sigma^2 = \mu_2' - (\mu_1')^2$

By the method of moments the estimation of population moments are obtained by equating $\mu_1' = m_1'$ = sample mean

$\hat{\sigma}^2 = m_2^{'} - (m_1^{'})^2$ = sample variance

$$= \frac{1}{N}\sum f x^2 - (\frac{\sum fx}{N})^2$$

Therefore the moment estimate of $\mu = \hat{\mu} = \dfrac{\sum fx}{N} = \dfrac{4585}{100} = 45.85$

$$\hat{\sigma}^2 = \frac{1}{N}\sum f x^2 - (\frac{\sum fx}{N})^2 = \frac{215875}{100} - (45.85)^2 = 56.5275$$

Therefore moment estimate of μ=45.850

Moment estimate of σ^2 = 56.5275