

## Summary

- The significance of the central limit theorem lies in the fact that it permits us to use sample estimators to make inferences about the population parameters without knowing anything about the shape of the frequency distribution of that population other than what we can get from the sample
- Central Limit Theorem States that Suppose  $X_1, X_2, \dots, X_n$  be  $n$  independent random variables having the same probability density function each with  $E(X_i) = \mu$  and  $V(X_i) = \sigma^2$ , for  $i=1, 2, \dots, n$  then  $S_n = X_1 + X_2 + \dots + X_n$  is approximately Normal with mean  $n\mu$  and variance  $n\sigma^2$

▪ Also

$$Z = \frac{\bar{X} - n\mu}{\sqrt{n}\sigma} \text{ is asymptotically } N(0, 1)$$

- In probability theory, the law of large numbers (LLN) is a theorem that describes the result of performing the same experiment a large number of times
- For the existence of WLLN we assume the following conditions

- $E(X_i)$  exists for all  $i$
- $B_n = V(X_1 + X_2 + \dots + X_n)$  exists and

○

$$\frac{B_n}{n^2} \rightarrow 0 \text{ as } n \rightarrow \infty$$

- The term "Weak" refers to the way the sample mean converges to the distribution mean
- Weak Law of Large Numbers states that the sample mean is a consistent estimator of the population mean. That is a sample mean converges in probability to the population mean
- The central limit theorem, one of the two fundamental theorems of probability, is a theorem about convergence in distribution

- A sequence  $\{X_n\}$  of random variables converges in probability towards  $X$  if for all  $\varepsilon > 0$ 

$$\lim_{n \rightarrow \infty} \Pr(|X_n - X| \geq \varepsilon) = 0.$$