Summary

- A random variable X is said to have a Standard Cauchy Distribution, if its pdf is given by,

$$f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$$ and X is termed as standard Cauchy variate.

- More generally, Cauchy distribution with parameters λ and μ has the pdf,

$$f(y) = \frac{\lambda}{\pi[\lambda^2 + (y-\mu)^2]}, -\infty < y < \infty; \lambda > 0$$ and we write X~C(λ,μ)

- By substituting X=(Y-μ)/λ in the general pdf of Cauchy distribution, we get the pdf of standard Cauchy distribution
- For Cauchy distribution median=mode=μ
- In general, moment of Cauchy distribution does not exist. Mean and variance also does not exist. By at principal value, mean of the distribution can be given by μ.
- To write the logistic distribution we make use of the results
    - tanh x = $(1-e^{-2x})^{-1}$
    - x cosec x = $1 + x^2/6 + (7/360)x^4+\ldots$
- A continuous random variable X is said to have logistic distribution with parameters α and β, if its distribution function is of the form $F_X(x)=[1+\exp\{-(x- \alpha)/ \beta\}]^{-1}$, $\beta>0$

$$= \frac{1}{2}\left[1 + \tanh\left\{\frac{1}{2}\frac{(x-\alpha)}{\beta}\right\}\right]; \beta > 0$$

- The pdf of logistic distribution with parameters α and β is given by

$$f(x) = \frac{1}{\beta}[1 + \exp(-(x-\alpha)/\beta]^{-2}[\exp(-(x-\alpha)/\beta] = \frac{1}{4\beta}\sec h^2\left\{\frac{1}{2}(x-\alpha)/\beta\right\}$$

- Logistic distribution is extensively used as growth function in population and demographic studies and in time series analysis.
- Theoretically, Logistic distribution can be obtained as
    - Limiting distribution (as n→∞) of the standardized mid-range, (average of the smallest and the largest sample observations), in random samples of size n
    - A mixture of extreme value distributions.
- For standard logistic distribution, $\mu_1'=0$ and all raw moments are equal to central moments. Also all odd order raw moments are equal to zero, in turn all odd order central moments are also equal to zero.
- Standard Logistic distribution is symmetric and has leptokurtic curve.