Frequently Asked Questions

1. Name some of Univariate continuous distributions.
   Answer:
   Some of the Univariate continuous distributions are,
   - Normal distribution
   - Exponential distribution
   - Beta distribution
   - Gamma distribution
   - Pareto distribution
   - Laplace distribution
   - Cauchy distribution and
   - Logistic distribution

2. How does normal distribution play an important role in statistical theory?
   Answer:
   Normal distribution plays a very important role in statistical theory because of the following reasons:
   i. Most of the distributions occurring in practice, e.g., Binomial, Poisson, and Hypergeometric distributions etc. can be approximated by normal distribution. Moreover, many of the sampling distributions, e.g., Student's t, Snedecor's F and Chi-square distributions, etc., tend to normality for large samples
   ii. Even if a variable is not normally distributed, it can sometimes be brought to normal form by simple transformation of variable. For example, if the distribution of X is skewed, the distribution of square root of x might come out to be normal
   iii. If $X \sim N(\mu, \sigma 2)$, then $P(\mu-3\sigma \leq X \leq \mu+3\sigma) = P(-3 \leq Z \leq 3) = 0.9973$. This property of normal distribution forms the basis of entire large sample theory
   iv. Many of the distributions of sample statistics (for example, the distributions of mean, sample variance etc.,) tend to normality for large samples and as such they can be best studied with the help of the normal curves
   v. The entire theory of small sample tests, namely t, F, χ^2 tests, etc., is based on the fundamental assumption that the parent populations form which the samples have been drawn follow normal distribution
   vi. Normal distribution finds large applications in Statistical Quality Control in industry for setting control limits

3. How W.J. Youden of the National Bureau of Standards describes the importance of the Normal distribution artistically?

   Answer:
   W. J. Youden of the National Bureau of Standards describes the importance of the Normal Distribution artistically in the following words:

   THE NORMAL
   LAW OF ERRORS
   STANDS OUT IN THE
   EXPERIENCE OF MANKIND
   AS ONE OF THE BROADEST

GENERALISATIONS OF NATURAL
PHILOSOPHY. IT SERVES AS THE
GUIDING INSTRUMENT IN RESEARCHES,
IN THE PHYSICAL AND SOCIAL SCIENCES
AND IN MEDICINE, AGRICULTURE AND
ENGINEERING. IT IS AN INDISPENSABLE TOOL FOR
THE ANALYSIS AND THE INTERPRETATION OF THE
BASIC DATA OBTAINED BY OBSERVATION AND EXPERIMENT.

4. Mention the properties of normal distribution.
Answer:
The normal distribution with parameters μ and σ^2 has the following properties:
- The curve of the distribution is bell-shaped and symmetrical about the line x=μ
- Mean, median and mode of the distribution coincide.
- As x increases numerically, f(x) decreases rapidly, the maximum probability occurring at the point x=μ and is given by, $1/\sigma\sqrt{2\pi}$
- $\beta_1=0$ and $\beta_2=3$
- All odd order central moments are equal to zero and even order central moments are given by, $\mu_{2r}=1.3.5\ldots(2r-1)\sigma^{2r}$,(r=0, 1, 2 …)
- Since f(x) being the probability, can never be negative, no portion of the curve lies below the x axis.
- Linear combination of independent normal variates is also a normal variate
- x-axis is an asymptote to the curve
- The points of inflexion of the curve are x = μ ± σ
- Mean deviation about mean $= \sigma\sqrt{\frac{2}{\pi}} \approx \frac{4}{5}\sigma$
- Quartiles are given by, $Q_1=\mu-0.6745\sigma$; $Q_3=\mu+0.6745\sigma$
- Q.D = $\frac{Q_3 - Q_1}{2} \approx \frac{2}{3}\sigma$ We have approximately
QD: MD: SD = (2/3) σ: (4/5) σ: σ = (2/3): (4/5): 1

5. Write the area property of the Normal distribution.
Answer:
Area property of the normal distribution is
P (μ – σ < X < μ + σ) = 0.6826; P (μ – 2 σ < X < μ + 2σ) = 0.9544 and
P (μ - 3σ < X < μ + 3σ) = 0.9973

6. Why uniform distribution is also called as rectangular distribution?
Answer:
Uniform distribution is also known as a rectangular distribution, since the curve of uniform distribution describes a rectangle over the X axis and between the ordinates at x=a and X = b if X ~ U (a, b)

7. Write the important property of exponential distribution.
Answer:
An important property of the exponential distribution is that it is memory less.
This means that if a random variable T is exponentially distributed, its conditional probability obeys

$$\Pr(T > s+t \mid T > s) = \Pr(T > t) \text{ for all } s, t \geq 0.$$

8. Give the relation between mean median and mode of beta distribution.
   Answer:
   If $1 < \alpha < \beta$ then mode ≤ median ≤ mean. Expressing the mode (only for $\alpha > 1$ and $\beta > 1$), and the mean in terms of α and β:

$$\frac{\alpha - 1}{\alpha + \beta - 2} \leq \text{median} \leq \frac{\alpha}{\alpha + \beta},$$

   If $1 < \beta < \alpha$ then the order of the inequalities are reversed.

9. Write the uses of gamma distribution.
   Answer:
   - The gamma distribution has been used to model the size of insurance claims and rainfalls. This means that aggregate insurance claims and the amount of rainfall accumulated in a reservoir are modeled by a gamma process
   - The gamma distribution is also used to model errors in multi-level Poisson regression models, because the combination of the Poisson distribution and a gamma distribution is a negative binomial distribution
   - In neuroscience, the gamma distribution is often used to describe the distribution of inter-spike intervals. Although in practice the gamma distribution often provides a good fit, there is no underlying biophysical motivation for using it

10. Write the 80-20 principle of Pareto.
    Answer:
    Pareto originally used this distribution to describe the allocation of wealth among individuals since it seemed to show rather well the way that a larger portion of the wealth of any society is owned by a smaller percentage of the people in that society. He also used it to describe distribution of income. This idea is sometimes expressed more simply as the Pareto principle or the "80-20 rule" which says that 20% of the population controls 80% of the wealth.

11. Give some examples for Pareto distribution.
    Answer:
    - The sizes of human settlements (few cities, many hamlets/villages)
    - File size distribution of Internet traffic which uses the TCP protocol (many smaller files, few larger ones)
    - Hard disk drive error rates
    - Clusters of Bose–Einstein condensate near absolute zero
    - The values of oil reserves in oil fields (a few large fields, many small fields)
    - The length distribution in jobs assigned supercomputers (a few large ones, many small ones)
    - The standardized price returns on individual stocks Sizes of sand particles
    - Sizes of meteorites
    - Numbers of species per genus (There is subjectivity involved: The tendency to divide a genus into two or more increases with the number of species in it)
    - Areas burnt in forest fires

12. Write the uses of Laplace distribution.
    Answer:
    - The Laplacian distribution has been used in speech recognition to model priors on DFT coefficients

- The addition of noise drawn from a Laplacian distribution, with scaling parameter appropriate to a function's sensitivity, to the output of a statistical database query is the most common means to provide differential privacy in statistical databases

13. How the Cauchy distribution used in statistics?

Answer:

The Cauchy distribution is often used in statistics as the canonical example of a "pathological" distribution. The Cauchy distribution is an example of a distribution which has no mean, variance or higher moments defined.

14. Write the applications of logistic distribution in different fields.

Answer:

- Biology – to describe how species populations grow in competition
- Epidemiology – to describe the spreading of epidemics
- Psychology – to describe learning
- Technology – to describe how new technologies diffuse and substitute for each other
- Marketing – the diffusion of new-product sales
- Energy – the diffusion and substitution of primary energy sources, as in the Hubbert curve

15. Write the application of Laplace distribution in Physics.

Answer:

The cdf of this distribution describes a Fermi gas and more specifically the number of electrons within a metal that can be expected to occupy a given quantum state. Its range is between 0 and 1, reflecting the Pauli exclusion principle. The value is given as a function of the kinetic energy corresponding to that state and is parameterized by the Fermi energy and also the temperature (and Boltzmann constant). By changing the sign in front of the "1" in the denominator, one goes from Fermi–Dirac statistics to Bose–Einstein statistics. In this case, the expected number of particles (bosons) in a given state *can* exceed unity, which is indeed the case for systems such as lasers.