Summary

- The mathematical expectation of a function g(x, y) of two dimensional variable (X, y) with pdf f(x, y) is given by $E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f(x,y)dxdy$ (If X and Y are continuous r.v.s)

  $E[g(X,Y)] = \sum_i \sum_j x_i y_j P(X = x_i \cap Y = y_j)$ (If X and Y are discrete r.v.s) provided the expectations exists.
- In particular, the r^{th} and s^{th} product moment about origin of the random variables X and Y is defined as $\mu_{rs}' = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^r y^s f(x,y)dxdy$ or

  $\mu_{rs}' = \sum_i \sum_j x_i^r y_j^s P(X = x_i \cap Y = y_j)$
- The joint r^{th} central moment of X and s^{th} central moment of Y is given by $\mu_{rs} = E[\{X - E(X)\}^r \{Y - E(Y)\}^s] = E[\{X - \mu_X\}^r \{Y - \mu_Y\}^s]$
- We can also write, the central moments using the raw moments, in particular variance and covariance as follows:
  - $V(X)=E(X^2)-\{E(X)\}^2$
  - $V(Y)= E(Y^2)-\{E(Y)\}^2$
  - $Cov(X,Y)=E(XY)-E(X)E(Y)$
- For any two random variables X and Y, $V(aX \pm bY)=a^2V(X)+b^2V(Y) \pm 2abCov(X, Y)$