Frequently Asked Questions

1. Define bivariate moment of discrete variable.

Answer:

The mathematical expectation of a function g(x, y) of two dimensional variable (X, y) is given by $E[g(X,Y)] = \sum_i \sum_j x_i y_j P(X = x_i \cap Y = y_j)$

Provided the expectation exists.

2. Define bivariate moment of continuous variable.

Answer:

The mathematical expectation of a function g(x, y) of two dimensional variable (X, y) with pdf f(x, y) is given by $E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f(x,y)dxdy$, provided the expectation exists.

3. Write an expression for bivariate raw moment of discrete random variables.

Answer:

In particular, the r^{th} and s^{th} product moment about origin of the random variables X and Y is defined as $\mu_{rs}' = \sum_i \sum_j x_i^{\,r} y_j^{\,s} P(X = x_i \cap Y = y_j)$

4. Write an expression for bivariate raw moment of continuous random variables.

Answer:

In particular, the rth and sth product moment about origin of the random variables X and Y is defined as $\mu_{rs}' = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^r y^s f(x,y)dxdy$

5. Mention 1^{st}, 2^{nd} and 3^{rd} order raw moments.

Answer:

1^{st} order raw moments - E(X), E(Y)

2^{nd} order raw moments - $E(X^2)$,E(XY),$E(Y^2)$

3^{rd} order raw moments - $E(X^3)$, $E(X^2Y)$, $E(XY^2)$, $E(Y^3)$

6. Write an expression for bivariate central moment of random variables.

Answer:

The joint rth central moment of X and sth central moment of Y is given by

$\mu_{rs} = E[\{X - E(X)\}^r\{Y - E(Y)\}^s] \quad = E[\{X - \mu_X\}^r\{Y - \mu_Y\}^s]$, where $E(X)=\mu_X$, $E(Y)=\mu_y$

7. Mention 1^{st}, 2^{nd} and 3^{rd} order central moments.

Answer:

1^{st} order central moments - E[X-E(X)], E[Y-E(Y)]

2^{nd} order central moments - $E[X-E(X)]^2$, E{[X-E(X)][Y-E(Y)]} and $E[Y-E(Y)]^2$

3^{rd} order central moments - $E[X-E(X)]^3$, $E\{[X-E(X)]^2[Y-E(Y)]\}$, $E\{[X-E(X)][Y-E(Y)]^2\}$, $E[Y-E(Y)]^3$

8. Obtain an expression for variance in terms of expectations.

Answer:

$V(X)=E[X-E(X)]^2=E[X^2+\{E(X)\}^2-2XE(X)]$

$=E(X^2)+\{E(X)\}^2-2E(X)E(X)=\mathbf{E(X^2)-\{E(X)\}^2}$

9. Obtain an expression for covariance in terms of expectations.

Answer:

sCov(X,Y)=E[{(X-E(X)}{Y-E(Y)}]=E[XY-YE(X)-XE(Y)+E(X)E(Y)]
=E(XY)-E(Y)E(X)-E(X)E(Y)+E(X)E(Y)=**E(XY)-E(X)E(Y)**

10. For any two random variables X and Y, show that $V(aX \pm bY)=a^2V(X)+b^2V(Y)\pm 2abCov(X, Y)$.

Answer:

$$V(aX\pm bY)=E[(aX\pm bY)-E(aX\pm bY)]^2=E[(aX\pm bY)-\{aE(X)\pm bE(Y)\}]^2$$
$$=E[a\{X-E(X)\}\pm b\{Y-E(Y)\}]^2$$
$$=a^2E[X-E(X)]^2+b^2E[Y-E(Y)]^2\pm 2abE[X-E(X)]E[Y-E(Y)]$$
$$= a^2V(X)+b^2V(Y)\pm 2abCov(X, Y)$$

11. Write an expression for V(X+Y).

Answer:

In general we know that, $V(aX\pm bY)=a^2V(X)+b^2V(Y)\pm 2abCov(X, Y)$

Putting a=1 and b=1 and considering only plus we get,

$V(1.X+1.Y)=1^2V(X)+1^2V(Y)+2.1.1.Cov(X, Y)= V(X+Y)=V(X)+V(Y)+2Cov(X, Y)$.

12. Write an expression for V(X-Y)

Answer:

In general we know that, $V(aX\pm bY)=a^2V(X)+b^2V(Y)\pm 2abCov(X, Y)$

Putting a=1 and b=1 and considering only plus we get,

$V(1.X-1.Y)=1^2V(X)+1^2V(Y)-2.1.1.Cov(X, Y)= V(X-Y)=V(X)+V(Y)-2Cov(X, Y)$.

13. Show that when X and Y are independent Cov(X, Y)=0.

Answer:

We know that, Cov(X, Y)= E(XY) – E(X)E(Y)
We know that if X and Y are independent, E(XY) = E(X)E(Y)
Therefore, Cov(X, Y)= E(X)E(Y) – E(X)E(Y)=0

14. Two random variables X and Y have the following joint probability density function. $f(x,y)=k(4-x-y);0\le x\le 2,\ 0\le y\le 2$. Find constant k, E(X), E(Y), V(X), V(Y) and Cov(X, Y).

Answer:

Since f(x,y) is a pdf, $\iint f(x,y)dx\,dy=1$

$$k\int_0^2\int_0^2(4-x-y)dxdy=1 \Rightarrow k\int_0^2(6-2y)dy=1 \Rightarrow k(8)=1 \Rightarrow k=\frac{1}{8}$$

Before we find Expectation and variance, we find marginal distribution of X and Y.
Marginal distribution of X is given by,

$$f(x)=\frac{1}{8}\int_0^2(4-x-y)dy=\frac{1}{8}(6-2x)=\frac{1}{4}(3-x);0\le x\le 2$$

Marginal distribution of Y is given by,

$$f(y)=\frac{1}{8}\int_0^2(4-x-y)dx=\frac{1}{8}(6-2y)=\frac{1}{4}(3-y);0\le y\le 2$$

Now, let us obtain Expectation.

$$E(x)=\int_0^2 xf(x)dx=\frac{1}{4}\int_0^2 x(3-x)dx=\frac{5}{6}$$

$$E(y)=\int_0^2 yf(y)dx=\frac{1}{4}\int_0^2 y(3-y)dy=\frac{5}{6}$$

To find variance, first we find $E(X^2)$ and $E(Y^2)$

$$E(X^2)=\int_0^2 x^2f(x)dx=\frac{1}{4}\int_0^2 x^2(3-x)dx=1$$

$$E(Y^2)=\int_0^2 y^2f(y)dy=\frac{1}{4}\int_0^2 y^2(3-y)dy=1$$

Variances are given as follows:
$V(X)=E(X^2)-\{E(X)\}^2=1-(^5/_6)^2={}^{11}/_{36}$
$V(Y)=E(Y^2)-\{E(Y)\}^2=1-(^5/_6)^2={}^{11}/_{36}$
We know that Cov(X,Y)=E(XY)-E(X)E(Y)
We have already found E(X) and E(Y). Now let us obtain E(XY).

$$E(XY)=\int_0^2\int_0^2 xyf(,y)dxdy\;=\frac{1}{8}\int_0^2 y\left[\int_0^2 x(4-x-y)dx\right]dy=\frac{1}{8}\int_0^2 y\left(\frac{16}{3}-2y\right)dy=\frac{2}{3}$$

Hence $Cov(X,Y)={}^2/_3-(^5/_6)(^5/_6)=-^1/_{36}$

15. X and Y have a bivariate distribution given by $P(X=x\cap Y=y)=\dfrac{x+3y}{24};(x,y)=(1,1),(1,2),(2,1),(2,2)$. Find E(X), E(Y), V(X), V(Y) and Cov(X,Y).

Answer:

First let us obtain marginal distribution of X and Y.
Marginal distribution of X is given by

$$P(X=x)=\sum_{y=1}^{2}\frac{x+3y}{24}=\frac{x+3.1}{24}+\frac{x+3.2}{24}=\frac{2x+9}{24};x=1,2$$

Marginal distribution of Y is given by,

$$P(Y=y)=\sum_{x=1}^{2}\frac{x+3y}{24}=\frac{1+3y}{24}+\frac{2+3y}{24}=\frac{3+6y}{24};y=1,2$$

Now let us find the expectations.

$$E(X)=\sum_{x=1}^{2}xP(X=x)=\sum_{x=1}^{2}x\frac{2x+9}{24}$$

$$=1\times\frac{2.1+9}{24}+2\times\frac{2.2+9}{24}=1\times\frac{11}{24}+2\times\frac{13}{24}=\frac{37}{24}$$

$$E(Y)=\sum_{y=1}^{2}y(P(Y=y)=\sum_{y=1}^{2}y\frac{3+6y}{24}$$

$$=1\times\frac{3+6.1}{24}+2\times\frac{3+6.2}{24}=1\times\frac{9}{24}+2\times\frac{15}{24}=\frac{39}{24}=\frac{13}{8}$$

Before finding the variance, let us find $E(X^2)$ and $E(Y^2)$

$$E(X^2)=\sum_{x=1}^{2}x^2P(X=x)=\sum_{x=1}^{2}x^2\frac{2x+9}{24}$$

$$=1^2\times\frac{2.1+9}{24}+2^2\times\frac{2.2+9}{24}=1\times\frac{11}{24}+4\times\frac{13}{24}=\frac{63}{24}=\frac{21}{8}$$

$$E(Y^2) = \sum_{y=1}^{2} y^2 P(Y = y) = \sum_{y=1}^{2} y^2 \frac{3 + 6y}{24}$$

$$= 1^2 \times \frac{3 + 6.1}{24} + 2^2 \times \frac{3 + 6.2}{24} = 1 \times \frac{9}{24} + 4 \times \frac{15}{24} = \frac{69}{24} = \frac{23}{8}$$

V(X)= E(X^2)-{E(X)}^{2} =$^{21}/_{8}$-($^{37}/_{24}$)^2=0.2483

V(Y)= E(Y^2)-{E(Y)}^{2} =$^{23}/_{8}$-($^{13}/_{8}$)^2=0.2344

To find cov(X, Y), let us find E(XY)

$$E(XY) = \sum_{x=1}^{2} \sum_{y=1}^{2} xyP(X = x \cap Y = y) = \sum_{x=1}^{2} \sum_{y=1}^{2} xy \frac{x + 3y}{24}$$

$$= \sum_{x=1}^{2} x \left(\sum_{y=1}^{2} y \frac{x + 3y}{24} \right) = \sum_{x=1}^{2} x \left(1. \frac{x + 3.1}{24} + 2 \frac{x + 3.2}{24} \right)$$

$$= \sum_{x=1}^{2} x \left(\frac{3x + 15}{24} \right) = 1. \frac{3.1 + 15}{24} + 2 \frac{3.2 + 15}{24} = \frac{60}{24} = \frac{5}{2}$$

Hence, covariance

Cov(X, Y)=E(XY)-E(X)E(Y)

= $^{5}/_{2} - [^{37}/_{24} . ^{13}/_{8}]$

=-0.0052