Frequently Asked Questions

1. When can a variable be modeled as log normal?

Answer:

A variable might be modeled as log-normal if it can be thought of as the multiplicative product of many independent random variables each of which is positive.

2. Give some examples of variates which have approximately log normal distribution.

Answer:

Examples of variates which have approximately log normal distributions are:

- The size of silver particles in a photographic emulsion,
- The survival time of bacteria in disinfectants
- The weight and blood pressure of humans
- Number of words written in sentences by George Bernard Shaw, etc.

3. Name different fields, where log normal distribution is used.

Answer:

Log normal distribution arises in problems of economics, biology, geology, and reliability theory. In particular it arises in the study of dimensions of particle under pulverization.

4. Define log normal distribution.

Answer:

A positive random variable X is said to have a log normal distribution if $\log_e X$ is normally distributed.

Let $Y = \log_e X \sim N(\mu,\sigma^2)$. And the probability density function is given by,

$$f_X(x) = \begin{cases} \dfrac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\log x-\mu)^2}{2\sigma^2}} & x>0 \\ 0, \; x\le 0 \end{cases}$$

5. Obtain distribution function of log normal distribution.

Answer:

Cumulative distribution function of log normal distribution is given by,

$$F_X(x) = \int_{-\infty}^{x} f_X(x)dx \; = \frac{1}{2} erfc\left[-\frac{\log_e x-\mu}{\sigma\sqrt{2}}\right] = \Phi\left(\frac{\log_e x-\mu}{\sigma}\right)$$

Where erfc is the complementary error function given by $erfc(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ and Φ is the cumulative distribution function of the standard normal distribution.

6. Find the median of log normal distribution.

Answer:

If M is the median of the distribution, then it divides the whole distribution into two equal parts. Or if we consider the cumulative distribution function, $F_X(x)=½$. That is, $\Phi\left(\frac{\log_e x-\mu}{\sigma}\right) = \frac{1}{2}$

Since Φ is standard normal variate, $\Phi(0)= ½$. i.e. $(\log_e x-\mu)/\sigma=0$.

$\Rightarrow \log_e x=\mu$ or $x=e^{\mu}$

$\therefore$Median of log normal distribution is, $M=e^{\mu}$

7. Obtain expressions for mode of log normal distribution.

Answer:

Mode is the value of x for which the pdf attains its maximum. Hence it is a solution of f'(x)=0 and f"(x) <0. pdf of log normal distribution is given by, $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log_e x-\mu)^2}{2\sigma^2}}$

Instead of differentiating the pdf f(x), we can work with logarithm. Hence, $\log_e f(x) = \log_e c - \log_e x - \frac{(\log_e x - \mu)^2}{2\sigma^2}, c = \frac{1}{\sqrt{2\pi}}$

By differentiating with respect to x, we get,

$$\frac{f'(x)}{f(x)} = -\frac{1}{x} - \frac{\log_e x - \mu}{\sigma^2}.\frac{1}{x} \Rightarrow f'(x) = -\frac{f(x)}{x}\left[1 - \frac{\log_e x - \mu}{\sigma^2}\right]$$

$$f'(x) = 0 \Rightarrow 1 - \frac{\log_e x - \mu}{\sigma^2} = 0$$

$$\Rightarrow \log_e x - \mu = \sigma^2 \Rightarrow \log_e x = \mu + \sigma^2 \Rightarrow x = e^{\mu+\sigma^2}$$

Now let us obtain 2nd derivative, f"(x)

$$f''(x) = \frac{d}{dx}\left\{-\frac{f(x)}{x}\left[1 - \frac{\log_e x - \mu}{\sigma^2}\right]\right\}$$

$$= f(x)\left[\frac{1}{x^2} - \frac{1}{\sigma^2}\left\{\frac{1-(\log_e x - \mu)}{x^2}\right\}\right] - \left[\frac{1}{x} - \frac{\log_e x - \mu}{x\sigma^2}\right].f'(x)$$

Observe that the above function is less than zero at $x = e^{\mu+\sigma^2}$

Hence, mode of the log normal distribution is given by, $x = e^{\mu+\sigma^2}$

8. Obtain an expression for rth raw moment of log normal distribution.

Answer:

If Y ~N(μ, σ^2), then $X=e^Y$ is called a log normal random variable. rth raw moment is given by, $\mu_r' = E(X^r) = E[(e^Y)^r] = E[e^{Yr}] = M_Y(r)$, that is mgf of Y, the normal distribution with parameters μ, σ^2.

We know that mgf is given by, $M_Y(r) = e^{r\mu + r^2\sigma^2/2}$

Hence, rth raw moment is given by, $\mu_r' = e^{r\mu + r^2\sigma^2/2}$

9. Obtain an expression for mean of log normal distribution using the expression of rth raw moment.

Answer:

We know that the expression for rth raw moment is, $\mu_r' = e^{r\mu + r^2\sigma^2/2}$

By putting r=1 we get mean of the log normal distribution as $\mu_1' = e^{\mu + \sigma^2/2}$

10. Find the variance of the log normal distribution.

Answer:

We know that the expression for rth raw moment is, $\mu_r' = e^{r\mu + r^2\sigma^2/2}$

By putting r=1 we get mean of the log normal distribution as $\mu_1' = e^{\mu + \sigma^2/2}$

By putting r=2, we get,

Variance of the distribution is given by,

$\mu_2 = \mu_2' - \mu_1'^2 = e^{2\mu + 2\sigma^2} - [e^{\mu+\sigma^2/2}]^2 = e^{2\mu+\sigma^2}[e^{\sigma^2} - 1]$

11. Find skewness of log normal distribution.
Answer:
Coefficient of skewness is given by,

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{\left[e^{3\mu+3\sigma^2/2}(e^{3\sigma^2} - 3e^{\sigma^2} + 2)\right]^2}{\left[e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)\right]^3} = \frac{\left[e^{3\sigma^2} - 3e^{\sigma^2} + 2\right]^2}{\left[e^{\sigma^2} - 1\right]^3} > 0$$

Hence, log normal distribution is positively skewed.

12. What is the kurtosis of log normal distribution?
Answer:
Coefficient of kurtosis is given by,

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{e^{4\mu+2\sigma^2}[e^{6\sigma^2} - 4e^{3\sigma^2} + 6e^{\sigma^2} - 3]}{\left[e^{2\mu+\sigma^2}[e^{\sigma^2} - 1]\right]^2}$$

$$= \frac{[e^{6\sigma^2} - 4e^{3\sigma^2} + 6e^{\sigma^2} - 3]}{\left[e^{\sigma^2} - 1\right]^2} = e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 3 > 3$$

Hence, log normal distribution has leptokurtic curve.

13. Write the expressions for obtaining the μ and σ^2 when we have mean and variance of log normal distribution.
Answer:
Given the mean and variance of log normal distribution, we can find μ and σ^2 as follows.

$\mu_1' = e^{\mu+\sigma^2/2}$ and $\mu_2 = e^{2\mu+\sigma^2}[e^{\sigma^2} - 1]$

Taking logarithms and simplifying the equations, we get,

$\mu = \log_e \mu_1' - \frac{1}{2}\log_e\left(1 + \frac{\mu_2}{(\mu_1')^2}\right)$ and $\sigma^2 = \log_e\left(1 + \frac{\mu_2}{(\mu_1')^2}\right)$

14. Obtain an expression for coefficient of variation for log normal distribution.
Answer:
We know that variance of the distribution is,

$\mu_2 = e^{2\mu+\sigma^2}[e^{\sigma^2} - 1] S.D. = e^{\mu+\sigma^2/2}[e^{\sigma^2} - 1]^{\frac{1}{2}}$

Also mean is given by $\mu_1' = e^{\mu+\sigma^2/2}$

Hence coefficient of variation is given by,

$$C.V. = \frac{S.D}{Mean} = \frac{e^{\mu+\sigma^2/2}[e^{\sigma^2} - 1]^{\frac{1}{2}}}{e^{\mu+\sigma^2/2}} = [e^{\sigma^2} - 1]^{\frac{1}{2}}$$

15. Mention the relation between Log Normal and Pareto distributions.
Answer:
The Pareto distribution and log-normal distribution are alternative distributions for describing the same types of quantities. One of the connections between the two is that they are both the distributions of the exponential of random variables distributed according to other common distributions, respectively the exponential distribution and normal distribution.